MATHEMATICS OF COMPUTATION
VOLUME 36, NUMBER 154
APRIL 1981

A Population of Linear, Second Order, Elliptic Partial Differential Equations on Rectangular Domains. Part I*

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Abstract. We present a population of 56 linear, two-dimensional elliptic partial differential equations (PDEs) suitable for evaluating numerical methods and software. Forty-two of the PDEs are parametrized which allows much larger populations to be made; 189 specific cases are presented here along with solutions (some are only approximate). Many of the PDEs are artificially created so as to exhibit various mathematical behaviors of interest; the others are taken from "real world" problems in various ways. The population has been structured by introducing measures of complexity of the operator, boundary conditions, solution and problem. The PDEs are first presented in mathematical terms along with contour plots of the 189 specific solutions. Machine-readable descriptions are given in Part 2; many of the PDEs involve lengthy expressions and about a dozen involve extensive tabulations of approximate solutions.

CONTENTS

1. Introduction. The motivation for creating this PDE population is for use in the evaluation of numerical methods and PDE software. The need and rationale for a systematic approach to such evaluations is given in Rice [10], Houstis and Rice [8], Crowder, Dembo and Mulvey [2]; it suffices here to say that a properly chosen problem population is an essential ingredient for a sound evaluation of numerical methods and software.

A useful population of PDEs is inevitably very lengthy and this one is no exception as one sees from the last two appendices. Thus in the body of this paper we discuss the sources of the PDEs, how they are described in the appendices and

Received July 29, 1980.
1980 *Mathematics Subject Classification.* Primary 65N99.
Key words and phrases. Elliptic partial differential equations, numerical methods, software evaluation, population of problems, linear, second order.
*Sponsored by the United States Army under contract Nos. DAAG29-75-C-0024 and DAAG29-80-C-0041. This material is based upon work supported by the National Science Foundation under Grant Nos. MCS77-01408 and MCS79-01437.

0025-5718/81/0000-0057/$03.50

how a structure has been created in the population through the use of quantitative (but subjective) measures of features.

It is important that one be able to create relevant subpopulations as one inevitably wants to evaluate methods for particular subclasses of PDEs (e.g., separable, with singularities or with mixed boundary conditions). Experience shows that no one universal method is best for all PDEs (even in this rather restricted context) and one of the important tasks of research is to create and/or identify methods that are especially efficient for particular classes of PDEs. Once one embarks on such a task one sees that this population, which originally might seem large and bulky, is actually rather small for the uses to be made of it. It is only the fact that it can be substantially expanded in various directions through the parametrization that gives one hope that it is adequate for a wide variety of evaluations.

2. Characteristics of the Problems. A source parameter is assigned to each PDE which ranges from 0 (artificial problem) to 100 (actual real world problem). This feature, as the others introduced later, is subjective in nature and the values given must be taken as approximate indications of our intuitive feelings. The PDE $u_{xx} + u_{yy} = 1$ might be completely artificial for one person and be the actual applications PDE for another. We have at least tried to be consistent in these values.

2.1. *Sources.* Many problems have been normalized so the maximum value of the solution is 1.0 and almost all have this value between .1 and 100. Many of the domains have been standardized to the unit square, $0 \leqslant x, y \leqslant 1$. The sources of the PDEs are:

A. *Problems used in previous studies.* Nine problems are included which were used by Eisenstat and Schultz [3] or Houstis et al. [4] and [5]. Subsets of this populaton have been used by Houstis and Papatheodorou [6] and [7] and Lynch and Rice [9]. Some of these PDEs have had parameters added and all have been normalized so the maximum value of the solution is about 1.0.

B. *Artificial problems.* Many problems have been created just to exhibit various mathematical behaviors of interest (e.g. singularities, oscillations or wave fronts). Such behaviors are important for theory or application (or both) and one needs to have them present in the population in an easily identifiable manner.

C. *Problems adapted from the "real world".* A persistent difficulty is the desire to have PDEs which represent the "real world" and the necessity to know their true solutions. Among the strategies to adapt real world problems we have used:

(i) choosing explicit functions which model the physical solutions and then determining appropriate boundary conditions and/or right side to make this the true solution;

(ii) using truncated series expansions (of high accuracy) with appropriate small modifications in the boundary conditions or right side;

(iii) solving nonlinear problems approximately, then substituting the tabulated numerical solution into the operator (using quadratic interpolation from a 10 by 10 grid) to obtain a linear problem which is, in turn, solved approximately. In these cases the true solution is not known, but the machine-readable population contains tabulated values of a hopefully accurate numerical solution.

2.2. *Problem Features and Complexity Classifications.* We identify as problem features the *smoothness* and *local variation* of operator, the boundary conditions and the solution. These features are quantified on a one-dimensional scale of 0 to 100 even though there are rather independent properties that can be called smoothness or local variation. These features are measured subjectively from the following descriptions of the scale.

Smoothness. This refers to the mathematical properties of the functions or operators involved. Key points on the scale are

00 = entire functions or constants,

10 = analytic; very well behaved,

30 = very smooth, some higher derivative (5 or so) discontinuity possible,

50 = still smooth, third derivative discontinuity possible,

70 = not rough to the eye, but possibly only 1 continuous derivative,

80 = continuous, functions might be theoretically smooth but rough on a gross scale,

90 = possibly discontinuous, nearly singular functions or operators,

100 = strong singularities like $1/x$ or $1/x^2$.

Local variation. This refers to how much a function changes (relative to its size) in a small part of its domain. These variations might be oscillations, wave fronts, peaks or boundary layers. Key points on the scale are

00 = very smooth, uniform,

10 = mild variation, probably convex, some nonuniformity, e.g. $\sin(2x)$, e^{3x} on [0, 1],

25 = modest variation of oscillation; mild wave front or peak, e.g. $\sin(6x)$, $1/(1 + 100x^4)$ on [0, 1],

40 = considerable peak or oscillation; change of magnitude occurs within 10–15% of domain,

60 = sharp peaks, wave fronts, boundary layers or oscillations; 100% change in magnitude occurs within 5% of domain,

75 = practically a discontinuity in magnitude; continuity observable only with a fine scale examination,

90 = actual discontinuity in magnitude; extreme oscillation, step functions, e.g. $\sin(300x)$ on [0, 1].

The overall problem complexity is represented by the average of the above six feature measures. The problems in this population do not have complexities exceeding 58 (only one exceeds 50), a level which might be interpreted as "rather messy with one or two substantial complications". The problem feature measures are included in the descriptions along with the source parameter.

Appendix 1 presents some summary information about the population. Tables are given which

A. group the PDEs according to types of the operator and boundary conditions (e.g. Helmholtz and Dirichlet or constant coefficients and mixed),

B. list the 56 PDEs with abbreviated feature descriptions,

C. group the PDEs according to the smoothness of the operator and right side,

D. group the PDEs according to the smoothness of the solution.

3. Format of Problem Descriptions. Appendix 2 contains a mathematical description of each PDE along with contour plots for each specific instance included in the set of 189 PDEs. An example is shown in Figure 1. The description begins with a problem number and source followed by a mathematical description of the PDE. Then brief comments are given for the operator, right side, boundary conditions, solution and parameters (if any). Sometimes functions appearing in the mathematical description are defined in these comments.

Four generic functions are used:

$f(x, y)$ = right side of PDE determined so that the given true solution is correct.

$f(x)$, $g(y)$ = right sides of boundary conditions determined so that the given true solution is correct.

$T(x, y)$ = the true solution, used in the coefficients of some PDEs derived from nonlinear problems.

$r(x, y)$ = an approximate solution used in some PDEs whose true solution is unknown.

Contour plots are given for one or more particular PDEs for each problem. The border of the plots contains the following information:

(i) values of the parameters, the variables A, B, C, etc. denote α, β, γ, etc.

(ii) maximum and minimum values of the solution; the contours are equispaced between these values.

(iii) the classification parameters in the form

S.P O1.O2 B1.B2 S1.S2

where

S = source parameter P = problem complexity
$\alpha 1$ = smoothness feature $\alpha 2$ = local variation feature
and α = O for the operator, B for the boundary conditions,
S for the solution.

The machine-readable description of the PDE population consists of two files: EQNFIL and MACFIL. EQNFIL has 189 entries which are either complete statements of the PDE in the ELLPACK language (see Boisvert, Houstis and Rice [1]) or a reference to an entry in MACFIL with values given for parameters. See Figure 2 for a short example. The information given starts with the problem number, feature parameter values and a code for various attributes of the PDE which are used within the ELLPACK system. The machine-readable description uses A, B, C, etc. for α, β, γ, etc. Then ELLPACK language code is given for the operator and boundary conditions; this code should be self explanatory once one sees the UXX$ represents u_{xx}, etc. Finally, there is a Fortran code for any functions that appear in the operator, right side or boundary conditions. This latter code averages about 20 lines and can be as much as 150 lines (excluding tables that are part of some problems). These descriptions are given in Part 2 of this report.

MACFIL entries are just like EQNFIL descriptions of a PDE except that the places where parameter values are to be substituted are indicated by &A, &B, etc. A refers to the first parameter, B the second and so on. There are somewhat more than 8500 lines in these two files.

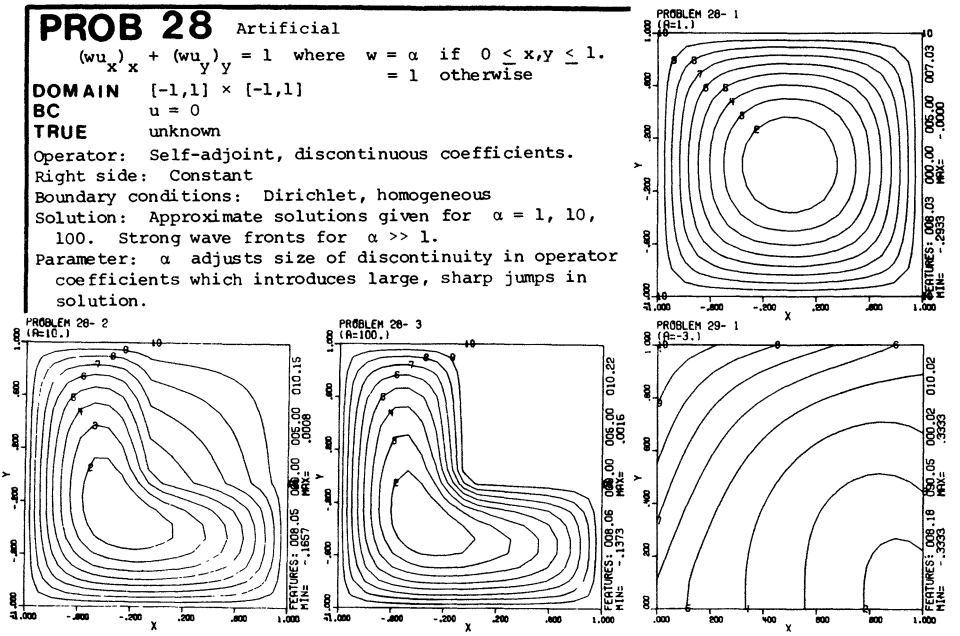

FIGURE 1

An example of the mathematical description of a PDE along with some contours

```
'EOR
'EOF
***************
* PROBLEM   2 *
***************
'EOR
*
*         000.04    000.00    004.05    010.02
*         2000200000020
1         TWO DIMENSIONS
1         UXX$ + (1.+Y*Y)UYY$ - UX$ - (1.+Y*Y)UY$ = F(X,Y)
2         MIXED
2         X=0. , MIXED = (1.)U + ( 1.)UX  =  0.27*EXP(Y)
2         X=1. , MIXED = (1.)U + (-1.)UX  =  0.
2         Y=0. , MIXED = (1.)U + ( 1.)UY  =  0.27*EXP(X)
2         Y=1. , MIXED = (1.)U + (-1.)UY  =  0.135*(ALOG(2.)-1.)*(X*X-X)**2
3     FUNCTION TRUE(X,Y)
3     TRUE = 0.135*(EXP(X+Y)+(X*X-X)**2*ALOG(1.+Y*Y))
3     RETURN
3     END
3     FUNCTION F(X,Y)
3     F = 0.135*( (-4.*X*X*X+18.*X*X-14.*X+2.)*ALOG(1.+Y*Y)
3    $    - 2.*((X*X-X)**2)*(Y*Y+Y**3+Y-1.)/(1.+Y*Y) )
3     RETURN
3     END
'EOR
'EOF
***************
* PROBLEM   3 *
***************
'EOR
*PARAMETER SET   1(A=1.5)
*         000.43    090.60    000.00    070.40
EXPAND 3/1.5/
'EOR
*PARAMETER SET   2(A=2.5)
*         000.35    080.50    000.00    060.20
EXPAND 3/2.5/
'EOR
*PARAMETER SET   3(A=3.5)
*         000.28    070.30    000.00    050.15
EXPAND 3/3.5/
'EOR
*PARAMETER SET   4(A=4.5)
*         000.23    055.20    000.00    040.20
EXPAND 3/4.5/
```

FIGURE 2

A sample from EQNFIL *showing a short PDE description in machine-readable form and a reference to a similar description in* MACFIL

Acknowledgements. This work has been supported in part as follows: National Science Foundation grant MCS77-01408 (Rice, Houstis and Dyksen), United States Army Contract DAAG29-75-C-0024 (Rice) and National Science Foundation grant MCS79-01437 (Houstis).

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Appendix 1: Tabulations of Population Characteristics

Table 1

Classifications of Problems According to Operator and Boundary Conditions

Operator	Constant Coefficients			Non-Constant Coefficients		
	Dirichlet	Neumann	Mixed	Dirichlet	Neumann	Mixed
Laplace	3, 4, 7, 8, 10, 11, 17, 33, 34, 35, 47, 50		4, 31, 35, 38, 55			
Helmholtz Type	9, 41, 53			6, 20, 39, 44, 45, 48, 49		
Self-Adjoint	5			1, 13, 22, 25, 28, 54		1, 19, 23, 52
General	14, 46	42	43	12, 15, 16, 18, 21, 26, 27, 29, 30, 32, 36, 37, 56	24	2, 23, 24, 40, 51

Note that problems 1, 4, and 35 appear in two places in the table since they have boundary conditions of the form $u + \alpha u_N = g$ and hence have Dirichlet boundary conditions for $\alpha = 0$. Problem 24 appears in two places since it has boundary conditions of the form $\beta u + u_x + u_y = 0$ and is Neumann for $\beta = 0$.

Table 2

Problem Characteristics

The principal characteristics are tabulated below using the following encodings:

A	Analytic	N	Neumann Boundary Condition
BL	Boundary Layer	NS	Nearly Singular
C	Constant (coefficients)	O	Oscillatory
CC	Computationally Complex	P	Parameterized or Peaked
D	Dirichlet Boundary Condition	S	Singular (infinite)
E	Entire	SD	Singular Derivative
H	Homogeneous	U	Unknown
J	Jump Discontinuity	VS	Variable Smoothness
M	Mixed Boundary Condition	WF	Wave Front

Problem Number	Operator	Right Side	Solution	Boundary Conditions	Domain
1P	A	E	E	M	Unit Square
2	E	A	A	M,H	Unit Square
3P	C	S,SD	S,SD	D,H	Unit Square
4P	C	E	E	M	Unit Square
5P	C	E	E	D,H	Unit Square
6	E,NS	A	A,O	D,H	Unit Square
7	C	C	SD	D,H	Unit Square
8P	C	SD	SD,WF	D	Unit Square
9P	C,NS	E,NS	E,BL	D	Unit Square
10P	C	E,P	E,P	D,H	Unit Square
11P	C	A,O	A,O	D	Unit Square
12P	E,O	E,O	E,O	D	Unit Square
13	J	S	SD	D	Unit Square
14P	C	S	S	D	Unit Square
15P	A,NS	S	SD	D	Unit Square
16P	A,NS	C	U,BL	D,H	Variable Square
17P	C	A,NS	A,NS,WF	D	Unit Square
18P	E	A,NS	A,NS,WF	D	Unit Square
19P	S	S	E	M,H	Square
20P	NS,P,CC	P	E,P	D	Rectangle
21	E	E	E	D	Unit Square
22	SD	S	E	D	Unit Square
23P	SD	SD	SD,WF	M,H	Unit Square
24P	S,NS	S,NS	U,P	M,H	Square
25P	SD	S	E	D,H	Unit Square

Table 2

Problem Characteristics

Problem Number	Operator	Right Side	Solution	Boundary Conditions	Domain
26P	A	A	U,SD	D,H	Variable Square
27	A,NS	C	U,BL	D,H	Square
28P	J	C	U,WF	D,H	Square
29P	S	H	U,VS,BL	D	Unit Square
30P	A,CC	A,CC	A,NS	D	Unit Square
31	C	C	E,(SD)	M	Square
32	A	A	E	D,H	Rectangle
33	C	E	E,O	D	Rectangle
34	C	C	E,(SD)	D	Square
35P	C	H	E,O,BL	M	Square
36P	S	S	A,BL	D	Unit Square
37	E	E	E	D	Unit Square
38P	C	H	E,O,VS	D	Rectangle
39P	CC,S	CC,S	U,BL	D,C	Unit Square
40P	E	A	A	M	Unit Square
41P	C,NS	SD,NS	SD	D,H	Square
42P	C	H	A,O	N	Variable Rectangle
43	C	H	E	M	Square
44P	CC	CC	U,BL	D,H	Unit Square
45P	C,NS	H	U,BL	D	Unit Square
46P	C,NS	H	U,BL	D	Variable Rectangle
47P	C	S	SD,VS	D	Unit Square
48P	CC	CC	U	U	Unit Square
49P	CC	CC	U,SD,BL	D,C	Unit Square
50	C	H	E,O	D	Rectangle
51P	S	C	U,SD,WF	M,H	Unit Square
52P	CC	H	U,O	M,C	Unit Square
53P	C,NS	E,O	E,O	D	Unit Square
54P	E,CC	S,CC	SD,VS	D	Unit Square
55P	C	H	S,VS,BL	M	Rectangle
56P	S	CC	U,O,(SD)	M	Rectangle

Table 3

Classifications of Problems According to Smoothness of the Operator and Right-Side

(A=Analytic; C=Constants; CC=Computationally Complicated; DD=Discontinuous Derivatives; E=Entire; O=Oscillatory; P=Peak; S=Singular)

Smoothness Operator	Smoothness Right-Side	Problem Numbers
C	C	7, 31, 34, 35, 38, 42, 43, 45, 46, 50, 55
C	E	4, 5, 9, 10, 33, 53
C	A	11, 17
C	DD	3, 8, 41
C	S	3, 14, 47
C	O	6, 11, 53
C	P	10
E	E	12, 21, 37
E	A	2, 6, 18, 40
E	S	54
A	C	16, 27
A	E	1
A	A	26, 30, 32
A	S	15
DD	C	28
DD	DD	13, 23, 25
DD	S	22, 25
S	C	29, 51, 56
S	S	19, 24, 36
O	A	6
O	O	12
C	CC	17, 18
S	CC	56
CC	C	52
CC	P	20
CC	CC	30, 39, 44, 48, 49, 54

Table 4

Classifications of Problems According to Smoothness of the Solution

Solution Smoothness	Problem Numbers
Entire	1, 4, 5, 9, 10, 12, 19, 20, 21, 22, 25, 31, 32, 33, 34, 35, 37, 38, 43, 50, 53
Analytic	2, 6, 11, 17, 18, 30, 36, 40, 42
Singular Derivatives	3, 7, 13, 14, 15, 41, 47, 51, 54, 56
Oscillatory	6, 11, 12, 33, 35, 38, 42, 50, 53, 56
Wave Front	8, 17, 18, 23, 28, 51
Discontinuous Derivatives	8, 23
Singular	54, 55
Boundary Layer	7, 9, 15, 16, 27, 29, 44, 45, 46, 49
Peak	10, 20, 24
Tabled Solution	16, 24, 26, 27, 28, 29, 39, 44, 45, 46, 48, 49, 51, 52

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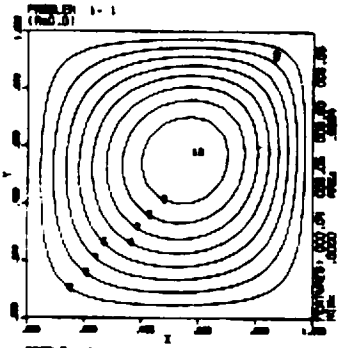
APPENDIX TWO: MATHEMATICAL DESCRIPTIONS AND SOLUTION CONTOURS

PROB 1 Artificial [7,12,13]

$$(e^{xy}u_x)_x + (e^{-xy}u_y)_y - u/(1 + x + y) = f$$

DOMAIN unit square
BC $u + \alpha u_N = g$
TRUE $.75e^{xy}\sin(\pi x)\sin(\pi y)$

Operator: Self-adjoint, analytic
Right side: Entire
Boundary conditions: Mixed except for $\alpha = 0$.
Solution: Entire, independent of α.
Parameter: α introduces normal derivative into boundary conditions. Problems 1-2 to 1-4 have $\alpha = 0.1$, 1.0 and 10.

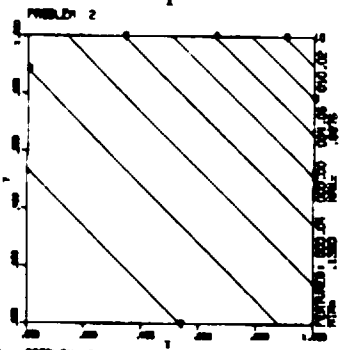


PROB 2 Artificial [12,13]

$$u_{xx} + (1 + y^2)u_{yy} - u_x - (1 + y^2)u_y = f$$

DOMAIN unit square
BC $u - u_N = g$
TRUE $.135(e^{x+y} + (x^2 - x)^2\log(1 + y^2))$

Operator: Entire
Right side: Analytic
Boundary conditions: Mixed
Solution: Analytic
Parameter: None

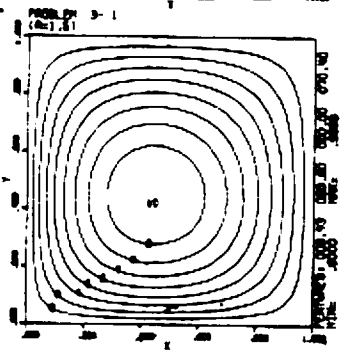


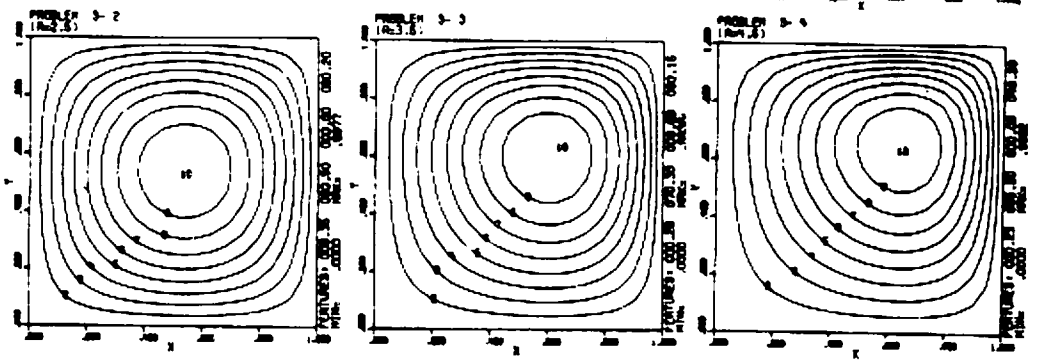
PROB 3 Artificial [13]

$$u_{xx} + u_{yy} = f$$

DOMAIN unit square
BC $u = 0$
TRUE $c(x^\alpha - x)(y^\alpha - y)$, $c = 1/(\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)})^2$

Operator: Laplace
Right side: singular for $\alpha < 3$
Boundary condition: Dirichlet, homogeneous
Solution: Singularity of variable strength
Parameter: α adjusts singularity strength



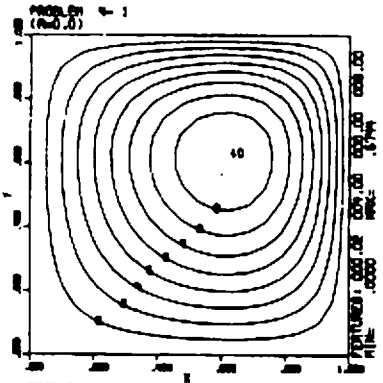


PROB 4 Artificial [7,12,13]

$$u_{xx} + u_{yy} = 6xy\, e^{x+y}(xy + x + y - 3)$$

DOMAIN unit square
BC $u = 0$ for $x \neq 0$; $u - \alpha(y - y^2)u_x = g$ for $x = 0$
TRUE $3e^{x+y}(x - x^2)(y - y^2)$

Operator: Laplace
Right side: Entire
Boundary conditions: Mixed except for $\alpha = 0$
Solution: Entire, independent of α
Parameter: α introduces normal derivative into boundary conditions. Problems 4-2 to 4-4 have $\alpha = 0.1$, 1.0 and 10.

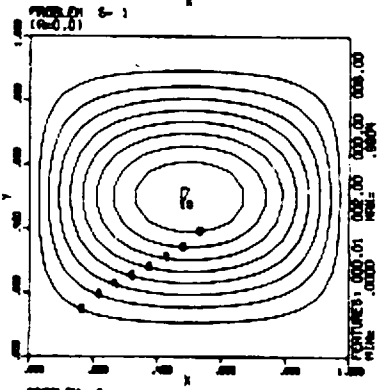


PROB 5 Artificial [13,14]

$$4u_{xx} + u_{yy} - \alpha u = f$$

DOMAIN unit square
BC $u = 0$
TRUE $2(x^2 - x)(\cos(2\pi y) - 1)$

Operator: Constant coefficient, separable
Right side: Entire
Boundary conditions: Dirichlet, homogeneous
Solution: Entire, independent of α
Parameter: α makes operator more singular without affecting solution. Problems 5-2 to 5-7 have $\alpha = 5, 8, 10, 20, 64$ and 100.



PROB 6 Stratospheric physics [13,14,16]

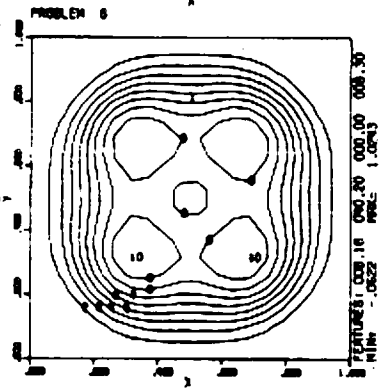
$$u_{xx} + u_{yy} - (100 + \cos(2\pi x) + \sin(3\pi y))u = f$$

DOMAIN unit square
BC $u = 0$
TRUE

$$-0.31(5.4 - \cos(4\pi x))\sin(\pi x)(y^2 - y)(5.4 - \cos(4\pi y))(1/(1+\varphi^4) - .5)$$

$$\varphi = 4(x - .5)^2 + 4(y - .5)^2$$

Operator: Entire, oscillatory, somewhat singular
Right side: Analytic
Boundary conditions: Dirichlet, homogeneous
Solution: Analytic, oscillatory
Parameter: None

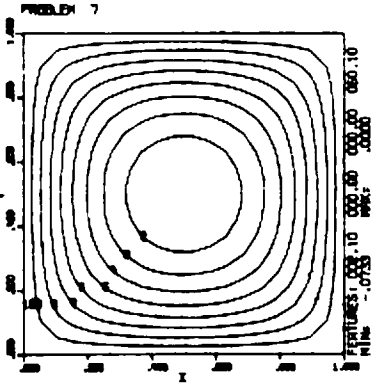


PROB 7 Artificial [6]

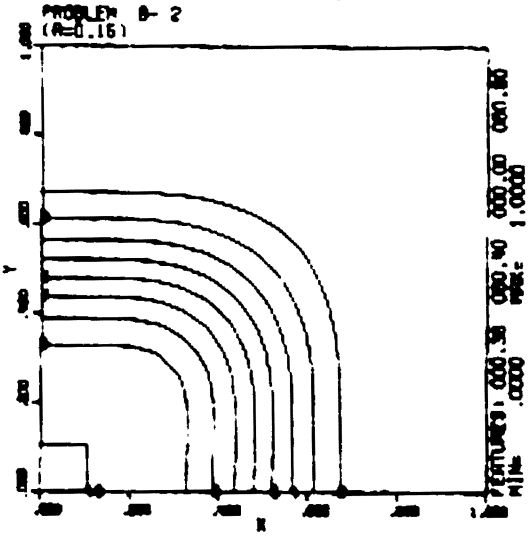
$$u_{xx} + u_{yy} = 1$$

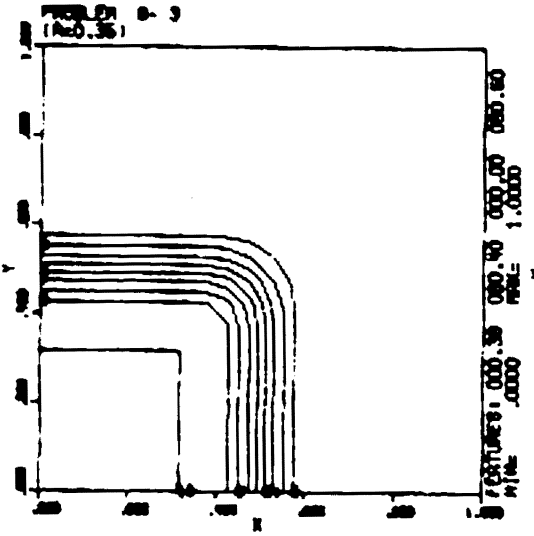
DOMAIN unit square
BC $u = 0$
TRUE Approximate series solution gives 10^{-9} accuracy.

Operator: Laplace
Right side: Constant
Boundary condition: Dirichlet, homogeneous
Solution: Has logarithmic singularities at corners in second derivatives; approximate solution is a polynomial.
Parameter: None



Boundary conditions: Dirichlet
Solution: Wave front along a right angle joining two regions where it is constant.
Parameter: α adjusts width and sharpness of wave front.



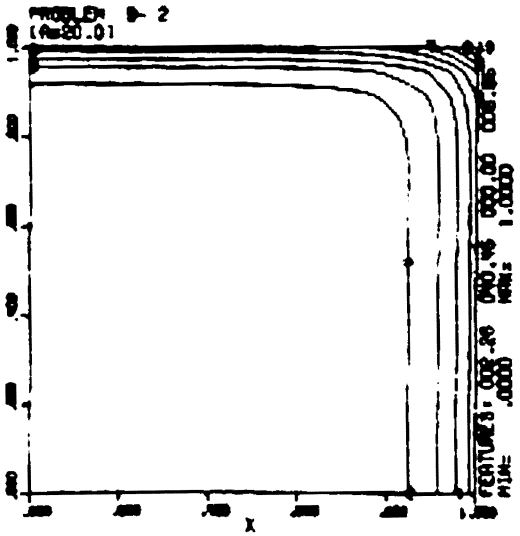


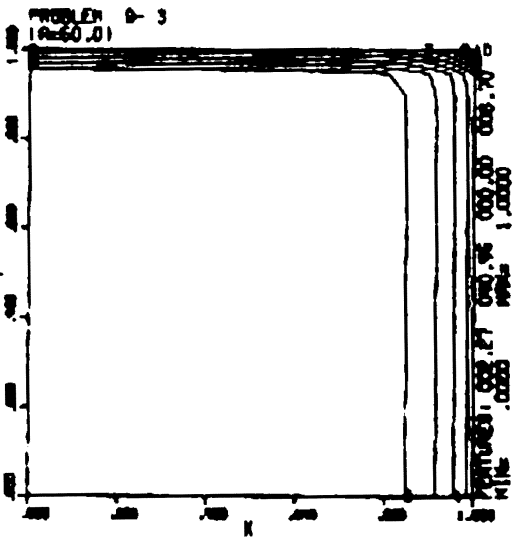
PROB 9 Artificial [13]

$$u_{xx} + u_{yy} - 100u = .5(\alpha^2 - 100)\cosh(\alpha y)/\cosh\alpha$$

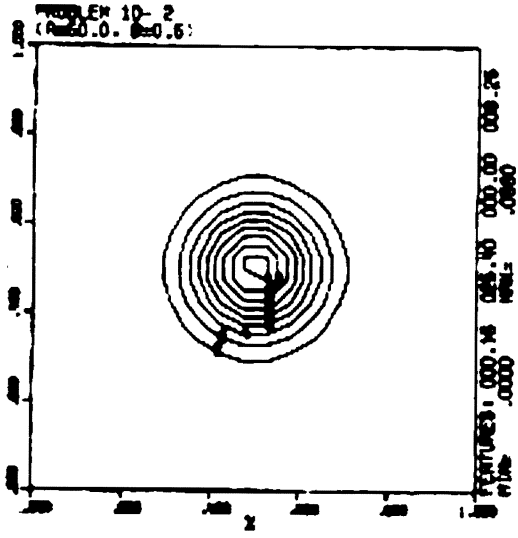
DOMAIN unit square
BC $u = g$
TRUE $.5(\cosh 10x/\cosh 10 + \cosh \alpha y/\cosh \alpha)$

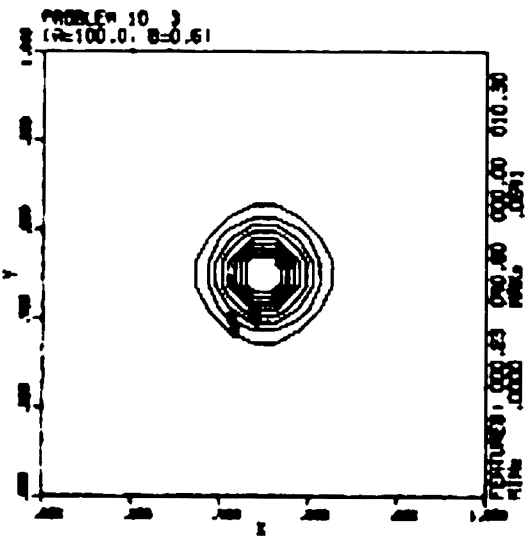
Operator: Helmhotz, constant coefficients, somewhat singular.
Right side: Entire but nearly singular for $\alpha \neq 10$.
Boundary conditions: Dirichlet
Solution: Boundary layer, nearly singular.
Parameter: α adjusts strength of y-side boundary layer.

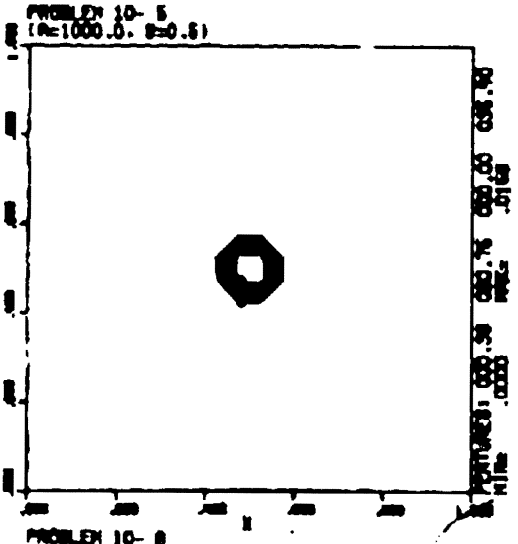


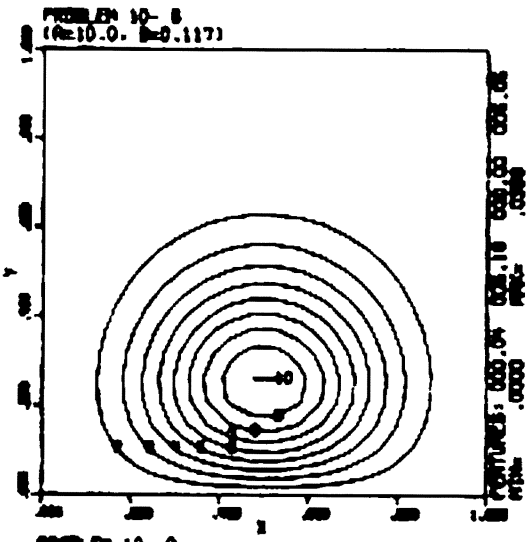


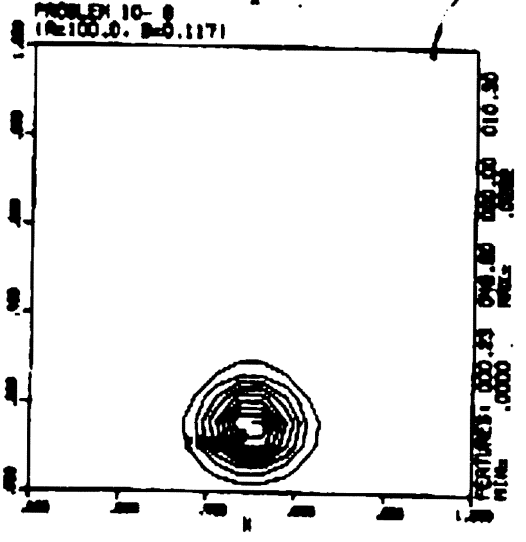
Solution: Strongly peaked for large α.
Parameters: α adjusts strength of the peak, β moves it in the y-direction.

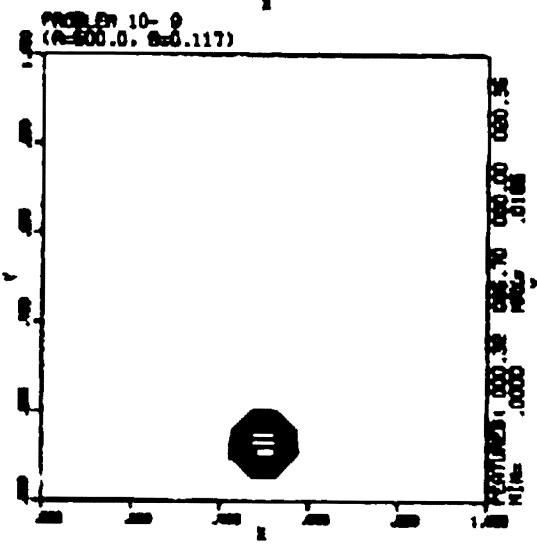




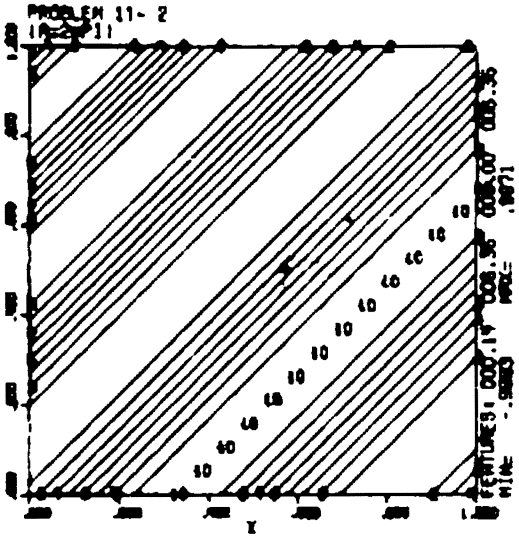


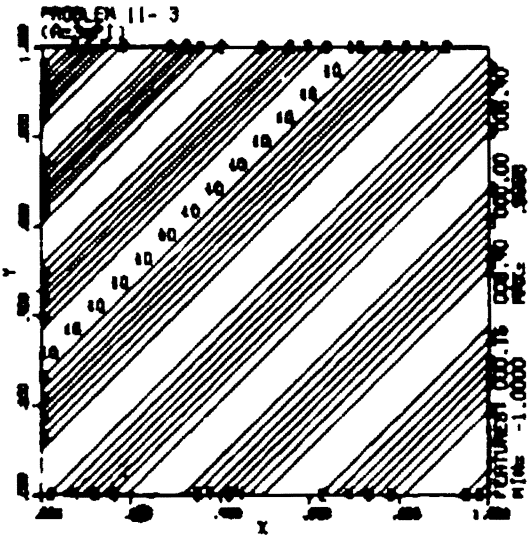






Solution: Oscillatory
Parameter: α adjusts frequency of oscillations



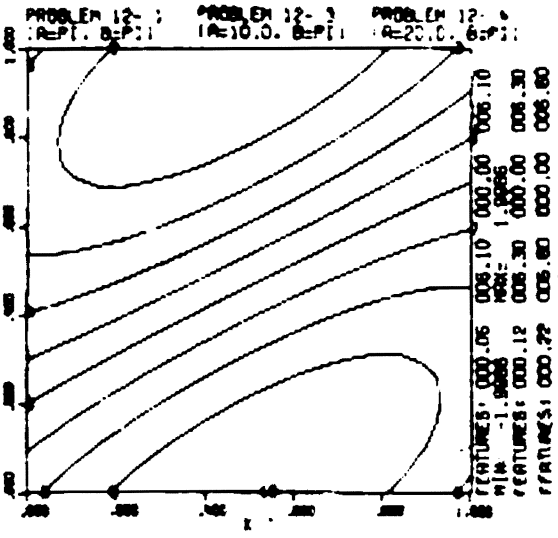


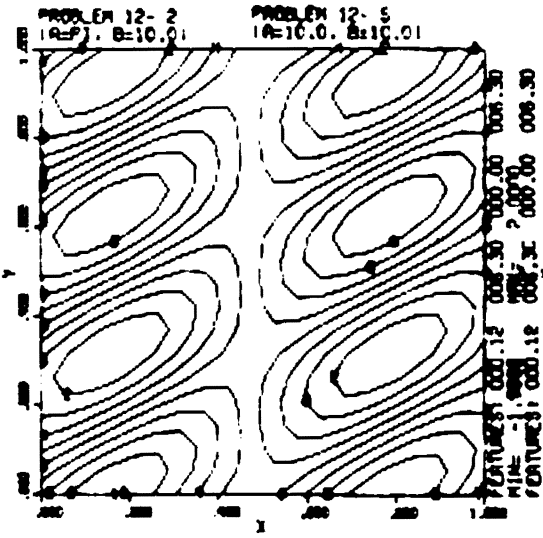
PROB 12 Artificial

$$u_{xx} + u_{yy} + (1 + \sin(\alpha x))u_x - \cos(\alpha y)u = f$$

DOMAIN unit square
BC $u = g$
TRUE $\cos(\beta y) + \sin \beta(x - y)$

Operator: Oscillatory, Laplacian plus lower terms
Right side: Oscillatory, analytic
Boundary conditions: Dirichlet
Solution: Oscillatory, entire
Parameter: α adjusts oscillation in PDE coefficients.
β adjust oscillation in the solution.





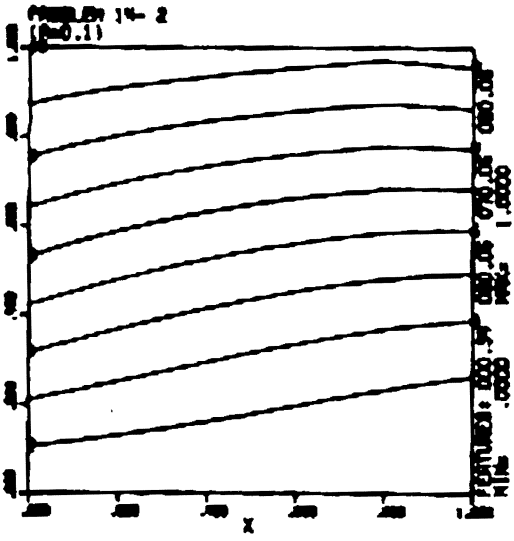
Solution: Derivative in x is singular.
Parameter: None

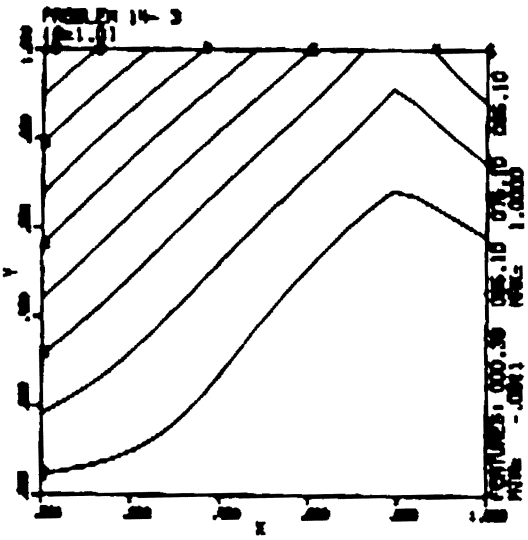
PROB 14 Artificial

$$u_{xx} + 2u_{yy} + 3u_x - 4u_y - u = f$$

DOMAIN unit square
BC u=0, y=0; u=y, x=0; u=g, x=1; u=1-.8α+α|x-.8| for y=1.
TRUE $y(1 - .8\alpha^{2-y} + \alpha|x - .8|^{2-y}) + xye^{-xy}(y - 1)$

Operator: Constant coefficients
Right side: Line of singularities at x = .8
Boundary conditions: Dirichlet, discontinuous derivative
Solution: Line of singularities of variable strength along x = .8.
Parameter: α adjusts strength of the singularity





PROB 15 Artificial

$$u_{xx} + u_{yy} + \alpha/(y + \gamma)u_y = f$$

DOMAIN unit square
BC u = g
TRUE $[y^{\beta} + \cos(xy^2) - 1]x^2(x - 1)^2$

Operator: Laplace plus nearly singular derivative term
Right side: Singularity in β - 1 y-derivative, nearly singular for small γ.
Boundary conditions: Dirichlet
Solution: Boundary layer at y = 0, derivative singular.
Parameters: α, γ adjust operator singularity, β adjusts unrelated derivative singularity in solution.

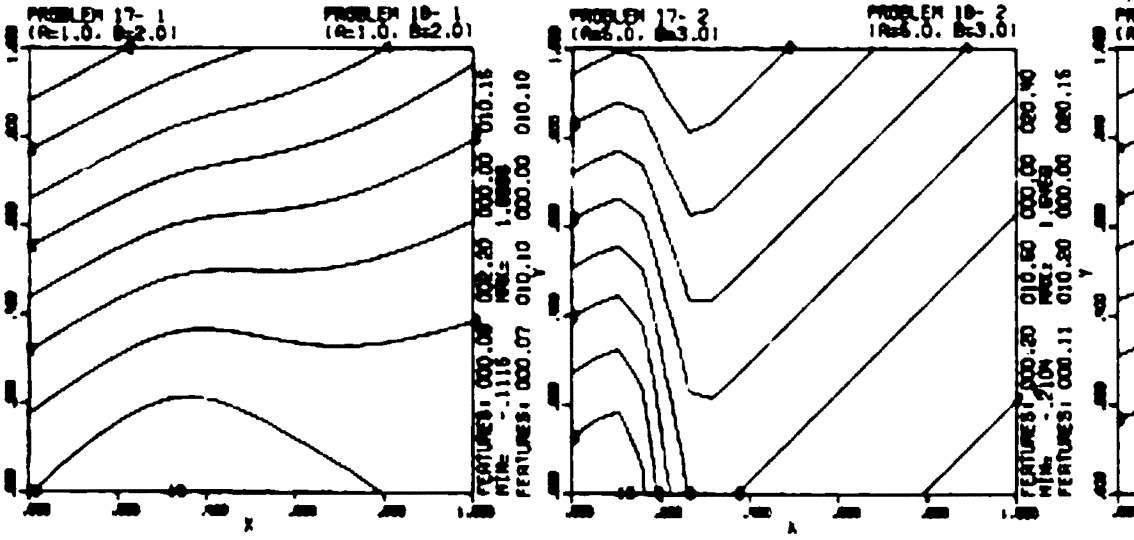
Solution: Approximate solutions given for $\beta = 1, 10$.
Parameter: β adjusts the size of the domain and right side.

PROB 17 Artificial

$$u_{xx} + u_{yy} = f$$

DOMAIN unit square
BC $u = g$
TRUE $e^{[y^2+(\alpha(\beta x)^3/(1+(\beta x)^3))^2]} + \sin(x - y + .5)$

Operator: Laplace
Right side: Large values for x near .15
Boundary conditions: Dirichlet
Solution: Sharp wave front near $x = .15$, entire.
Parameters: α, β adjust the strength and shape of the wave front.



PROB 18 Artificial

$$u_{xx} + (1 + xy)u_{yy} + \cos(x)u_x - e^{-x}u_y + 3u = f$$

DOMAIN unit square
BC $u = g$
TRUE $e^{-[y^2+(\alpha(\beta x)^3/(1+(\beta x)^3))^2]} + \sin(x - y + .5)$

Operator: Entire
Right side: Large values for x near .15
Boundary conditions: Dirichlet
Solution: Sharp wave front near $x = .15$, entire.
Solution is the same as in the preceding problem.
Parameters: α, β adjust the strength and shape of the wave front.

problem.
Parameter: $-.5 < \alpha < 1$ is a physical parameter.
Problem 19-2 has $\alpha = 0.25$

PROB 20

From $u_{xx} + u_{yy} = e^{u_T}$ [1]
$u_{xx} + u_{yy} - wu = f$, $w = e^T$

DOMAIN $[0,.5] \times [0,.75]$
BC $u = g$
TRUE $10\varphi(x)\varphi(y) + \alpha$ where $\varphi(x) = e^{-100(x-.5)^2}(x^2 - x)$

Operator: Helmholtz type, approximates nonlinear operator.
Right side: Sharp, large values near $x = y = .5$.
Boundary conditions: Dirichlet, homogeneous.
Solution: T has a peak at $x = y = .5$.
Parameter: α adjusts singularity of operator.
Problem 20-2 has $\alpha = 10$.

PROB 21

Artificial

$Au_{xx} + Bu_{xy} + Cu_{yy} = f$, $A = C = 1 + T^2$, $B = -2T^2$

DOMAIN unit square
BC $u = g$
TRUE e^{x+y}

Operator: Entire, has mixed derivative term.
Right side: Entire
Boundary conditions: Dirichlet
Solution: T is entire
Parameter: None

PROB 22

Elastic-plastic torsion [15]

$w(u_{xx} + u_{yy}) + w_x u_x + w_y u_y = f$, w defined below

DOMAIN unit square
BC $u = g$
TRUE $[17.06 + 3.62(x^2 + y^2)](x^2 - 1)(y^2 - 1)$

Operator: Expanded form of self-adjust problem, discontinuous coefficients. $w = 1/7996$ if $A \leq .0025$
$w = 1/(236 + 19.4/A)$ if $A > .0025$ where $A = \sqrt{T_x^2 + T_y^2}$
Right side: Singular
Boundary conditions: Dirichlet
Solution: T is a quartic polynomial
Parameter: None

Right side: Analytic

Boundary conditions: Mixed

Solution: Has jumps in third y-derivatives.

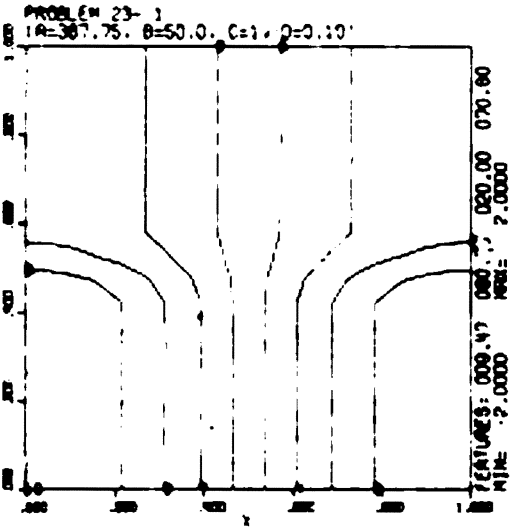
Parameters: Three cases for w given in terms of

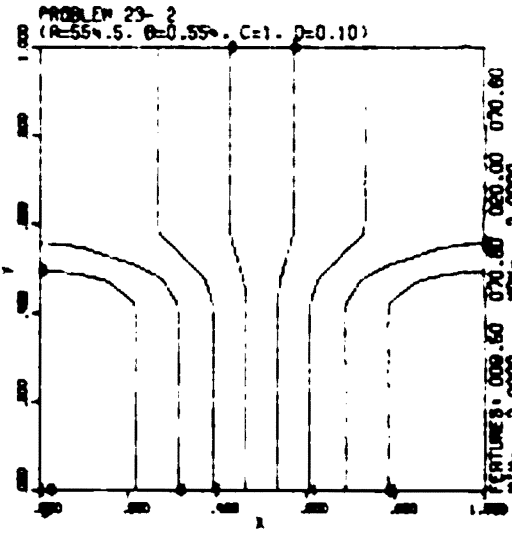
$$A = \sqrt{T_x^2 + T_y^2}, \quad c = 1. \quad w = 1/(\alpha + \beta A)$$

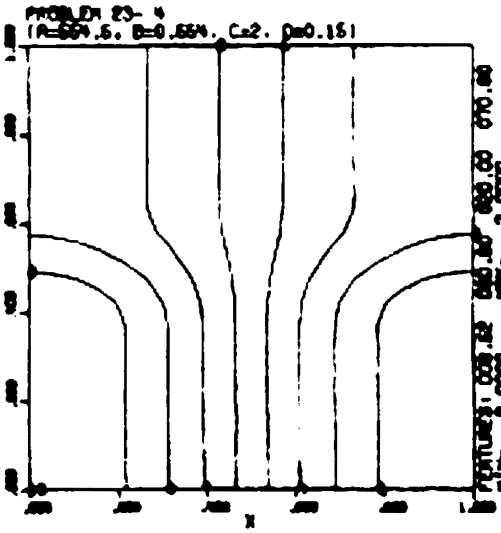
$$c = 2. \quad w = e^{[A/(\alpha+\beta A)]}/A$$

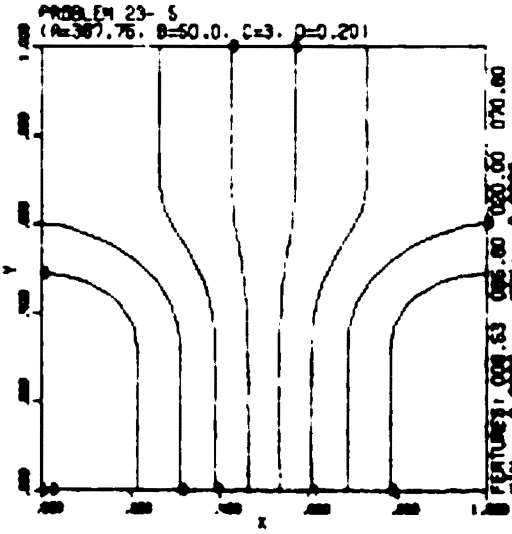
$$c = 3. \quad w = \alpha \tanh(\beta A)/A$$

Physical parameters α,β of (387.75,50) and (554.5,.544) have been used in practice.



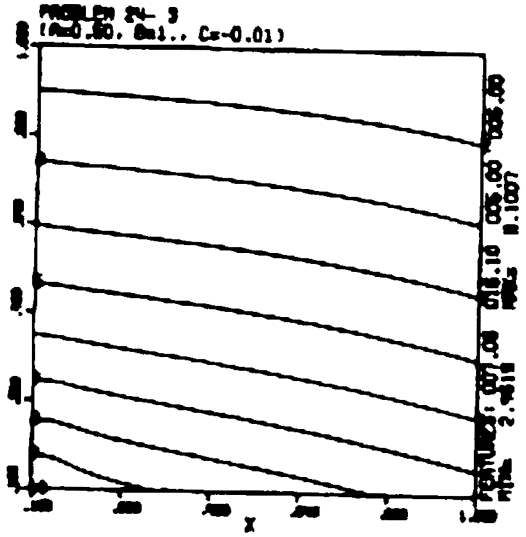


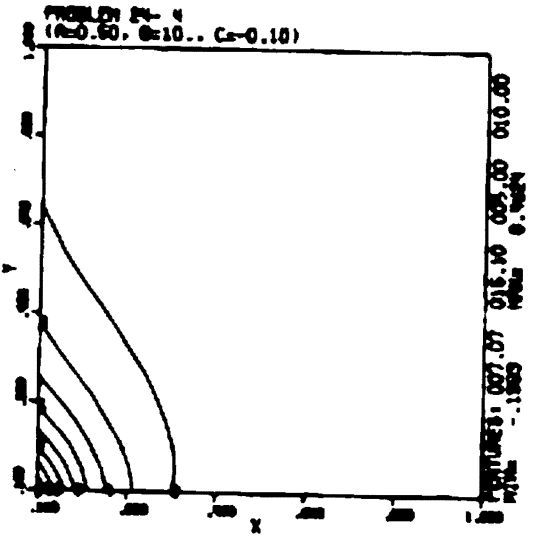


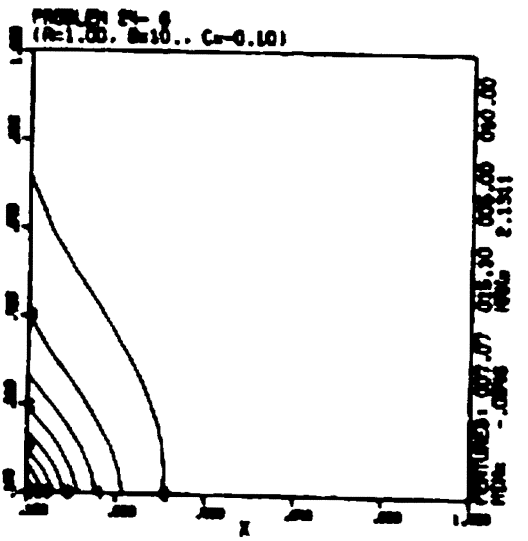


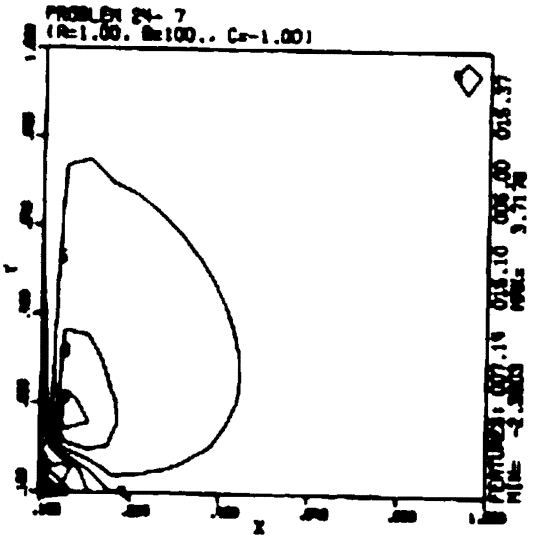
Boundary conditions: Mixed, homogeneous.
Solution: Approximate solutions given for 8 cases.
Parameters: α, β and γ are physical parameters,

1.	$\alpha = .25$	$\beta = 100$	$\gamma = -.1$
2.	$\alpha = .25$	$\beta = 1000$	$\gamma = .1$
3.	$\alpha = .5$	$\beta = 1$	$\gamma = -.01$
4.	$\alpha = .5$	$\beta = 10$	$\gamma = -.1$
5.	$\alpha = 1$	$\beta = 1$	$\gamma = -.1$
6.	$\alpha = 1$	$\beta = 10$	$\gamma = -.1$
7.	$\alpha = 1$	$\beta = 100$	$\gamma = -1.$
8.	$\alpha = 1$	$\beta = 1000$	$\gamma = -1.$









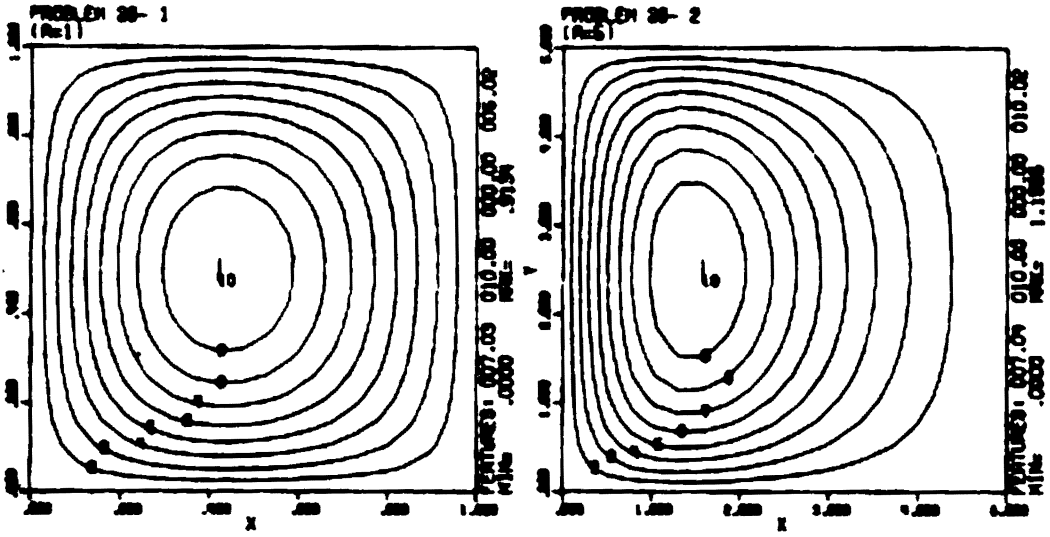
Parameter: α affects smoothness of operator and right side without affecting solution. Problems 25-2 to 25-4 have α = 2.5, 3.5 and 4.5

PROB 26 Viscous flow [3]

$u_{xx}+u_{yy}+Au_x=-60\alpha x/B$ where $B=(\alpha+x^2)^3$, $A=6x(1+x^2)^2/B$

DOMAIN $[0,\alpha] \times [0,\alpha]$
BC $u = 0$
TRUE unknown

Operator: Laplacian plus u_x term. For $\alpha = 1$ it is expansion of a self-adjoint operator.
Right side: Analytic
Boundary conditions: Dirichlet, homogeneous
Solution: Approximate solutions found for α = 1,5 and 10.
Parameter: α is a physical parameter adjusting the domain and entering the coefficients.



PROB 27 Distribution of diffused particles [3]

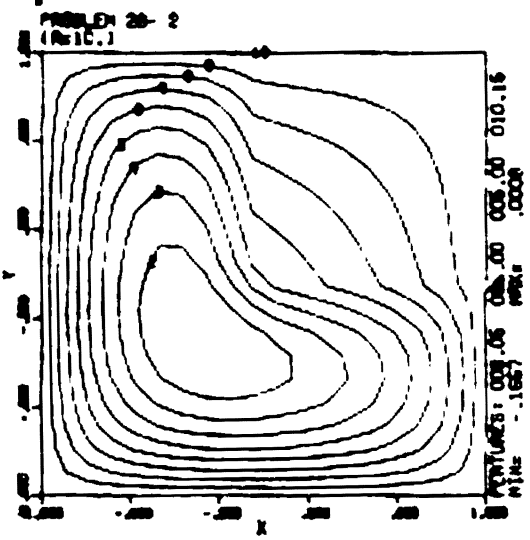
$$u_{xx} + \frac{2}{x}u_x + \frac{1}{x^2}u_{yy} + \frac{1}{x^2}(\cot y)^3 u_y = -100$$

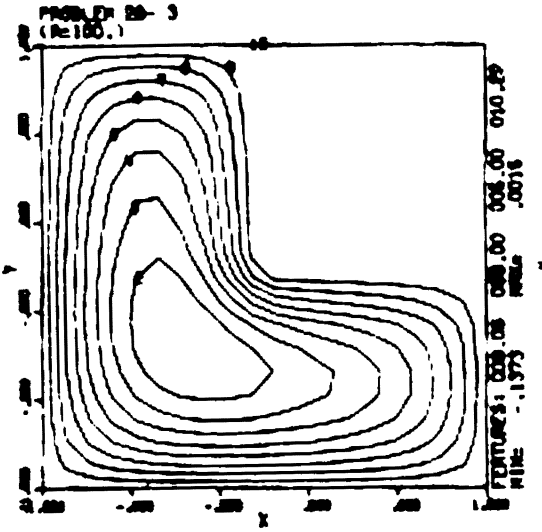
DOMAIN $[.1,1] \times [.1,1]$
BC $u = 0$
TRUE unknown

Operator: Nearly singular, analytic
Right side: Constant
Boundary conditions: Dirichlet, homogeneous
Solution: Approximate solution given.
Parameter: None

100. Strong wave fronts for $\alpha \gg 1$.

Parameter: α adjusts size of discontinuity in operator coefficients which introduces large, sharp jumps in solution.





PROB 29

Many physical interpretations [10]

$$u_{xx} + u_{yy} + \frac{\alpha}{y} u_y = 0$$

DOMAIN unit square

BC $u = (x - y)/\alpha$

TRUE unknown

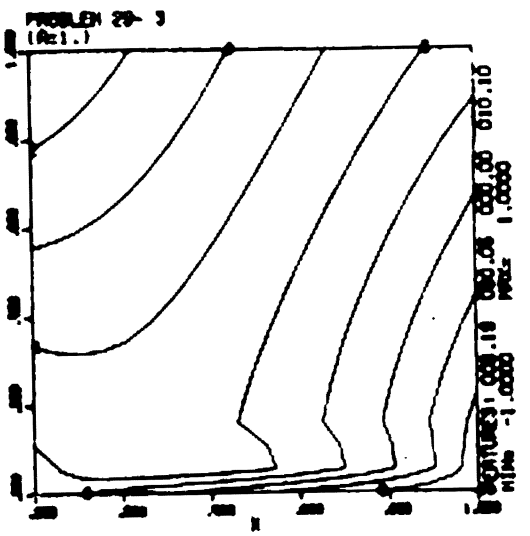
Operator: Laplace plus singular u_y term.

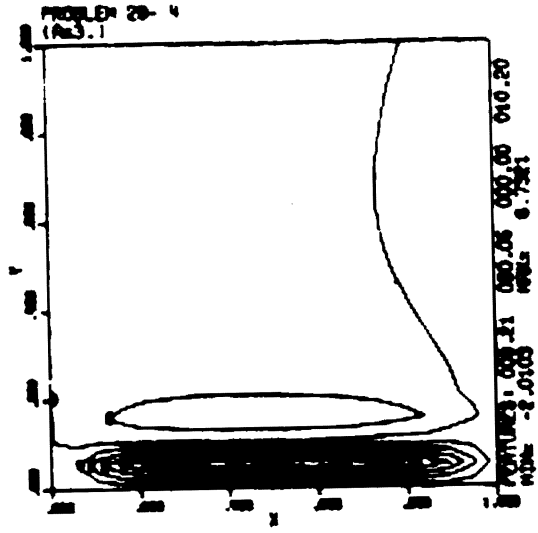
Right side: Homogeneous

Boundary conditions: Dirichlet

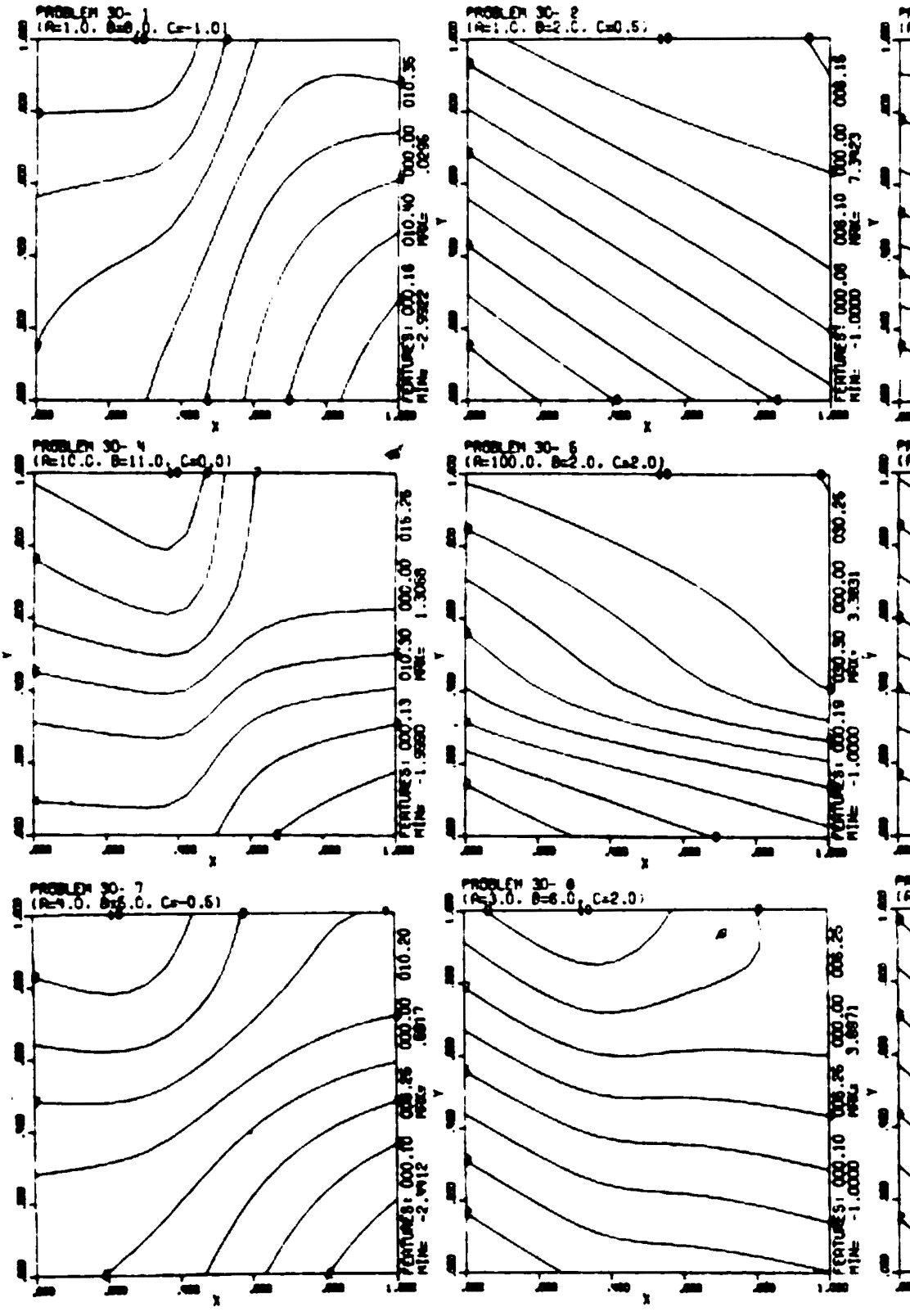
Solution: Five approximate solutions given, some are difficult.

Parameter: α changes physical application: $\alpha = 1$, potentials; $\alpha = -1$ streamlines; $\alpha = 3$, torsion and $\alpha = -3$ or 5, stresses.





Solution: Complicated behavior, with wave fronts, etc.
Parameters: α, β, γ adjust the contribution of 3 independent complexities of the problem.



Right side: Constant
Boundary conditions: Mixed
Solution: Harmonic poly. expansion for homo. BC.
Parameter: None

PROB 32 Stress in helical spring [5]

$$u_{xx} + u_{yy} + \frac{3}{5 - y} u_y = f$$

DOMAIN $[-.5,.5] \times [-1,1]$
BC $u = 0$
TRUE $(1-y^2)(1-4x^2)(5-y^3)(.0004838y + .0010185)$
Operator: Analytic
Right side: Analytic
Boundary conditions: Dirichlet, homogeneous
Solution: Polynomial obtained by Ritz method for a physical problem.
Parameter: The 5 in the operator is a value of a physical parameter.

PROB 33 Torsion on a shaft [5]

$$u_{xx} + u_{yy} = f$$

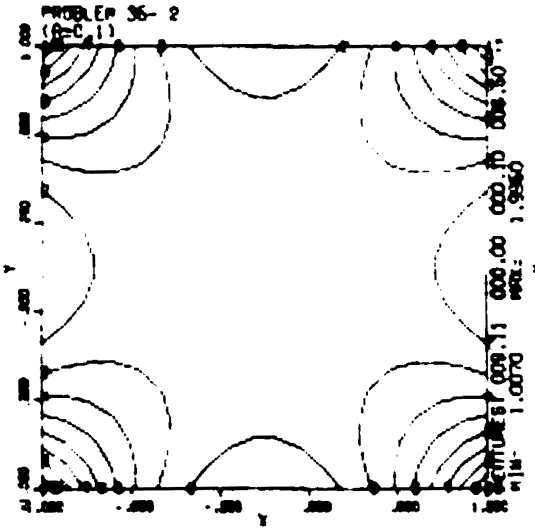
DOMAIN $[0,1] \times [-1,1]$
BC $u = g$
TRUE $p = 14 + \sqrt{133}$, $q = 14 - \sqrt{133}$, $r = (7-q)/(r\sqrt{133})$, $t(y) = 1-y^2$, $C(x) = e^{\sqrt{p}x} - e^{\sqrt{q}x}$, $B(x) = (7-p)r/16C(x)$, $A(x) = rC(x) + e^{\sqrt{q}x}$, $TRUE = t(y)[A(x) + t(y)B(x)]$
Operator: Laplace
Right side: Entire
Boundary conditions: Dirichlet
Solution: Entire

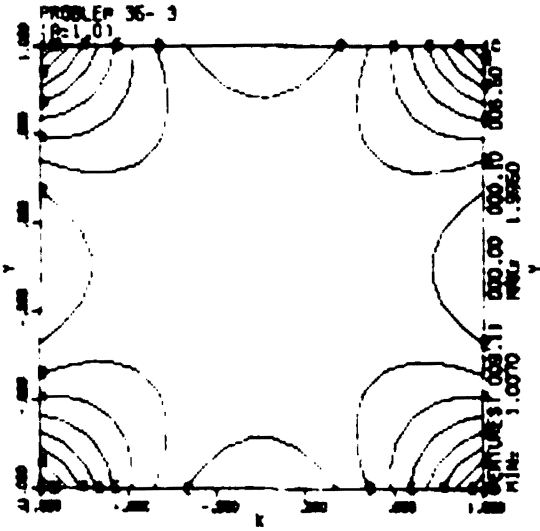
PROB 34 From infinite region problem [5]

$$u_{xx} + u_{yy} = -1$$

DOMAIN $[-1,1] \times [-1,1]$
BC $u = g$
TRUE $.295776 - (x^2+y^2)/4 - 14476(x^4-6x^2y^2+y^4)/319424 + 429(x^8-28x^6y^2+70x^4y^4-28x^2y^6+y^8)/319424$
Operators: Laplace
Right side: Constant
Boundary conditions: Dirichlet
Solution: Harmonic polynomial expansion for homogeneous boundary conditions.
Parameter: None

Solution: Harmonic polynomial combination.
Parameter: α adjusts contribution of mixed boundary condition; $\alpha = 0$ is the physical problem.





PROB 36

Adapted from Problem 27

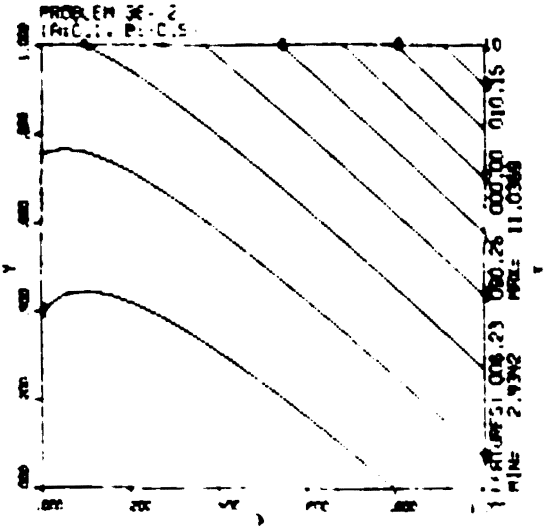
$$(1+\beta)u_{xx} + \frac{2}{x+\alpha}u_x + \frac{1}{(x+\alpha)^2}u_{yy} + \frac{\cot y}{x+\alpha}u_y = f$$

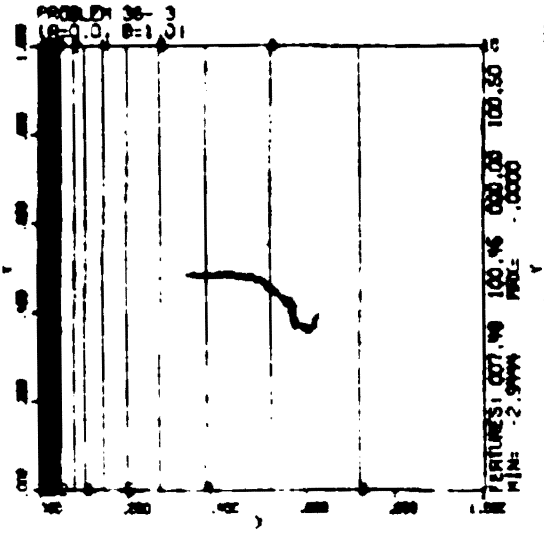
DOMAIN unit square

BC $u = g$

TRUE $(1-\beta)e^{x+y} + \beta\log_e(x+\alpha)$

Operator: Possibly singular coefficients for $\alpha = 0$.
Right side: Analytic except for $\alpha = 0$; then singular.
Boundary conditions: Dirichlet
Solution: Logarithmic singularity for $\alpha = 0$.
Parameters: α adjusts distance of singularity from domain, β adjusts relative size of exponential and logarithmic terms in solution.





Boundary conditions: Dirichlet
Solution: Entire
Parameter: None

PROB 38 Electrostatics [11]

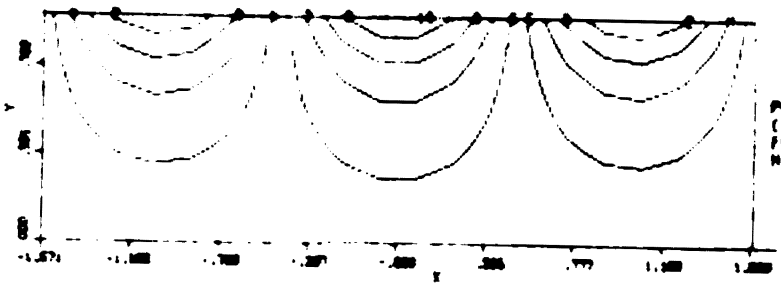
$$u_{xx} + u_{yy} = 0$$

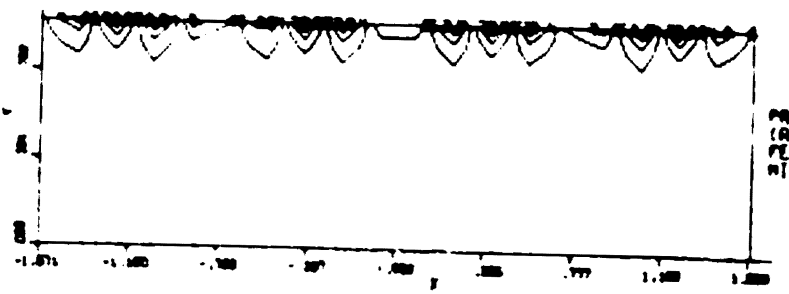
DOMAIN $[-\pi/2,\pi/2] \times [0,1]$

BC $u = g$ for $x = \pm\pi/2$, $y = 1$; $u_y = g$ for $y = 0$

TRUE $e^{-\sqrt{2\alpha+1}} \cos[(2\alpha+1)x]\sinh[(2\alpha+1)y]/(2\alpha+1)$

Operator: Laplace, homogeneous
Right side: Zero
Boundary conditions: Mixed
Solution: Entire, may be oscillatory.
Parameter: α adjusts the oscillations.

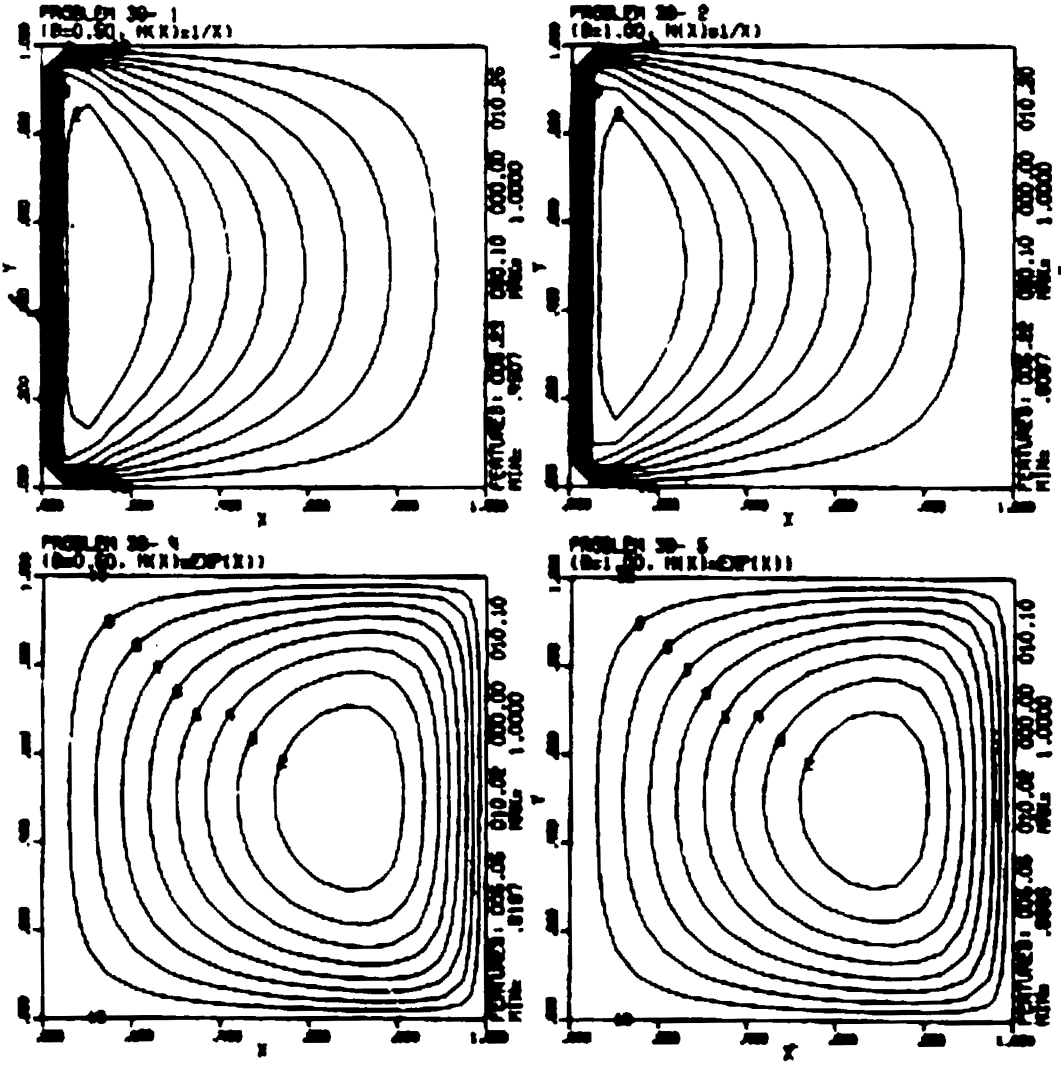




tabulated for 5 cases.

Parameters: $h(x) = 1/x$ for $\beta = .5,1$ (Cases 1 and 2)

$h(x) = e^x$ for $\beta = .25,.5,1$ (Cases 3, 4 and 5)



PROB 40 Hadamard's example [17]

$$u_{xx} + (1+x^2)u_{yy} - yu_x = f$$

DOMAIN unit square

BC $u = g$ for $y = 0$ or 1, $\alpha u + \beta u_x = g$ for $x = 0$ or 1.

TRUE $\log_{10}[(x+1)/(y+1)] + e^{2(x+y)/(2+x-y)-2}$

Operator: Entire
Right side: Analytic
Boundary conditions: Mixed
Solution: Analytic
Parameters: α and β adjust the contributions to the mixed boundary condition on two sides. Problem 40-2 has $\alpha = 0.15$, $\beta = 0.85$ and 40-3 has $\alpha = 0.85$, $\beta = 0.15$

Right side: Series for function with singularities.
Boundary conditions: Dirichlet, homogeneous.
Solution: Infinite series converging like $1/k^3$. The solution has derivative singularities.
Parameters: α adjust u term, possibly makes operator nearly singular. β is number of terms in series.

PROB 42 Artificial [20]

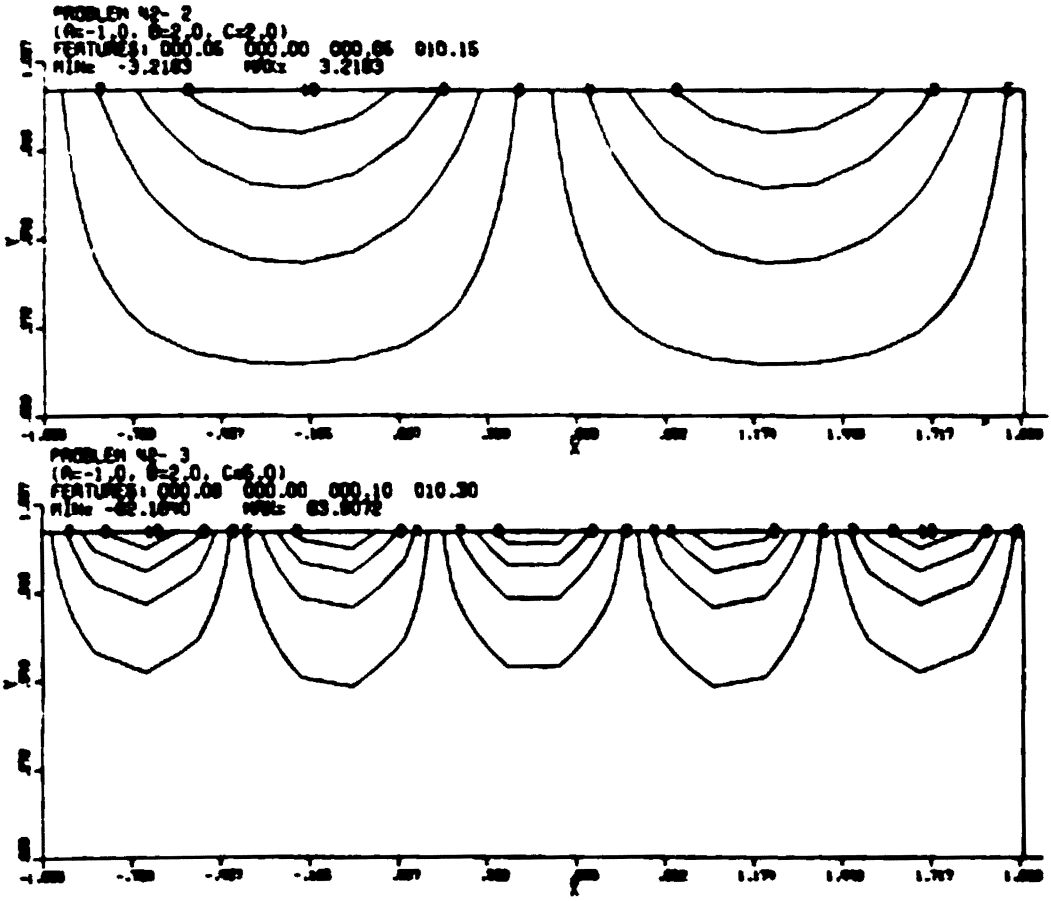
$$u_{xx} + u_{yy} + u_y - u = 0$$

DOMAIN $[\alpha,\beta] \times [0,1]$

BC $u_N = g$

TRUE $$e^{-y/2} \sinh\sqrt{\frac{5}{4} + \frac{\gamma^2\pi^2}{(\beta-\alpha)^2}}\, y \sin\left[\frac{\gamma\pi(x-\alpha)}{(\beta-\alpha)}\right]$$

Operator: Constant coefficients, homogeneous.
Right side: Zero
Boundary conditions: Neumann (but PDE solution unique)
Solution: Analytic, can oscillate as γ increases.
Parameters: (α,β) for domain, γ adjusts oscillations.



Right side: Zero
Boundary conditions: Mixed
Solution: Entire
Parameters: None

PROB 44 From nonlinear problem [20]

$$u_{xx} + u_{yy} + wu = w$$

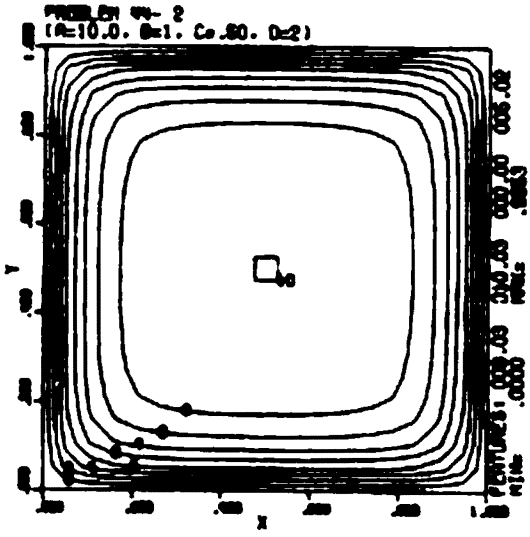
DOMAIN unit square
BC $u = 0$
TRUE unknown

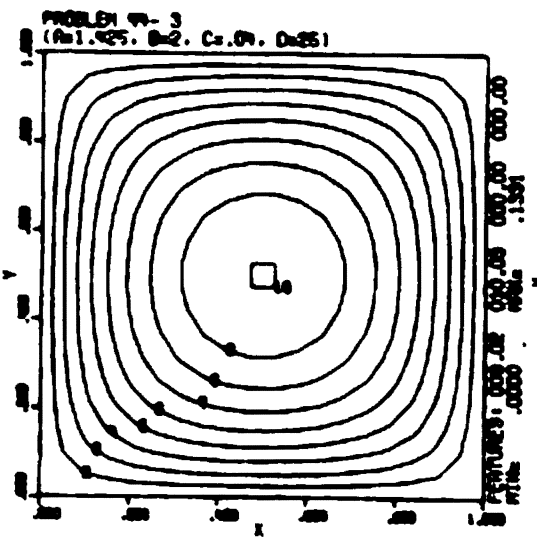
Operator: Helmholtz type
Right side: Complicated
Boundary conditions: Dirichlet, homogeneous
Solution: Approximate solution given for $r = r(x,y)$ tabulated from a solution to the nonlinear problem; r should be u,

$$w(x,y) = -\alpha^2 (1-r)^{\beta-1} e^{[\gamma\delta r/(1+\gamma r)]}.$$

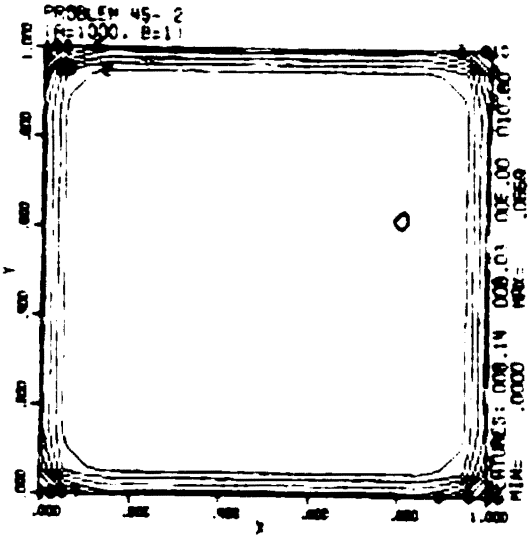
Parameters: α, β, γ and δ are physical parameters. Four cases are given:

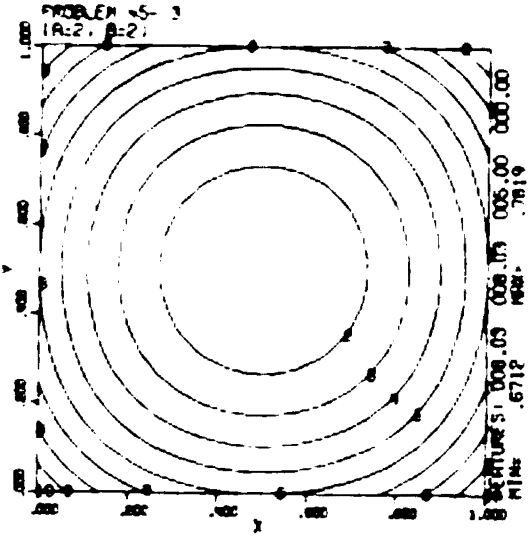
1.	$\alpha = 1.425$	$\beta = 1$	$\gamma = .5$	$\delta = 2$
2.	$\alpha = 10$	$\beta = 1$	$\gamma = .5$	$\delta = 2$
3.	$\alpha = 1.425$	$\beta = 2$	$\gamma = .04$	$\delta = 25$
4.	$\alpha = 1.425$	$\beta = 2$	$\gamma = .5$	$\delta = 2$





Parameters: α and β are physical parameters. Three cases are given: (1,2), (1,1000) and (2,2).





PROB 46 Magnetohydrodynamics [19]

$$u_{xx} + u_{yy} - \beta u_y = 0$$

DOMAIN $[0,\alpha] \times [0,1]$

BC $u = 0$ for $x = 0,\alpha$; $u = 1$ for $y = 1$; $u = -1$ for $y = 0$

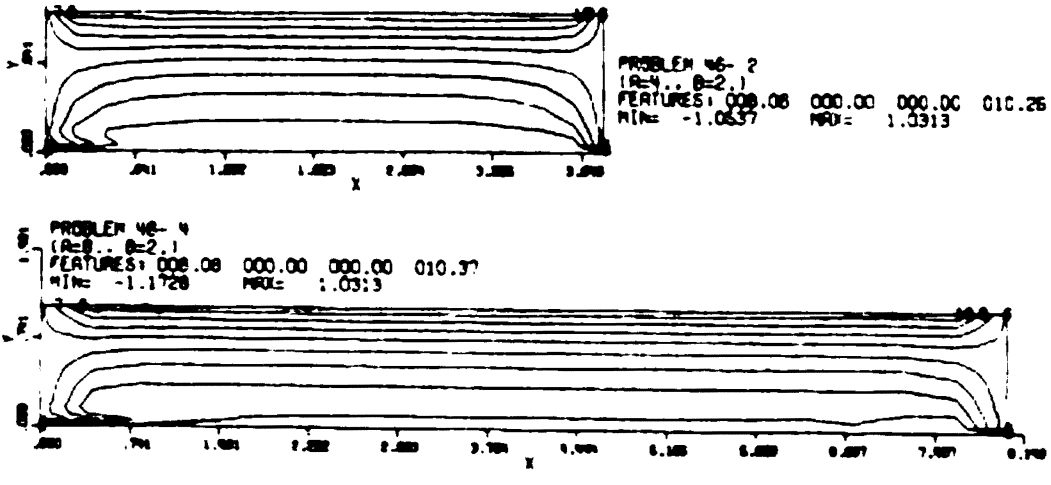
TRUE Unknown

Operator: Constant coefficients, homogeneous.

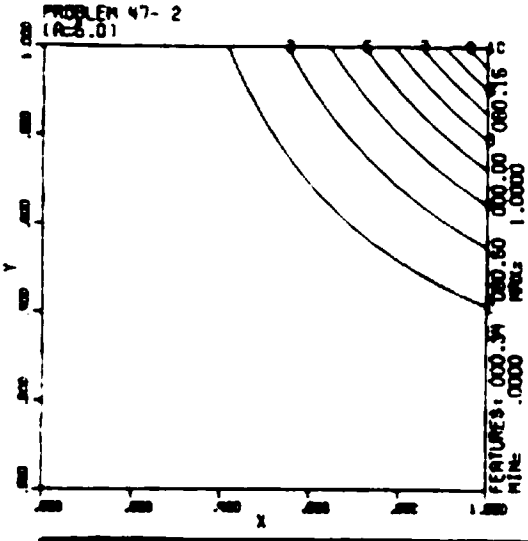
Right side: Zero

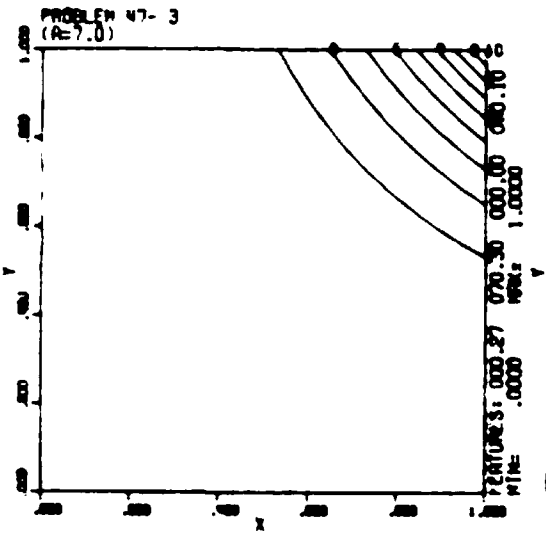
Boundary conditions: Dirichlet

Solution: Approximate solution given for 4 cases: $\alpha = 1$ and $\beta = 2$, $\alpha = 4$ and $\beta = 2$, $\alpha = 4$ and $\beta = 10$, $\alpha = 8$ and $\beta = 2$.



Solution: Singularity of variable strength.
Parameter: α adjusts singularity strength.

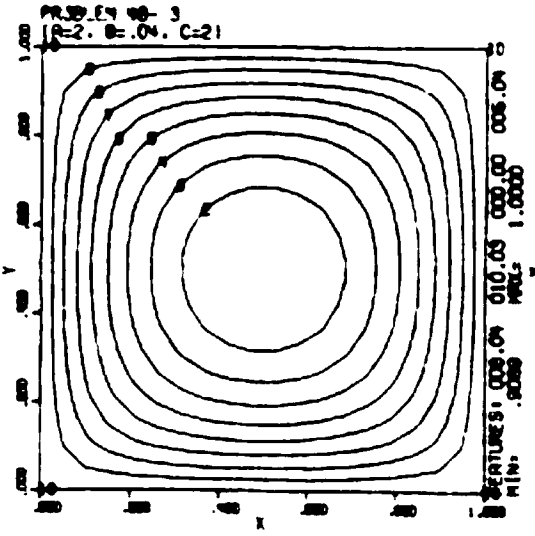


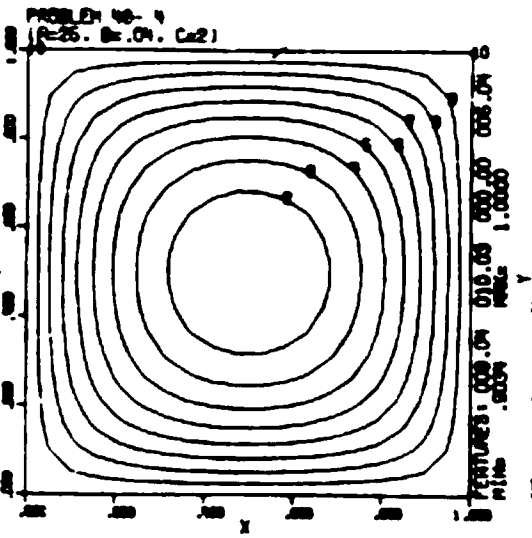


PROB 48 Nonlinear diffusion in catalysts [2]

$$u_{xx} + u_{yy} - 1.425r^{(\gamma-1)}e^{(\alpha\beta(1-r))/(1+\beta(1-r))}u = 0$$

DOMAIN unit square
BC u = 1
TRUE Unknown
Operator: Helmholtz type, homogeneous
Right side: Zero
Boundary conditions: Dirichlet
Solution: Approximate solution given for r = r(x,y) tabulated from a nonlinear PDE solver; r should be u.
Parameters: (α,β,γ) are physical parameters. 5 cases given: (1,.04,2), (1,.04,25), (2,.04,2), (2,.04,25) and (2,.5,2).

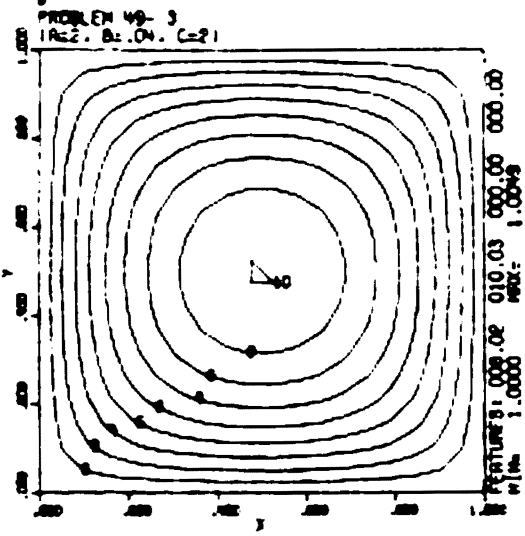


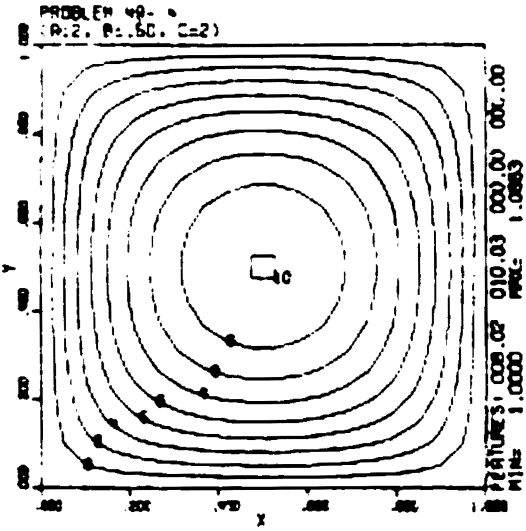


tabulated from a solution to the [illegible]
should be u,

$$w(x,y) = -(1.425)^2 [(1+\beta-r)/\beta]^{\alpha-1} e^{\gamma(r-1)/r}.$$

Parameters: (α,β,γ) are physical parameters. Four cases are given: (1,.5,2), (1,.5,25), (2,.04,2) and (2,.5,2).





PROB 50 Artificial [20]

$$u_{xx} + u_{yy} = 0$$

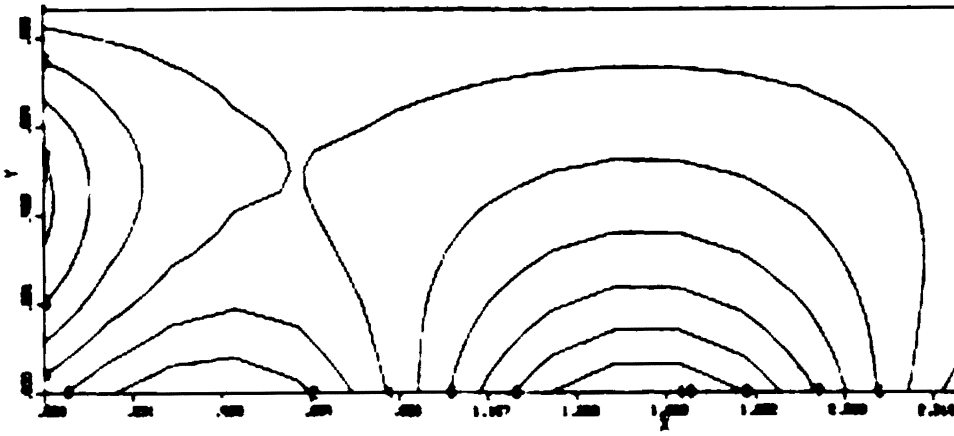
DOMAIN $[0,\pi] \times [0,1]$

BC $u=3\sin(x)/4-\sin(3x), y=0;\ u=0, x=\pi, y=1;\ u=\sin\pi y,\ x=0$

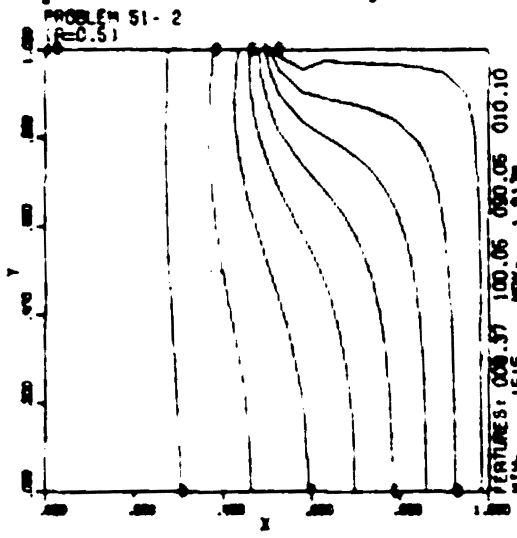
TRUE

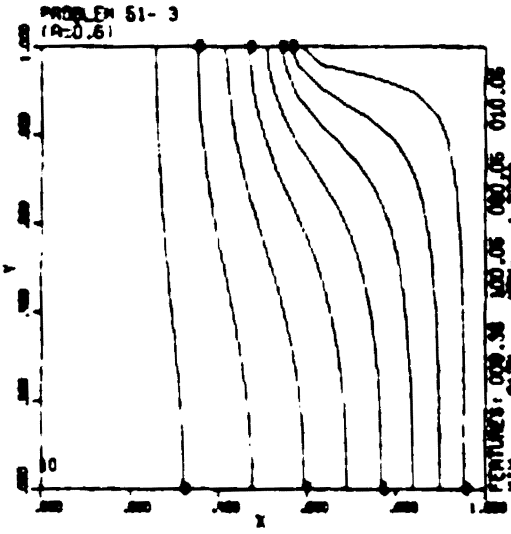
$$\frac{3\sinh(1-y)\sin x}{4\sinh 1} - \frac{\sinh 3(1-y)\sin 3x}{\sinh 3} + \frac{\sinh\pi(1-x)\sin\pi y}{\sinh\pi^2}$$

Operator: Laplace, homogeneous
Right side: Zero
Boundary conditions: Dirichlet
Solution: Entire
Parameters: None



Solution: Has singularity, unusual behavior.
Parameters: α adjusts position of change in boundary condition for $y = 1$.



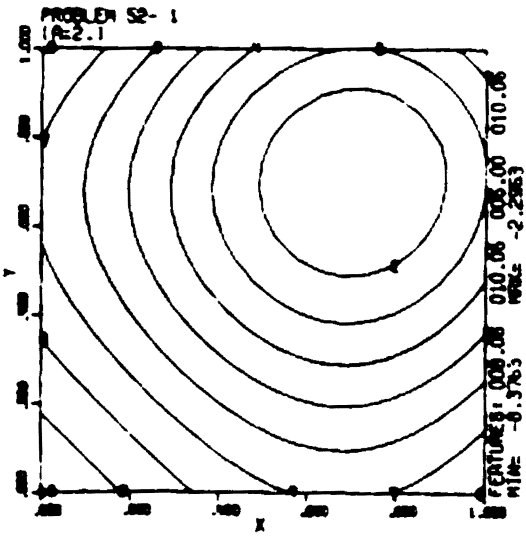


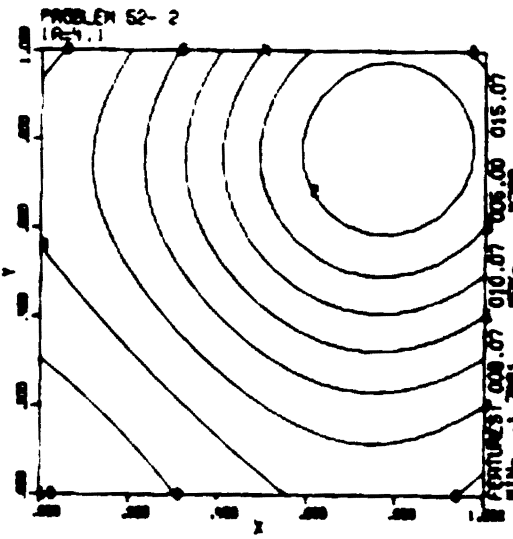
PROB 52 Nonlinear reaction [2]

$$r(u_{xx} + u_{yy}) + r_1 u_x + r_2 u_y - \alpha u = 0$$

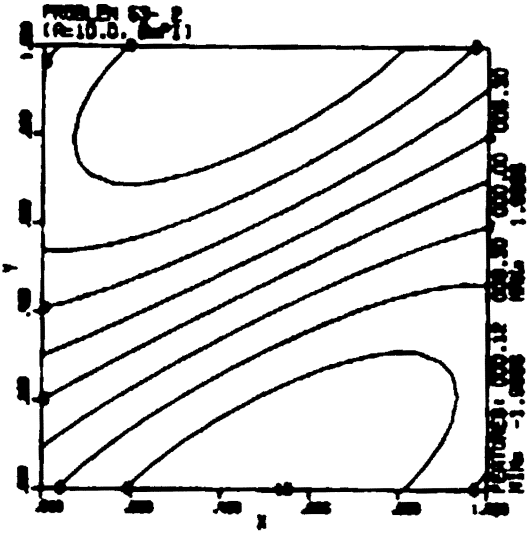
DOMAIN unit square
BC $u + u_N = 1$
TRUE Unknown

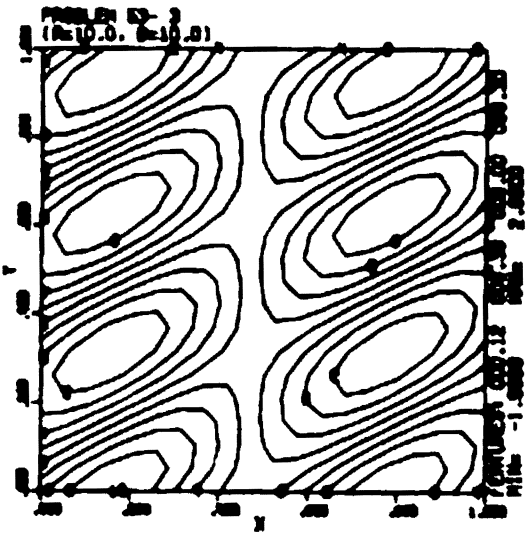
Operator: Expanded from self-adjoint PDE, homogeneous.
Right side: Zero
Boundary conditions: Mixed
Solution: Approximate solution for $r(x,y)$ tabulated from nonlinear PDE solver; r should be $11/(1 + 10u)$; r_1, r_2 are finite differences for r_x, r_y.
Parameter: α = physical parameter.





Solution: Entire, oscillatory
Parameters: α can make operator nearly singular. β adjusts the oscillations of the solution.





PROB 54 Artificial

$$(1+x^2)u_{xx}+(1+A^2)u_{yy}+2xu_x+16yAu_y-(1+(8y-x-4)^2)u = f$$

$A(y) = 4y^2 + \alpha$

DOMAIN unit square

BC $u = g$

TRUE $B = \max[0,(3-x/A(y))^3]$, $C = \max[0,x-A(y)]$

$D = 0$ if $C < .02$, $D = e^{-B/C}$ if $C \geq .02$

$u(x,y) = 2.25x(x-A(y))^2(1-D)/(4A(y)^3)+1/(1+(8y-x-4)^2)$

Operator: Expanded form of self-adjoint operator. Analytic.
Right side: Complicated with possible wild behavior.
Boundary conditions: Dirichlet
Solution: Wildly behaving for α possible, has singularities for $x - 4y^2 = \alpha$ or $4y^2 = -\alpha$.

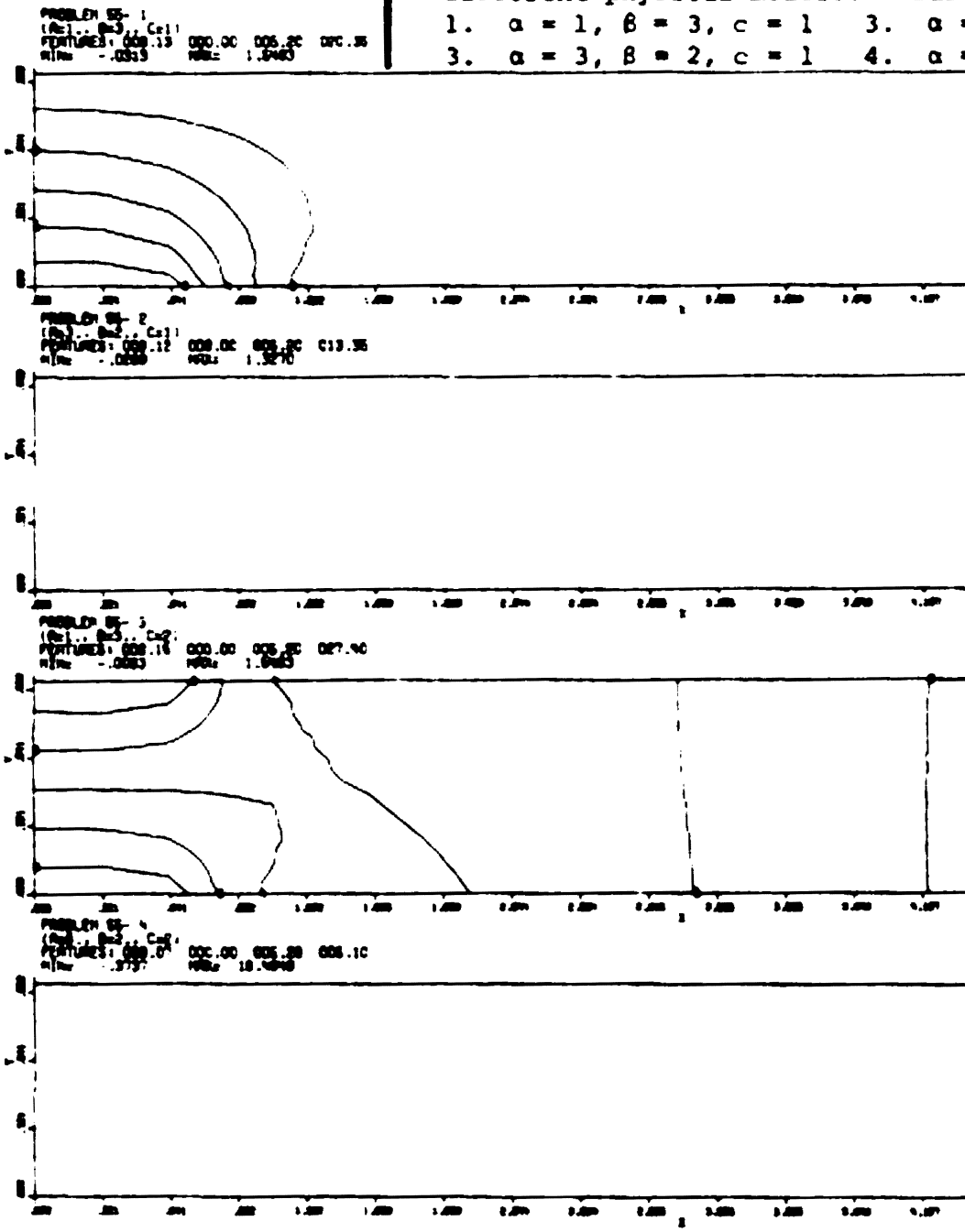
$$A(x) = \begin{cases} 1 & x < \alpha \\ x & x \geq \alpha \end{cases} \qquad B(x) = \begin{cases} 0 \\ 1 \end{cases}$$

$$C(x) = \begin{cases} 1 & x < \alpha \text{ or } c = 1 \\ 0 & x \geq \alpha \text{ and } c = 2 \end{cases} \qquad D(x$$

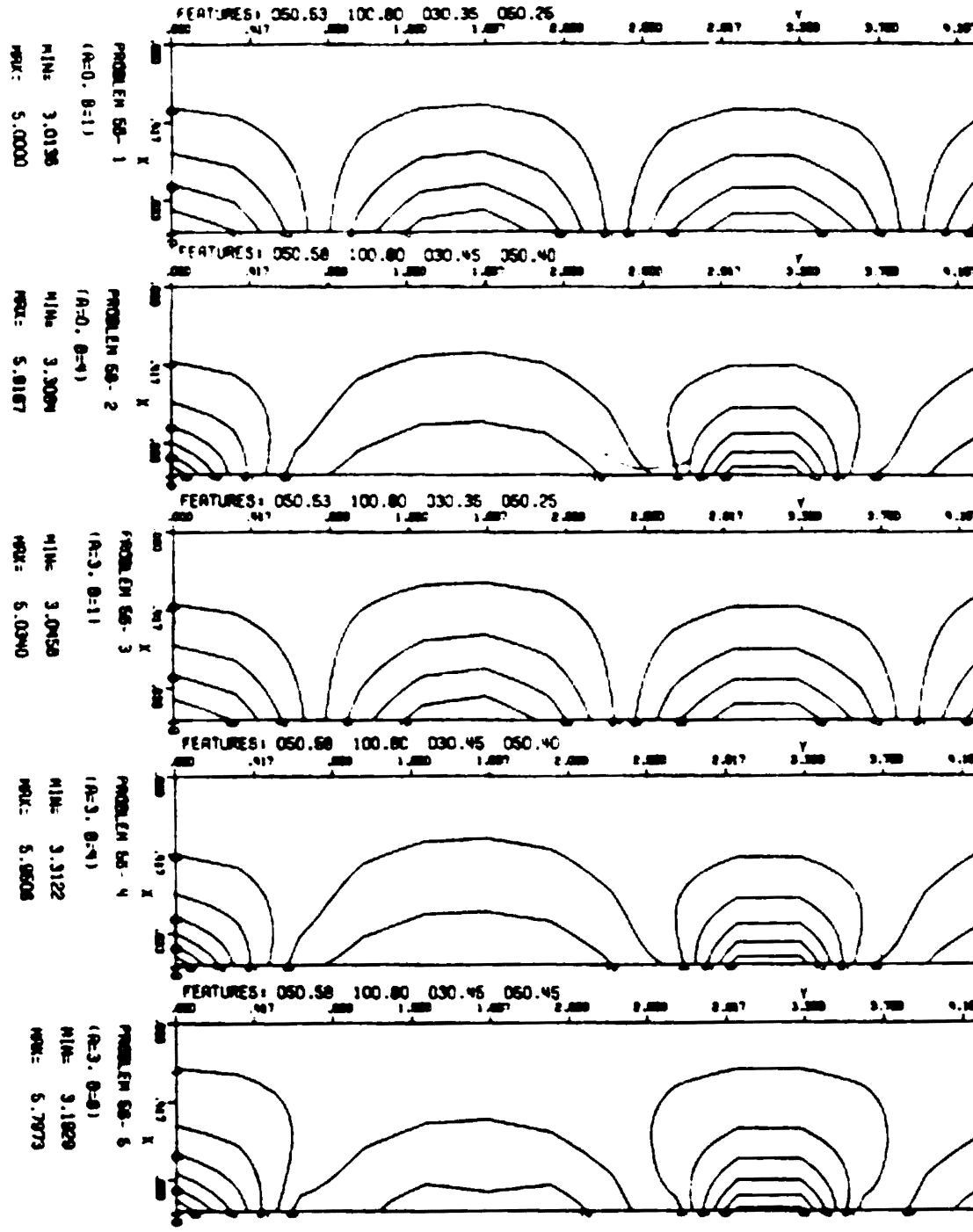
$$G(x) = \begin{cases} 1/\alpha^3 & x<\alpha \\ e^{2(\alpha-x)}/\alpha^3 & x>\alpha \end{cases} \qquad H(x) = \begin{cases} 0 \\ e \end{cases}$$

Solution: Has singularities at bou
ing behavior.

Parameters: α, β are physical par
different physical models. Four
1. $\alpha = 1$, $\beta = 3$, $c = 1$ 3. α
3. $\alpha = 3$, $\beta = 2$, $c = 1$ 4. α



Right side: Constant
Boundary conditions: Mixed
Solution: Series expansion approximates electrostatics soluti
Parameters: α = order of Gauss quadrature for integral, β = no.



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C                PURDUE UNIVERSITY
C                AUGUST 4, 1978 (REVISED JANUARY 29, 1979)
C
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C      REVISED:  WAYNE R. DYKSEN
C                DEPARTMENT OF MATHEMATICS
C                PURDUE UNIVERSITY
C                NOVEMBER 1, 1979
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CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
C  GENERAL DESCRIPTION:
C  --------------------
C
C     GENPGM GENERATES AN ELLPACK PROGRAM ACCORDING TO THE SPEC
C  TIONS GIVEN IN AN ENCODED INPUT STRING.  THE RESULTING ELLPA
C  PROGRAM IS WRITTEN ON FILE PGMFIL.
C
C
C  ENCODED ELLPACK PROGRAMS:
C  -------------------------
C
C     THE ELLPACK PROGRAM TO BE GENERATED IS ENCODED AS FOLLOWS
C
C           N1-N2/N3/N4/N5/N6/N7/N8/COMMENT/
C
C     WHERE  N1   IS THE NUMBER OF THE PROBLEM TO BE SOLVED
C            N2   IS THE PARAMETER SET NUMBER (IF ANY)
C            N3   IS THE OPTIONS SEGMENT NUMBER.
C            N4   IS EITHER N1 OR N1,N2 OR N1,N2,N3 GIVING EITHE
C                 GRID NUMBER OR THE SIZE OF A 2 OR 3 DIMENSIONA
C                 UNIFORM GRID RESPECTIVELY.
C            N5   IS EITHER N1 OR N1,PARAMETER LIST. WHERE N1 IS
C                 MODULE NUMBER OF THE DISCRETIZATION MODULE AND
C                 PARAMETERS ARE OPTIONAL PARAMETERS WHICH ARE T
C                 PASSED TO THE MODULE.
C            N6   GIVES THE MODULE NUMBER AND PARAMETERS OF THE
C                 INDEXING MODULE AS IN N5.
C            N7   GIVES THE MODULE NUMBER AND PARAMETERS OF THE
C                 SOLUTION MODULE AS IN N5.
C            N8   IS THE OUTPUT SEGMENT NUMBER.
C            COMMENT   IS A COMMENT OF .LE. 50 CHARACTERS.  IF T
C                      FIRST CHARACTER OF THE COMMENT IS $, THEN
C                      SUMMARY OF THE RUN IS PLACED ON THE FILE
C
C        THE PROBLEM NUMBER IS A FILE NUMBER IN FILE EQNFIL.
C     THE PROBLEM, GRID, OPTIONS AND OUTPUT NUMBERS ARE RECORD
C     NUMBERS IN THE FILES MACFIL, GRDFIL, OPTFIL AND OUTFIL RE
C     TIVELY. THE PARAMETERS N6 AND N7 MAY BE NULL IF NO INDEXI
C     SOLUTION MODULE IS REQUIRED.  THE PARAMETERS N3 AND N8 MA
C     NULL IN WHICH CASE THE FOLLOWING DEFAULT SEGMENTS ARE USE
C              OPTIONS.   TIME   $  MEMORY     $  LEVEL=0
C              OUTPUT.    MAX-U  $  MAX(20,20)-ERROR
C
C
C  FILES USED BY GENPGM:
C  ---------------------
C
C     FORTRAN IO UNIT NUMBERS FOR THE FILES USED BY GENPGM ARE
C  ASSIGNED IN THE BLOCK DATA SUBPROGRAM.  THE PURPOSE AND FORM
C  THESE ARE AS FOLLOWS:
C
```

```
C     EDNFIL IS DIVIDED UP INTO (SUB) FILES WHICH ARE IN TURN DI
C   UP INTO RECORDS.  THE FILES ARE NUMBERED 0, 1, 2,...ETC. AND
C   RECORDS WITHIN EACH FILE ARE SIMILARLY NUMBERED 0, 1, 2,...ET
C   THE END-OF-FILE AND END-OF-RECORD ARE DESIGNATED BY *EOF AND
C   RESPECTIVELY ('EOF AND 'EOF RESPECTIVELY ON NON-CDC INSTALLA
C
C     FILE 0 CONTAINS A DESCRIPTION OF THE PURPOSE AND ORGANIZAT
C   EDNFIL.  FILES 1, 2, 3,... CONTAIN THE INFORMATION NEEDED TO
C   ATE THE ELLPACK PROGRAM CORRESPONDING TO PROBLEMS 1, 2, 3,...
C   RESPECTIVELY OF THE ELLPACK PDE POPULATION. AS MENTIONED ABO
C   FILES ARE DIVIDED INTO RECORDS WHICH HAVE THE FOLLOWING FORM
C
C        RECORD 0 CONTAINS THE ELLPACK PDE POPULATION PROBLEM N
C     RECORDS 1, 2,... MAY TAKE ONE OF THE FOLLOWING FORMATS:
C
C     ALTERNATIVE 1:
C     ..............
C
C     LINE 1:  LINE 1 IS EITHER BLANK OR CONTAINS DESCRIPTIVE I
C     TION SOME OF WHICH BECOMES A COMMENT IN THE GENERATED ELL
C     PROGRAM.  IF NON-BLANK, LINE 1 CONTAINS THE PARAMETER SET
C     IN I3 FORMAT STARTING IN COLUMN 16. FOLLOWED BY THE PARAM
C     OF THE PROBLEM IN FREE FORMAT STARTING IN COLUMN 19.
C
C     LINE 2:  LINE 2 CONTAINS THE COMPLEXITY MEASURES OF VARIO
C     PROBLEM FEATURES IN FREE FORMAT STARTING IN COLUMN 11.  T
C     INFORMATATION BECOMES A COMMENT IN THE GENERATED ELLPACK
C     GRAM.
C
C     LINE 3:  LINE 3 CONTAINS THE PROBLEM TYPE INFORMATION STA
C     IN COLUMN 11 WHICH IS USED TO TEST THE COMPATIBILIY OF TH
C     BLEM WITH THE ELLPACK ROUTINES SELECTED IN THE ENCODED PR
C     (SEE ROUTINE COMPAT).  THIS INFORMATION BECOMES A COMMENT
C     GENERATED ELLPACK PROGRAM.
C
C     REMAINING LINES:  THE REMAINING LINES OF THE RECORD CONTA
C     INFORMATION COPIED INTO THE EQUATION, BOUNDARY AND FORTRA
C     MENTS OF THE GENERATED ELLPACK PROGRAM IN THE ABOVE ORDER
C     THE EQUATION, BOUNDARY AND FORTRAN CARDS ARE MARKED BY A
C     OR 3 IN COLUMN 1 RESPECTIVELY.  NOTE THAT FORTRAN CODE SH
C     BEGIN IN COLUMN 7 AS USUAL AND THAT A FORTRAN COMMENT CAN
C     WRITTEN BY TYPING A C IN COLUMN 2.  FOR PORTABILITY PURPO
C     THE END-OF-RECORD IS DETECTED BY THE PRESENCE OF A '-' (D
C     IN COLUMN 1.
C
C     ALTERNATIVE 2:
C     ..............
C
C     LINE 1:  LINE 1 CONTAINS THE PARAMETER SET NUMBER IN I3 F
C     STARTING IN COLUMN 16, FOLLOWED BY THE PARAMETERS OF THE
C     BLEM IN FREE FORMAT STARTING IN COLUMN 19.
C
C     LINE 2:  LINE 2 CONTAINS THE COMPLEXITY MEASURES OF VARIO
C     PROBLEM FEATURES IN FREE FORMAT STARTING IN COLUMN 11.  T
C     INFORMATION BECOMES A COMMENT IN THE GENERATED ELLPACK PR
C     GRAM.
C
C     LINE 3:  LINE 3 IS OF THE FORM EXPAND N/N1,N2,....NK/ STA
C     IN COLUMN 1.  THIS SPECIFIES THAT MACRO N (THE NTH RECORD
C     MACFIL) SHOULD BE EXPANDED WITH ACTUAL PARAMETERS N1,N2,..
C     REPLACING THE DUMMIES &A,&B,&C,...ETC.  THE PARAMETERS MA
C     CONTAIN ANY CHARACTERS EXCEPT A BLANK, COMMA OR SLASH AND
C     MAY BE NO LONGER THAN 30 CHARACTERS.
```

```
C  GENERATE THE ELLPACK PROGRAMS CORRESPONDING TO PROBLEMS 1, 2,
C  OF THE ELLPACK PDE POPULATION.  ALTHOUGH SOME OF THE PROBLEMS
C  NOT PARAMETERIZED AND HENCE HAVE NO MACRO CALLS, YET, THERE A
C  DUMMY RECORDS FOR THESE PROBLEMS ON MACFIL SO THAT THE PROBLE
C  MACRO NUMBERS MAY BE IN A ONE-TO-ONE CORRESPONDENCE.  RECORDS
C  1, 2, 3... HAVE THE FOLLOWING FORMAT:
C
C     LINES 1-3:  LINES 1 THROUGH 3 CONTAIN THE MACRO NUMBER.
C
C     LINE 4:  LINE 4 CONTAINS THE PROBLEM TYPE INFORMATION STAR
C     IN COLUMN 11 WHICH IS USED TO TEST THE COMPATIBILIY OF THE
C     BLEM WITH THE ELLPACK ROUTINES SELECTED IN THE ENCODED PRO
C     (SEE ROUTINE COMPAT).  THIS INFORMATION BECOMES A COMMENT
C     GENERATED ELLPACK PROGRAM.
C
C     REMAINING LINES:  THE REMAINING LINES OF THE RECORD CONTAI
C     INFORMATION COPIED INTO THE EQUATION, BOUNDARY AND FORTRAN
C     MENTS OF THE GENERATED ELLPACK PROGRAM IN THE ABOVE ORDER.
C     THE EQUATION, BOUNDARY AND FORTRAN CARDS ARE MARKED BY A 1
C     OR 3 IN COLUMN 1 RESPECTIVELY.  NOTE THAT FORTRAN CODE SHO
C     BEGIN IN COLUMN 7 AS USUAL AND THAT A FORTRAN COMMENT CAN
C     WRITTEN BY TYPING A C IN COLUMN 2.  FOR PORTABILITY PURPOS
C     THE*END-OF-RECORD IS DETECTED BY THE PRESENCE OF A '-' (DA
C     IN COLUMN 1.
C
C  OPTFIL
C  ......
C
C     OPTFIL CONTAINS INFORMATION WHICH MAY BE USED TO GENERATE
C  OPTIONS SEGMENT OF THE ELLPACK PROGRAM.
C
C     OPTFIL IS DIVIDED UP INTO RECORDS WHICH ARE NUMBERED 0, 1,
C  ETC.  THE END-OF-RECORD IS DESIGNATED OF *EOR ('EOR ON NON-CI
C  INSTALLATIONS).
C
C     RECORD 0 CONTAINS A DESCRIPTION OF THE PURPOSE AND ORGANIZ
C  OF OPTFIL.  RECORDS 1, 2, 3.... HAVE THE FOLLOWING FORMAT:
C
C     LINE 1:  LINE 1 CONTAINS THE RECORD NUMBER.
C
C     LINE 2:  LINE 2 MUST CONTAIN THE 8 CHARACTERS OPTIONS. IN
C     COLUMNS 1-8, FOLLOWED ON THE SAME LINE BY ALL OF THE OPTIO
C     SELECTED.
C
C  GRDFIL
C  ......
C
C     GRDFIL CONTAINS INFORMATION WHICH MAY BE USED TO GENERATE
C  GRID SEGMENT OF THE ELLPACK PROGRAM.
C
C     GRDFIL IS DIVIDED UP INTO RECORDS WHICH ARE NUMBERED 0, 1,
C  ETC.  THE END-OF-RECORD IS DESIGNATED OF *EOR ('EOR ON NON-CI
C  INSTALLATIONS).
C
C     RECORD 0 CONTAINS A DESCRIPTION OF THE PURPOSE AND ORGANIZ
C  OF GRDFIL.  RECORDS 1, 2, 3.... HAVE THE FOLLOWING FORMAT:
C
C     LINE 1:  LINE 1 CONTAINS THE RECORD NUMBER.
C
C     REMAINING LINES:  THE REMAINING LINES OF THE RECORD CONTAI
C     GRID SEGMENT EXACTLY AS IT WILL APPEAR IN THE GENERATED EL
C     PROGRAM.  TO SPECIFY AN ELLPACK CONTINUATION LINE, TYPE A
C     (DOT) IN COLUMN 2.  THE INFORMATION OF THIS RECORD IS COPI
```

```
C
C     RECORD 0 CONTAINS A DESCRIPTION OF THE PURPOSE AND ORGANI
C   OF OUTFIL.  RECORDS 1, 2, 3,... HAVE THE FOLLOWING FORMAT:
C
C     LINE 1:  LINE 1 CONTAINS THE RECORD NUMBER.
C
C     REMAINING LINES:  THE REMAINING LINES OF THE RECORD CONTA
C     OUTPUT SEGMENT EXACTLY AS IT WILL APPEAR IN THE GENERATED
C     PACK PROGRAM.  TO SPECIFY AN ELLPACK CONTINUATION LINE, T
C     '.' (DOT) IN COLUMN 2.  THE INFORMATION OF THIS RECORD IS
C     DIRECTLY INTO THE OUTPUT SEGMENT OF THE GENERATED ELLPACK
C     PROGRAM.  FOR PORTABILITY PURPOSES, THE END-OF-RECORD IS
C     DETECTED BY THE PRESENCE OF A '-' (DASH) IN COLUMN 1.
C
C   MODATA
C   ......
C
C     MODATA CONTAINS THE INFORMATION NEEDED TO DECODE ENCODED
C   PROGRAMS AND CHECK COMPATIBILITY.
C
C     LINE 1 HAS THE NUMBER OF MODULES (NRMODS) IN I3 FORMAT.
C   FOLLOWS NRMODS PAIRS OF LINES.  THE FIRST OF EACH CONTAINS A
C   INTEGER IN THE RANGE 1....,NRMODS IN I3 FORMAT AND AN ELLPAC
C   MODULE NAME STARTING IN COLUMN 6.  THESE GIVE THE CORRESPOND
C   BETWEEN MODULE NUMBERS USED IN ENCODED PROGRAMS AND THE NAME
C   OF MODULES TO BE USED IN THE GENERATED ELLPACK PROGRAM.  THE
C   LINE OF THE PAIR CONTAINS A LIST OF INTEGERS IN 4X,30I3 FORM
C   THE MODULE NUMBERS THAT MAY IMMEDIATELY FOLLOW THIS MODULE I
C   ELLPACK PROGRAMS.  AFTER THIS SET OF LINES THERE IS ONE LINE
C   CONTAINING A SINGLE INTEGER IN I3 FORMAT GIVING THE NUMBER O
C   DISCRETIZATION MODULES (NRDIS).  THIS IS FOLLOWED BY NRDIS L
C   EACH CONTAINING A MODULE NUMBER IN I3 FORMAT AND 15 DIGITS
C   STARTING IN COLUMN 5 GIVING MODULE COMPATIBILITY INFORMATION
C   (SEE ROUTINE COMPAT).
C
C   ******  ALL INPUT FILES MUST BE TERMINATED BY A CARD   ******
C   ******  WITH A $ IN COLUMN ONE.                        ******
C
C
C     THE OUTPUT FILES PRODUCED BY GENPGM ARE:
C
C     STANDARD OUTPUT = SUMMARY OF WORK DONE, MESSAGES.
C     PGMFIL = THE GENERATED ELLPACK PROGRAMS, SEPARATED BY *EO
C     HEADER = A SUMMARY OF THE PROBLEM, ITS CHARACTERISTICS, A
C              THE SOLUTION METHOD ARE SAVED HERE FOR LATER
C              PROCESSING.  THE COMMENT INCLUDED WITH THE ENCODE
C              INPUT IS ALSO WRITTEN HERE.  SEE FORMAT 2025.
C
C     FORTRAN IO UNIT NUMBERS FOR THESE FILES ARE SPECIFIED IN
C     BLOCK DATA SUBPROGRAM
C
C
C   NON-STANDARD FORTRAN
C   --------------------
C
C   THIS CODE CONFORMS TO STANDARD FORTRAN (1966) WHEREVER POSSI
C   SEVERAL SYSTEM FUNCTIONS ARE CALLED HOWEVER.  THEY ARE:
C
C     DATE   --   INTEGER FUNCTION RETURNING DATE IN A8 FORMAT.  C
C                 IN MAIN PROGRAM
C     CLOCK  --   INTEGER FUNCTION RETURNING TIME IN A9 FORMAT.  C
C                 IN MAIN PROGRAM
C        (IF THESE ARE REMOVED THE WRITE STATEMENT FOLLOWING TH
```

```
C
C     SKIP -- CDC ASSEMBLY LANGUAGE ROUTINE WHICH PROVIDES AUTO
C             FILE AND/OR RECORD SKIPPING.  SEE THE DISCUSSION
C             BELOW UNDER SKIPPING FILES AND RECORDS.
C
C  THE PROGRAM CARD IN THE MAIN PROGRAM MUST BE REMOVED FOR NON-
C  CDC INSTALLATIONS.
C
C
C  SKIPPING FILES AND RECORDS:
C  ---------------------------
C
C     THE METHOD OF GENERATING ELLPACK PROGRAMS USED IN GENPGM I
C  ITATES HAVING THE ABILITY TO SKIP FILES AND/OR RECORDS ON FI
C  EQNFIL, MACFIL, OPTFIL, GRDFIL AND OUTFIL.
C
C     FOR EXAMPLE, IF THE ENCODED PROGRAM WERE TO REQUEST PROBL
C  IT WOULD BE NECESSARY TO SKIP FILES (PROBLEMS) ON EQNFIL TO
C  AND THEN, WITHIN FILE 5, SKIP RECORDS (PARAMETER SETS) TO RE
C  3.  MOREOVER, SINCE PROBLEM 5 CALLS MACRO 5, IT WOULD ALSO B
C  NECESSARY TO SKIP RECORDS (MACROS) ON MACFIL TO MACRO 5.  SI
C  SKIPPING MAY BE DONE ON OPTFIL, GRDFIL OR OUTFIL DEPENDING O
C  THE ENCODED PROGRAM.
C
C     IN ORDER TO AVOID THE SLOW PROCESS OF SCANNING THROUGH TH
C  FILES LINE BY LINE TO FIND THE DESIRED FILES AND/OR RECORDS,
C  HAVE TAKEN ADVANTAGE OF SOME NON-PORTABLE FEATURES OF THE PU
C  CDC INSTALLATION.  THE END-OF-FILE AND END-OF-RECORD MARKERS
C  *EOF AND *EOR RESPECTIVELY (OR 'EOF AND 'EOR RESPECTIVELY) H
C  SPECIAL MEANING TO THE CDC FILE SYSTEM.  WE PROVIDE FOR CDC
C  INSTALLATIONS THE CDC ASSEMBLY LANGUAGE (COMPASS) PROGRAM SK
C  WHICH USES THESE MARKERS TO GIVE ESSENTIALLY 'AUTOMATIC' SKI
C  OF FILES AND/OR RECORDS.
C
C     ALTHOUGH THESE FEATURES ARE NON-PORTABLE, MANY MACHINES H
C  SIMILAR CONVENTIONS.  WHERE POSSIBLE WE STRONGLY SUGGEST THA
C  ON NON-CDC INSTALLATIONS THE 'EOF AND 'EOR MARKERS BE REPLAC
C  THE LOCAL END-OF-FILE AND END-OF-RECORDS MARKERS AND THAT TH
C  COMPASS PROGRAM FILES BE REPLACED BY A SIMILAR LOCAL ASSEMBL
C  LANGUAGE PROGRAM WHICH WOULD PROVIDE FOR FAST SKIPPING OF FI
C  AND/OR RECORDS.
C
C     SINCE THESE FEATURES ARE NOT AVAILABLE ON SOME MACHINES,
C  PROVIDE AS COMMENTS IN THE COMPASS PROGRAM FILES, TWO FORTRA
C  ROUTINES, SKIPF AND SKIPR, WHICH SKIP FILES AND RECORDS RESP
C  IVELY BY SCANNING FOR THE CHARACTERS 'EOF AND 'EOR IN A4 FOR
C  STARTING IN COLUMN 1.  ON THE PURDUE CDC INSTALLATION, THIS
C  METHOD OF SKIPPING IS UP TO AN ORDER OF MAGNITUDE SLOWER THA
C  THE 'AUTOMATIC' SKIPPING DISCUSSED ABOVE.
C
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
      INTEGER CODE(400),CODIM,CLEN,PTR,LINE(80),PTYPE(15),PCHAR
     $        DISMOD,INDMOD,SOLMOD,EQNREC,OPTREC,OUTREC,GRDREC,
     $        ALGST,ALGEND,COMENT(100),DISPAR,PROBNO,PRPARM(50)
     $        CHARC,CHAR,PNCHEK
C
C  USAGE OF CERTAIN LOCAL VARIABLES -----
C
C     CODE   = THE ENCODED INPUT PROGRAM. (VECTOR)
C     CODIM  = MAX LENGTH OF AN ENCODED INPUT STRING
```

```
C                IN THE ENCODED INPUT STRING CODE.
C       COMENT = THE COMMENT IN THE ENCODED INPUT STRING CODE. (VE
C       FATAL  = SWITCH INDICATING WHETHER AN ERROR HAS BEEN DETEC
C       PROBNO = PROBLEM NUMBER FROM EQNFIL.
C       PRPARM = PROBLEM PARAMETERS FROM EQNFIL (VECTOR).
C
C  GLOBAL VARIABLES
C
       INTEGER EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,PARAM,CO
      $        SLASH,LPAREN,RPAREN,BLANK,AMP,E,ZERO,ONE,TWO,THREE
      $        MODNAM,DMTYPE,VALID,LEVEL,DOT,NULL,NRMODS,
      $        NUDCOL,STAR,DOLLAR,HEADER,DIGIT,LETTER,MODATA,DASH
       LOGICAL FATAL
       COMMON / GLOBAL / EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMF
      $                  HEADER,COMMA,SLASH,LPAREN,RPAREN,BLANK,A
      $                  DOT,DOLLAR,STAR,NULL,ZERO,ONE,TWO,THREE,
      $                  MODATA,DASH,
      $                  PARAM(10,35),MODNAM(70,10),DMTYPE(70,15
      $                  VALID(70,20),IZEROS(15),DIGIT(10),LETTE
      $                  LEVEL,NRMODS,NUDCOL,FATAL
C
       DATA CODIM/400/,NRTYPE/15/,CHARC/1HC/
C
C  READ TABLE OF MODULE INFORMATION
C
C      READ PAST FILE DOCUMENTATION
C
   2   READ(MODATA,2042)CHAR
       IF (CHAR .NE. DASH) GO TO 2
C
       READ(MODATA,2029)NRMODS
       IF (LEVEL .GT. 2)  WRITE(6,2032) NRMODS
       DO 5 I=1,NRMODS
          READ(MODATA,2030) N,(MODNAM(N,K),K=1,10)
          NP1 = N+1
          READ(MODATA,2036) (VALID(NP1,K),K=1,20)
          IF (LEVEL .LE. 2)  GO TO 5
             NONZ = 0
             DO 3 J=1,20
                NONZ = NONZ + 1
                IF (VALID(NP1,NONZ) .LE. 0)  GO TO 4
    3        CONTINUE
    4        CONTINUE
             WRITE(6,2033) N,(MODNAM(N,K),K=1,10),(VALID(NP1,K),
    5  CONTINUE
       READ(MODATA,2029) NRDIS
       IF (LEVEL .GT. 2)  WRITE(6,2034) NRDIS
       DO 10 I=1,NRDIS
          READ(MODATA,2031) N,(DMTYPE(N,K),K=1,NRTYPE)
          VALID(1,I) = N
          IF (LEVEL .GT. 2)  WRITE(6,2035) N,(DMTYPE(N,K),K=1,NR
   10  CONTINUE
       VALID(1,NRDIS+1) = 0
C
C  GET ENCODED PROBLEM
C
       FATAL = .FALSE.
       WRITE(6,2001)
       CALL INCODE(CODE,CODIM,CLEN,PTR)
   20  CONTINUE
       IF (CLEN .EQ. 0)  GO TO 900
C
C      GET PROBLEM NUMBER
```

```
      IF (PROBNO .NE. PNCHEK) CALL ERREND(5,EQNFIL)
C
C     CHECK FOR PARAMETER SET NUMBER
C
      IF ((CODE(PTR-1) .EQ. DASH ) .OR.
     $    (CODE(PTR-1) .EQ. DOT  ) .OR.
     $    (CODE(PTR-1) .EQ. COMMA))      GO TO 21
         NPMSET=1
         CALL SKIPR(0,EQNFIL)
         GO TO 23
21    CONTINUE
C
C     SKIP RECORDS TO PARAMETER SET
C
      NPMSET=NUMBER(CODE,PTR)
      IF (NPMSET .EQ. 0) NPMSET=1
      CALL SKIPR(NPMSET-1,EQNFIL)
C
C     VALIDATE PARAMETER SET NUMBER AND GET PARAMETER INFO
C
23    CONTINUE
      READ(EQNFIL,2041)NPSCHEK,(PRPARM(I),I=1,50)
      NPSCHEK=NPSCHEK+0
      IF(NPSCHEK .EQ. 0) NPSCHEK=1
      IF (NPMSET .NE. NPSCHEK) CALL ERREND(7,EQNFIL)
      CALL GETLN(EQNFIL,LINE)
      WRITE(PGMFIL,2000) (CODE(I),I=1,CLEN)
      WRITE(PGMFIL,2009) PROBNO,NPMSET,(PRPARM(I),I=1,50),
     $                   (LINE(I),I=7,50)
      J = 0
      DO 30 K=1,4
         IFIRST = 10*K+1
         ILAST = IFIRST+5
         DO 29 I=IFIRST,ILAST
            J = J+1
            PCHAR(J) = LINE(I)
29       CONTINUE
         J = J+1
         PCHAR(J) = BLANK
30    CONTINUE
C
C     CHECK FOR MACRO CALL
C
      CALL GETLN(EQNFIL,LINE)
      IF (LINE(1) .EQ. E) CALL EXPAND(LINE,FILE)
C
C     PICK UP PROBLEM TYPE INFORMATION (PTYPE)
C
      J = 0
      DO 35 I=11,23
         J = J + 1
         PTYPE(J) = LINE(I)
35    CONTINUE
      WRITE(PGMFIL,2011) (LINE(I),I=11,23)
C
C     GENERATE EQUATION SEGMENT
C
      WRITE(PGMFIL,2012)
      CALL COPYRC(FILE,PGMFIL,BLANK,TWO,LINE,.TRUE.)
C
C     GENERATE BOUNDARY SEGMENT
C
      WRITE(PGMFIL,2013)
```

```
   42    CONTINUE
C
C     PROCESS OPTIONS RECORD
C
C        NOTE--- THE PREPROCESSOR LIMITS OPTIONS SEGMENTS TO ONE
C                LINE EACH. HENCE WE ASSUME OPTIONS FILE RECORD
C                HAVE THE WORD OPTIONS. AT THE START OF EACH LI
C
         OPTREC = NUMBER(CODE,PTR)
         IF (OPTREC .EQ. 0)  GO TO 45
C
C        SKIP RECORDS TO OPTION
C
            REWIND OPTFIL
            CALL SKIPR(OPTREC,OPTFIL)
C
C        VALIDATE OPTION NUMBER
C
            READ(OPTFIL,2040)NUMOPT
            IF (NUMOPT .NE. OPTREC) CALL ERREND(9,OPTFIL)
            CALL COPYRC(OPTFIL,PGMFIL,NULL,DASH,LINE,.TRUE.)
            GO TO 50
C
C           USE DEFAULT SEGMENT
   45       CONTINUE
            WRITE(PGMFIL,2026)
   50    CONTINUE
C
C     PROCESS GRID SEGMENT
C
         GRDREC = NUMBER(CODE,PTR)
         IF (CODE(PTR-1) .EQ. SLASH)  GO TO 60
C
C           UNIFORM GRID SIZE IS GIVEN IN ENCODED PROGRAM
            NX = GRDREC
            NY = NUMBER(CODE,PTR)
            NZ = 0
            IF (CODE(PTR-1) .EQ. COMMA)  NZ = NUMBER(CODE,PTR)
            PTYPE(14) = ZERO
            PTYPE(15) = TWO
            IF ((NX .EQ. NY) .AND. ((NX .EQ. NZ) .OR. (NZ .EQ.
     $         PTYPE(14) = TWO
            WRITE(PGMFIL,2002) NX,NY
            IF (NZ .GT. 1)  WRITE(PGMFIL,2003) NZ
            GO TO 70
C
C           NON-UNIFORM GRID GIVEN IN FILE GRDFIL
C
   60       CONTINUE
            PTYPE(14)=ZERO
            PTYPE(15) = ZERO
C
C        SKIP RECORDS TO GRID
C
            REWIND GRDFIL
            CALL SKIPR(GRDREC,GRDFIL)
C
C        VALIDATE GRID NUMBER
C
            READ(GRDFIL,2040) NGRDRC
            IF (NGRDRC .NE. GRDREC) CALL ERREND(10,GRDFIL)
            WRITE(PGMFIL,2024)
            CALL COPYRC(GRDFIL,PGMFIL,BLANK,DASH,LINE,.TRUE.)
```

```
C
C     PROCESS INDEXING SEGMENT
C
          INDMOD = NUMBER(CODE,PTR)
          IF (INDMOD .GT. NRMODS)  GO TO 800
          IF (INDMOD .EQ. 0)  GO TO 80
             WRITE(PGMFIL,2005)
             CALL WRTMOD(INDMOD,CODE,CLEN,PTR)
    80    CONTINUE
C
C     PROCESS SOLUTION SEGMENT
C
          SOLMOD = NUMBER(CODE,PTR)
          IF (SOLMOD .GT. NRMODS)  GO TO 800
          IF (SOLMOD .EQ. 0)  GO TO 90
             WRITE(PGMFIL,2006)
             CALL WRTMOD(SOLMOD,CODE,CLEN,PTR)
    90    CONTINUE
          ALGEND = PTR-1
C
C     PROCESS OUTPUT RECORD
C
          OUTREC = NUMBER(CODE,PTR)
          IF (OUTREC .EQ. 0)  GO TO 95
C
C         SKIP RECORDS TO OUTPUT RECORD
C
             REWIND OUTFIL
             CALL SKIPR(OUTREC,OUTFIL)
C
C         VALIDATE OUTPUT RECORD
C
          READ(OUTFIL,2040) NOUTRCD
          IF (NOUTRCD .NE. OUTREC) CALL ERREND(11,OUTFIL)
             WRITE(PGMFIL,2007)
             CALL COPYRC(OUTFIL,PGMFIL,BLANK,DASH,LINE,.TRUE.)
             GO TO 100
C
C            USE DEFAULT SEGMENT
    95       CONTINUE
             WRITE(PGMFIL,2027)
   100    CONTINUE
C
C     WRITE A SEQUENCE SEGMENT ONTO THE ELLPACK INPUT PROGRAM
C
          WRITE(PGMFIL,2019)
          IF (DISMOD .NE. 0)  WRITE(PGMFIL,2020)
          IF (INDMOD .NE. 0)  WRITE(PGMFIL,2021)
          IF (SOLMOD .NE. 0)  WRITE(PGMFIL,2022)
          WRITE(PGMFIL,2023)
          IF (DISMOD .NE. 0)  WRITE(PGMFIL,2028)
C
C     CHECK PROB/DIS COMPATIBILITY AND MODULE SEQUENCE LEGALITY
C
          IF (LEVEL .LT. 1)  GO TO 105
             IF (DISMOD .EQ. 0)
     $          WRITE(PGMFIL,2010) PROBNO,(PTYPE(I),I=1,NRTYPE),
     $                             OPTREC,DISMOD,(IZEROS(I),I=1,
     $                             INDMOD,SOLMOD,OUTREC
             IF (DISMOD .NE. 0)
     $          WRITE(PGMFIL,2010) PROBNO,(PTYPE(I),I=1,NRTYPE),
     $                             OPTREC,DISMOD,(DMTYPE(DISMOD,
     $                             I=1,NRTYPE),INDMOD,SOLMOD,OUT
   105    CONTINUE
```

```
  120     CONTINUE
          IF ((CODE(PTR) .EQ. SLASH) .OR. (J .GE. 100))  GO TO 13
             J = J+1
             COMENT(J) = CODE(PTR)
             PTR = PTR+1
             GO TO 120
  130     CONTINUE
C
C         PICK UP TODAYS DATE AND CURRENT TIME
          RDATE = DATE(0)
          RTIME = CLOCK(0)
C
C         WRITE INFO ONTO HEADER FILE
          IF (COMENT(1) .EQ. DOLLAR)
     $    WRITE(HEADER,2025) PROBNO,NPMSET,RDATE,RTIME,
     $                       (COMENT(I),I=1,100),(PRPARM(I),I=1,5
     $                       (PTYPE(I),I=1,15),(PCHAR(I),I=1,27),
     $                       (CODE(I),I=ALGST,ALGEND)
C
C     READ NEXT PROBLEM
C
  200     CONTINUE
          CALL INCODE(CODE,CODIM,CLEN,PTR)
C
          GO TO 20
C
C  ABNORMAL TERMINATION OF GENERATED PROGRAM
C
  800 CONTINUE
      FATAL = .TRUE.
      WRITE(PGMFIL,2015) STAR,STAR,STAR
      WRITE(6,2015) BLANK,BLANK,BLANK
      WRITE(PGMFIL,2008)
      END FILE PGMFIL
      GO TO 200
C
C  PROGRAM EXIT
C
  900 CONTINUE
      CALL OFFSW(1)
      IF (FATAL)  CALL ERREND(1,IDUM)
      STOP
C
C-----------------------------------------------------------------
C
 2000 FORMAT(1H*/4H*    ,26(1H-)
     $       /30H*    PDE METHOD COMPARISON TEST
     $       /4H*    ,26(1H-)/1H*
     $       /24H*       ENCODED PROGRAM IS
     $       /(1H*,10X,60A1))
 2001 FORMAT(1H1)
 2002 FORMAT(5HGRID..5X,11HUNIFORM X =,I3,3X,1H*,3X,11HUNIFORM
 2003 FORMAT(21X,11HUNIFORM Z =,I3)
 2004 FORMAT(1H*/15HDISCRETIZATION.)
 2005 FORMAT(9HINDEXING.)
 2006 FORMAT(9HSOLUTION.)
 2007 FORMAT(1H*/7HOUTPUT.)
 2008 FORMAT(1H*/1H*,70(1H-)/4HEND.)
 2009 FORMAT(1H*/19H*    PROBLEM NUMBER ,I3,1H-,I3,50A1
     $       /19H*    PROBLEM TYPE = ,44A1)
 2010 FORMAT(1H*/1H*,70(1H-)/1H*
     $    /38H*    ELLPACK PROGRAM GENERATION SUMMARY  /1H*
     $    /1H*,10X,24HPROBLEM NUMBER          = ,I3,8X,6H(TYPE=,15
```

```
 2021 FORMAT(14X,8HINDEXING)
 2022 FORMAT(14X,8HSOLUTION)
 2023 FORMAT(14X,6HOUTPUT)
 2024 FORMAT(5HGRID.)
 2025 FORMAT(/2I4,2X,A8,1X,A9,2X,100A1/4X,50A1,3X,15A1,5X,27A1
     $       /80A1)
 2026 FORMAT(41HOPTIONS.  TIME   $   MEMORY   $   LEVEL=0     )
 2027 FORMAT(1H*/40HOUTPUT.     MAX-U   $   MAX(20,20)-ERROR
 2028 FORMAT(14X,4HTEST)
 2029 FORMAT(I3)
 2030 FORMAT(I3,2X,10A4)
 2031 FORMAT(I3,2X,30A1)
 2032 FORMAT(1H1,100(1H-)/38X,21HM O D U L E   D A T A/1X,100(1
     $       5X,20HNUMBER OF MODULES = ,I5///
     $       2X,6HNUMBER,12X,4HNAME,31X,
     $       15HLEGAL FOLLOWERS/2X,6(1H-),12X,4(1H-),31X,15(1H-
 2033 FORMAT(3X,I4,3X,10A4,1X,20I3)
 2034 FORMAT(//1X,9HTHERE ARE ,I3,34H DISCRETIZATION MODULES.
     $       //2X,13HMODULE NUMBER,4X,18HCOMPATIBILITY INFO/
     $       2X,13(1H-),4X,18(1H-)/)
 2035 FORMAT(5X,I4,10X,30A1)
 2036 FORMAT(4X,30I3)
 2037 FORMAT(80A1)
 2038 FORMAT(/T11,I3/)
 2040 FORMAT(T8,I3)
 2041 FORMAT(T16,I3,50A1)
 2042 FORMAT(A1)
 9000 FORMAT(I1)
C
      END
      BLOCK DATA
C
C   THIS BLOCK DATA SUBPROGRAM PERFORMS TABLE INITIALIZATIONS FO
C   THE PROGRAM GENPGM
C
C
C   GLOBAL VARIABLES
C
       INTEGER EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,PARAM,C
     $         SLASH,LPAREN,RPAREN,BLANK,AMP,E,ZERO,ONE,TWO,THRE
     $         MODNAM,DMTYPE,VALID,LEVEL,DOT,NULL,NRMODS,
     $         NUDCOL,STAR,DOLLAR,HEADER,DIGIT,LETTER,MODATA,DAS
       LOGICAL FATAL
       COMMON / GLOBAL / EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMF
     $                   HEADER,COMMA,SLASH,LPAREN,RPAREN,BLANK,
     $                   DOT,DOLLAR,STAR,NULL,ZERO,ONE,TWO,THREE
     $                   MODATA,DASH,
     $                   PARAM(10,35),MODNAM(70,10),DMTYPE(70,15
     $                   VALID(70,20),IZEROS(15),DIGIT(10),LETTE
     $                   LEVEL,NRMODS,NUDCOL,FATAL
C
C   INITIALIZE CONSTANTS
C
C      MEANING OF SELECTED CONSTANTS ----
C           EQNFIL,GRDFIL,OPTFIL,MACFIL,PGMFIL,HEADER,MODATA =
C                    FORTRAN IO UNIT NUMBERS
C           LEVEL = DEBUG PRINT LEVEL. MAY BE 1 TO 5.
C           NUDCOL = NUMBER OF COLUMNS IN THE ARRAY VALID
C
       DATA
     $      EQNFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,GRDFIL,HEADER,MOD
     $     /     1,     2,     3,     4,     7,     8,     9,
     $      COMMA,SLASH,LPAREN,RPAREN,BLANK,AMP,E  ,DOT,STAR,DOL
```

```
      SUBROUTINE INCODE (CODE,CODIM,CLEN,PTR)
C
C  INCODE READS THE ENCODED ELLPACK PROGRAM FROM IO UNIT 5. THE
C  IS ASSUMED TO START ON A NEW CARD AND CARDS ARE READ UNTIL TH
C  INPUT IS COMPLETE (I.E. 8 SLASHES ARE FOUND). THE VARIABLE CO
C  GIVES THE MAX ALLOWABLE LENGTH OF THE INPUT STRING. BLANKS AR
C  SQUEEZED OUT OF THE INPUT AND THE RESULTING STRING IS PLACED
C  THE ARRAY CODE. THE LENGTH OF THE STRING IS THEN CLEN.
C  PTR IS INITIALIZED TO 1.
C
      INTEGER CODE(400),CODIM,CLEN,PTR
C
C  GLOBAL VARIABLES
C
      INTEGER EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,PARAM,CO
     $        SLASH,LPAREN,RPAREN,BLANK,AMP,E,ZERO,ONE,TWO,THREE
     $        MODNAM,DMTYPE,VALID,LEVEL,DOT,NULL,NRMODS,
     $        NUDCOL,STAR,DOLLAR,HEADER,DIGIT,LETTER,MODATA,DASH
      LOGICAL FATAL
      COMMON / GLOBAL   EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMF
     $                  HEADER,COMMA,SLASH,LPAREN,RPAREN,BLANK,(
     $                  DOT,DOLLAR,STAR,NULL,ZERO,ONE,TWO,THREE,
     $                  MODATA,DASH,
     $                  PARAM(10,35),MODNAM(70,10),DMTYPE(70,15
     $                  VALID(70,20),IZEROS(15),DIGIT(10),LETTE
     $                  LEVEL,NRMODS,NUDCOL,FATAL
C
      PTR = 1
      IBEGIN = 1
      IEND = 80
      NSLASH = 0
      NONBLK = 0
   10 CONTINUE
C        CHECK IF INPUT IS TOO LONG
         IF (IEND .GT. CODIM)  GO TO 600
         READ(5,1000) (CODE(I),I=IBEGIN,IEND)
         IF (CODE(1) .EQ. DOLLAR)  GO TO 500
         DO 20 I=IBEGIN,IEND
C           IGNORE BLANKS
            IF (CODE(I) .EQ. BLANK)  GO TO 20
            NONBLK = NONBLK+1
            CODE(NONBLK) = CODE(I)
C           COUNT SLASHES
            IF (CODE(NONBLK) .EQ. SLASH)  NSLASH = NSLASH+1
   20    CONTINUE
         IBEGIN = NONBLK+1
         IEND = IBEGIN+79
C        CHECK IF INPUT IS COMPLETE
         IF (NSLASH .LT. 8)  GO TO 10
      CLEN = NONBLK
      IF (LEVEL .GT. 0)  WRITE(6,2000) (CODE(I),I=1,CLEN)
      RETURN
C
C  END OF DATA
C
  500 CONTINUE
      CLEN = 0
      RETURN
C
C  ERROR IN INPUT
C
  600 CONTINUE
      FATAL = .TRUE.
```

```
C
C  NUMBER CONVERTS CHARACTERS IN THE ARRAY CODE (80A1) TO A DECI
C  INTEGER.  CONVERSION BEGINS WITH THE CHARACTER CODE(PTR) AND
C  STOPS WHEN ONE OF THE CHARACTERS /,()$ IS ENCOUNTERED.
C
      INTEGER CODE(400),PTR,CHAR
C
C  GLOBAL VARIABLES
C
      INTEGER EOMFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,PARAM,CO
     $        SLASH,LPAREN,RPAREN,BLANK,AMP,E,ZERO,ONE,TWO,THREE
     $        MODNAM,DMTYPE,VALID,LEVEL,DOT,NULL,NRMODS,
     $        NUDCOL,STAR,DOLLAR,HEADER,DIGIT,LETTER,MODATA,DASH
      LOGICAL FATAL
      COMMON / GLOBAL / EOMFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFI
     $                  HEADER,COMMA,SLASH,LPAREN,RPAREN,BLANK,A
     $                  DOT,DOLLAR,STAR,NULL,ZERO,ONE,TWO,THREE,
     $                  MODATA,DASH,
     $                  PARAM(10,35),MODNAM(70,10),DMTYPE(70,15)
     $                  VALID(70,20),IZEROS(15),DIGIT(10),LETTER
     $                  LEVEL,NRMODS,NUDCOL,FATAL
C
      N = 0
      K = PTR
   10 CONTINUE
      CHAR = CODE(K)
C     CHECK FOR END OF NUMBER
      IF ((CHAR .EQ. SLASH ) .OR. (CHAR .EQ. COMMA ) .OR.
     $    (CHAR .EQ. RPAREN) .OR. (CHAR .EQ. LPAREN) .OR.
     $    (CHAR .EQ. DOLLAR) .OR. (CHAR .EQ. DASH  ) .OR.
     $    (CHAR .EQ. DOT   ))                                  GO TO
C        FIND WHICH DIGIT IS IN CHAR
         DO 15 J=1,10
            IDIGIT = J-1
            IF (CHAR .EQ. DIGIT(J))  GO TO 20
   15    CONTINUE
C        NOT A DIGIT
         CALL ERREND(2,CHAR)
   20    CONTINUE
C        DIGIT FOUND
         N = 10*N + IDIGIT
         K = K+1
         GO TO 10
   50 CONTINUE
      NUMBER = N
      PTR = K+1
      IF (LEVEL .GT. 3)  WRITE(6,3000) NUMBER
      RETURN
C
 3000 FORMAT(8H NUMBER=,I10)
C
      END
      SUBROUTINE GETLN (FILE,LINE)
C
C  GETLN RETURNS THE NEXT LINE (80A1) FROM THE GIVEN FILE.
C  IF FILE IS THE MACRO FILE THE LINE IS SCANNED FOR DUMMY
C  PARAMETERS WHICH ARE THEN EXPANDED.
C
      INTEGER FILE,LINE(80),NLINE(80),OUT
C
C  GLOBAL VARIABLES
C
      INTEGER EOMFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,PARAM,C
```

```
C
      READ(FILE,1000) (LINE(I),I=1,80)
      IF (LINE(1) .EQ. DOLLAR)  GO TO 900
      IF (LEVEL .GT. 3)  WRITE(6,3000) (LINE(I),I=1,80)
      IF (FILE .NE. MACFIL)  GO TO 500
C
C  EXPAND MACRO PARAMETERS
C
         IN = 1
         OUT = 0
   10    CONTINUE
         IF ((IN .GT. 80) .OR. (OUT .GE. 80))  GO TO 100
C           SCAN FOR AMPERSAND (SIGNALS PARAMETER)
            IF (LINE(IN) .EQ. AMP)  GO TO 50
               OUT = OUT+1
               NLINE(OUT) = LINE(IN)
               IN = IN+1
               GO TO 10
C              HAVE A PARAMETER
   50          CONTINUE
               INP1 = IN+1
               DO 55 I=1,10
                  NPARAM = I
                  IF (LINE(INP1) .EQ. LETTER(I))  GO TO 58
   55          CONTINUE
C              UNDEFINED PARAMETER
               CALL ERREND(6,LINE(INP1))
   58          CONTINUE
               K = 1
C              REPLACE DUMMY PARAMETER WITH ACTUAL PARAMETER
   60          CONTINUE
               IF ((K .GT. 30) .OR. (PARAM(NPARAM,K) .EQ. BLANK
     $             GO TO 80
                  OUT = OUT+1
                  NLINE(OUT) = PARAM(NPARAM,K)
                  K = K+1
                  GO TO 60
   80          CONTINUE
               IN = IN+2
               GO TO 10
  100    CONTINUE
C
C  RETURN LINE WITH EXPANDED PARAMETERS
C
      DO 120 I=1,80
         LINE(I) = BLANK
  120 CONTINUE
      DO 140 I=1,OUT
         LINE(I) = NLINE(I)
  140 CONTINUE
      IF (LEVEL .GT. 3)  WRITE(6,3001) (LINE(I),I=1,80)
  500 CONTINUE
      RETURN
C
C  ERROR EXIT
C
  900 CONTINUE
      CALL ERREND(3,FILE)
      RETURN
C
 1000 FORMAT(80A1)
 3000 FORMAT(6H LINE=,80A1)
 3001 FORMAT(15H EXPANDED LINE=,80A1)
```

```
C  GLOBAL VARIABLES
C
      INTEGER EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,PARAM,CL
     $        SLASH,LPAREN,RPAREN,BLANK,AMP,E,ZERO,ONE,TWO,THREE
     $        MODNAM,DMTYPE,VALID,LEVEL,DOT,NULL,NRMODS,
     $        NUDCOL,STAR,DOLLAR,HEADER,DIGIT,LETTER,MODATA,DASH
      LOGICAL FATAL
      COMMON / GLOBAL / EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMF
     $                  HEADER,COMMA,SLASH,LPAREN,RPAREN,BLANK,
     $                  DOT,DOLLAR,STAR,NULL,ZERO,ONE,TWO,THREE,
     $                  MODATA,DASH,
     $                  PARAM(10,35),MODNAM(70,10),DMTYPE(70,15
     $                  VALID(70,20),IZEROS(15),DIGIT(10),LETTE
     $                  LEVEL,NRMODS,NUDCOL,FATAL
C
C  PICK UP MACRO NUMBER
C
      LPTR = 8
      MACNO = NUMBER(LINE,LPTR)
      IF (LEVEL .GT. 1)  WRITE(6,3000) MACNO
C
C  ASSEMBLE TABLE OF ACTUAL PARAMETERS
C
C     FIRST BLANK OUT THE PARAMETER TABLE
      DO 5 I=1,10
         DO 5 J=1,30
            PARAM(I,J) = BLANK
    5 CONTINUE
      NCH = 0
      NPARAM = 1
   10 CONTINUE
C     CHECK FOR END OF LAST PARAMETER
      IF (LINE(LPTR) .EQ. SLASH)  GO TO 50
C        CHECK FOR END OF CURRENT PARAMETER
         IF (LINE(LPTR) .EQ. COMMA)  GO TO 40
            NCH = NCH+1
            PARAM(NPARAM,NCH) = LINE(LPTR)
            LPTR = LPTR+1
            GO TO 10
   40    CONTINUE
         NPARAM = NPARAM+1
         NCH = 0
         LPTR = LPTR+1
         GO TO 10
   50 CONTINUE
      IF (LEVEL .GT. 1)  WRITE(6,3001) NPARAM,((PARAM(I,J),
     $                                     J=1,30),I=1,NPARAM)
C
C   SKIP RECORDS TO MACRO
C
      FILE=MACFIL
      REWIND MACFIL
      CALL SKIPR(MACNO,MACFIL)
C
C   VALIDATE MACRO NUMBER
C
      READ(MACFIL,3002)MNCHEK
      IF (MACNO .NE. MNCHEK) CALL ERREND(8,FILE)
C
C  READ FIRST LINE OF MACRO
C
      CALL GETLN(FILE,LINE)
      RETURN
```

```
C  ARRAY LINE.
C
      INTEGER IFILE,OFILE,LINE(80),PUTC,ENDC
      LOGICAL NEEDLN
C
C  GLOBAL VARIABLES
C
      INTEGER EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,PARAM,C
     $        SLASH,LPAREN,RPAREN,BLANK,AMP,E,ZERO,ONE,TWO,THREE
     $        MODNAM,DMTYPE,VALID,LEVEL,DOT,NULL,NRMODS,
     $        NUDCOL,STAR,DOLLAR,HEADER,DIGIT,LETTER,MODATA,DASH
      LOGICAL FATAL
      COMMON / GLOBAL / EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMF
     $                  HEADER,COMMA,SLASH,LPAREN,RPAREN,BLANK,
     $                  DOT,DOLLAR,STAR,NULL,ZERO,ONE,TWO,THREE,
     $                  MODATA,DASH,
     $                  PARAM(10,35),MODNAM(70,10),DMTYPE(70,15
     $                  VALID(70,20),IZEROS(15),DIGIT(10),LETTE
     $                  LEVEL,NRMODS,NUDCOL,FATAL
C
      IF (NEEDLN)  CALL GETLN(IFILE,LINE)
   10 CONTINUE
      IF (LINE(1) .EQ. ENDC)  GO TO 30
         IF (PUTC .NE. NULL)  LINE(1) = PUTC
         IF (LINE(2) .NE. DOT)  GO TO 20
            LINE(1) = DOT
            LINE(2) = BLANK
   20    CONTINUE
         WRITE(OFILE,1000) (LINE(I),I=1,80)
         CALL GETLN(IFILE,LINE)
         GO TO 10
   30 CONTINUE
      RETURN
C
 1000 FORMAT(80A1)
C
      END
      SUBROUTINE WRTMOD (MODULE,CODE,CLEN,PTR)
C
C  WRTMOD GENERATES A MODULE SEGMENT (DIS, INDEX OR SCL) WHICH
C  MODULE NUMBER MODULE. THE ENCODED ELLPACK PROGRAM CODE IS CH
C  FOR THE PRESENCE OF MODULE PARAMETERS WHICH ARE ALSO WRITTEN
C
      INTEGER MODULE,CODE(CLEN),PTR,CLEN,PSTART,PSTOP
C
C  GLOBAL VARIABLES
C
      INTEGER EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,PARAM,C
     $        SLASH,LPAREN,RPAREN,BLANK,AMP,E,ZERO,ONE,TWO,THRE
     $        MODNAM,DMTYPE,VALID,LEVEL,DOT,NULL,NRMODS,
     $        NUDCOL,STAR,DOLLAR,HEADER,DIGIT,LETTER,MODATA,DAS
      LOGICAL FATAL
      COMMON / GLOBAL / EQNFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMF
     $                  HEADER,COMMA,SLASH,LPAREN,RPAREN,BLANK,
     $                  DOT,DOLLAR,STAR,NULL,ZERO,ONE,TWO,THREE
     $                  MODATA,DASH,
     $                  PARAM(10,35),MODNAM(70,10),DMTYPE(70,15
     $                  VALID(70,20),IZEROS(15),DIGIT(10),LETTE
     $                  LEVEL,NRMODS,NUDCOL,FATAL
C
C  WRITE OUT MODULE NAME
C
      WRITE(PGMFIL,2000) (MODNAM(MODULE,K),K=1,10)
```

```
C        PARAMETERS ARE NOW IN CODE(PSTART+1) TO CODE(PSTOP-1)
   20    CONTINUE
         PSTART = PSTART+1
         PSTOP = PSTOP-1
         IF (PSTART .GE. PSTOP)  GO TO 100
         WRITE(PGMFIL,2001) LPAREN,(CODE(I),I=PSTART,PSTOP),RPA
         PTR = PSTOP+2
C
  100 CONTINUE
      RETURN
C
 2000 FORMAT(1H ,14X,16A4)
 2001 FORMAT(1H ,20X,50A1)
C
      END
      SUBROUTINE COMPAT(PTYPE,DISMOD,NRTYPE,NX,NY,NZ,DISPAR,CODE
C
C  COMPAT USES THE PROBLEM TYPE INFO (PTYPE) AND DISRETIZATION
C  INFO (DMTYPE) TO DETERMINE WHETHER MODULE DISMOD IS COMPATIBL
C  WITH THE GIVEN PDE PROBLEM. THE NRTYPE (NOW 15) ITEMS IN THE
C  PTYPE AND IN EACH ROW OF THE TABLE DMTYPE HAVE THE FOLLOWING
C  MEANINGS --
C
C          VALUE        PTYPE MEANING              DMTYPE MEANING
C          -----    ----------------------   ----------------------
C            0       ITEM NOT PRESENT         ITEM MUST NOT BE PRES
C            1       ALWAYS MATCHES           ALWAYS MATCHES
C            2       ITEM PRESENT             ITEM MUST BE PRESENT
C
C  THE 14 ITEMS CURRENTLY CHECKED FOR COMPATIBILITY ARE
C
C       CONCERNING THE OPERATOR
C           1     TWO DIMENSIONAL
C           2     THREE DIMENSIONAL
C           3     POISSON EQUATION
C           4     LAPLACE EQUATION
C           5     UX OR UY TERMS
C           6     CONSTANT COEFFICIENTS
C           7     SELF-ADJOINT FORM
C           8     HOMOGENEOUS
C           9     UXY TERM
C       CONCERNING THE BOUNDARY CONDITIONS
C          10     DIRICHLET PROBLEM
C          11     SOME NORMAL DERIVATIVE CONDITIONS
C          12     SOME MIXED CONDITIONS
C          13     HOMOGENEOUS
C       CONCERNING THE GRID
C          14     HX=HY(=HZ)
C          15     UNIFORM GRID
C
C  OTHER CHECKING THAT IS DONE IS
C
C     1. FOR UNIFORM GRIDS, THE PROBLEM DIMENSION AND THE GRID
C        DIMENSION ARE CHECKED FOR CONSISTENCY.
C     2. FOR THE FFT 9-POINT DISCRETIZATION MODULE WE CHECK FOR
C           A. PARAMETERS MUST BE PRESENT
C           B. IF IORDER=6 IS THE PARAMETER PROBLEM MUST BE POI
C           C. THE GRID MUST BE A POWER OF 2
C
C
      INTEGER DISMOD,PTYPE(NRTYPE),NRBAD,INCOMP(20),DISPAR,CLEN
     $        CODE(CLEN)
C
```

```
     $                        LEVEL,NRMODS,NUDCOL,FATAL
C
C  CHECK IF A DISCRETIZATION MODULE IS PRESENT   --  IF NOT, WA
C
      IF (DISMOD .EQ. 0)  GO TO 600
C
C  DETERMINE WHICH ROW OF DMTYPE CONTAINS INFO ABOUT MODULE DIS
C
      DO 10 I=1,NUDCOL
         K = I
         IF (VALID(1,K) .EQ. DISMOD)  GO TO 20
   10 CONTINUE
      GO TO 510
C
C  COMPARE PTYPE(.) AND DMTYPE(K,.) FOR COMPATIBILITY
C
   20 CONTINUE
      NRBAD = 0
      DO 50 I=1,NRTYPE
         IF (PTYPE(I) .EQ. ONE)  GO TO 50
         IF (DMTYPE(K,I) .EQ. ONE)  GO TO 50
         IF (PTYPE(I) .EQ. DMTYPE(K,I))  GO TO 50
            NRBAD = NRBAD+1
            INCOMP(NRBAD) = I
   50 CONTINUE
      IF (NRBAD .NE. 0)  GO TO 500
C
C  NOW CHECK IF NON-UNIFORM GRID IS COMPATIBLE WITH PROBLEM DIM
C
         IF (PTYPE(15) .EQ. ZERO)  GO TO 95
         IF (((PTYPE(1) .EQ. TWO) .AND. (NZ .LE. 1)) .OR.
     $        ((PTYPE(2) .EQ. TWO) .AND. (NZ .GT. 1)))  GO TO 95
            FATAL = .TRUE.
            WRITE(PGMFIL,2018) STAR,STAR,STAR
            WRITE(6,2018) BLANK,BLANK,BLANK
   95    CONTINUE
C
C-----------------------------------------------------------------
C
C  THE FOLLOWING ARE SPECIAL CHECKS FOR PARTICULAR MODULES
C
C    --- THIS SECTION OF CODE MUST BE CHANGED WHEN MODULES ---
C    --- ARE RENUMBERED.                                  ---
C
C
C  CHECK FFT 9-POINT
C
      IF (DISMOD .NE. 9)  GO TO 110
         IF (CODE(DISPAR-1) .NE. COMMA)  GO TO 520
         IORDER = CODE(DISPAR+7)
         IF ((IORDER .EQ. SIX) .AND. (PTYPE(3) .NE. TWO))  GO T
         IF (MOD(NX-1,2) .NE. 0)  GO TO 540
  110 CONTINUE
C
C-----------------------------------------------------------------
C
C
C  NORMAL EXIT  --  MODULE COMPATIBLE WITH PROBLEM
C
      WRITE(PGMFIL,2000)
      GO TO 900
C
C  ABNORMAL EXITS
```

```
      WRITE(6,2002) BLANK,BLANK,BLANK,DISMOD
      GO TO 900
C
C     FFT 9-POINT PROBLEMS
  520 CONTINUE
      FATAL = .TRUE.
      WRITE(PGMFIL,2003) STAR,STAR,STAR
      WRITE(6,2003) BLANK,BLANK,BLANK
      GO TO 900
  530 CONTINUE
      FATAL = .TRUE.
      WRITE(PGMFIL,2004) STAR,STAR,STAR
      WRITE(6,2004) BLANK,BLANK,BLANK
      GO TO 900
  540 CONTINUE
      FATAL = .TRUE.
      WRITE(PGMFIL,2005) STAR,STAR,STAR
      WRITE(6,2005) BLANK,BLANK,BLANK
      GO TO 900
C
C     NO DISCRETIZATION MODULE PRESENT  -- PRINT WARNING
  600 CONTINUE
      WRITE(PGMFIL,2007) STAR,STAR,STAR
      WRITE(6,2007) BLANK,BLANK,BLANK
C
  900 CONTINUE
      RETURN
C
 2000 FORMAT(1H* / 41H*        PROBLEM/DIS MODULE ARE COMPATIBLE
 2001 FORMAT(A1/A1,40H------ ERROR IN ENCODED PROGRAM
     $       /A1,47H       PROBLEM/DIS MODULE INCOMPATIBLE IN I'
     $       ,2013)
 2002 FORMAT(A1/A1,40H------ ERROR IN ENCODED PROGRAM
     $       /A1,7X,6HMODULE,I3,31H IS NOT A DISCRETIZATION MODU
 2003 FORMAT(A1/A1,40H------ ERROR IN ENCODED PROGRAM
     $          /A1,40H       FFT 9-POINT MUST HAVE A PARAMETER
 2004 FORMAT(A1/A1,40H------ ERROR IN ENCODED PROGRAM
     $          /A1,40H       FFT 9-POINT MUST HAVE POISSON EQN
     $             ,40H WHEN IORDER=6                           )
 2005 FORMAT(A1/A1,40H------ ERROR IN ENCODED PROGRAM
     $          /A1,40H       FFT 9-POINT MUST HAVE GRID OF SIZ
     $             ,40HE (2**N)-1                               )
 2007 FORMAT(A1/A1,40H------ WARNING
     $          /A1,40H       NO DISCRETIZATION PRESENT        )
 2018 FORMAT(A1/A1,22H------ ERROR IN INPUT
     $          /A1,44H       GRID DOES NOT MATCH PROBLEM DIMENS
C
      END
      SUBROUTINE LEGAL (DISMOD,INDMOD,SOLMOD)
C
C  LEGAL DETERMINES WHETHER THE SEQUENCE OF MODULES (DISMOD,INDM
C  SOLMOD) IS LEGAL, I.E. WHETHER THESE MODULES ARE COMPATIBLE.
C  THE SET OF ALL LEGAL SEQUENCES IS REPRESENTED BY THE CONNECT
C  TABLE VALID.  MODULE J MAY FOLLOW MODULE I IF AND ONLY IF J
C  APPEARS IN ROW I+1 OF THE ARRAY VALID. THE LAST ENTRY IN EAC
C  ROW IS ZERO. ROW 1 CONTAINS THE INDICES OF THOSE MODULES WHI
C  ARE PRECEEDED BY NO OTHERS.
C
      INTEGER DISMOD,INDMOD,SOLMOD,MODULE(4),SEQLEN
      LOGICAL FOUND
C
C  GLOBAL VARIABLES
C
```

```
C
      SEQLEN = 1
      MODULE(1) = 0
      IF (DISMOD .EQ. 0)  GO TO 10
         SEQLEN = SEQLEN+1
         MODULE(SEQLEN) = DISMOD
   10 CONTINUE
      IF (INDMOD .EQ. 0)  GO TO 20
         SEQLEN = SEQLEN+1
         MODULE(SEQLEN) = INDMOD
   20 CONTINUE
      IF (SOLMOD .EQ. 0)  GO TO 30
         SEQLEN = SEQLEN+1
         MODULE(SEQLEN) = SOLMOD
   30 CONTINUE
C
C  CHECK FOR LEGALITY
C
      IF (SEQLEN .EQ. 1)  GO TO 210
      NRINT = SEQLEN-1
      DO 50 K=1,NRINT
C        CHECK IF MODULE(K+1) MAY FOLLOW MODULE(K)
         NEXT = MODULE(K+1)
         NOW = MODULE(K)+1
         IF (LEVEL .GT. 3)  WRITE(6,3000) MODULE(K),(VALID(NOW,
     $                                          II=1,20),NEXT
         FOUND = .FALSE.
         I = 1
   40    CONTINUE
C        SEARCH FOR VALUE NEXT IN ROW NOW OF VALID
         IF (VALID(NOW,I) .EQ. 0)  GO TO 45
             IF (VALID(NOW,I) .EQ. NEXT)  FOUND = .TRUE.
             I = I+1
             GO TO 40
   45    CONTINUE
         IF (.NOT. FOUND)  GO TO 220
   50 CONTINUE
      N = MODULE(SEQLEN)+1
      IF (VALID(N,1) .NE. 0)  GO TO 220
C
C  NORMAL EXIT  --  EXECUTION SEQUENCE LEGAL
C
      WRITE(PGMFIL,2000)
      GO TO 300
C
C  ABNORMAL EXIT  --  EXECUTION SEQUENCE ILLEGAL
C
  210 CONTINUE
      WRITE(PGMFIL,2001) STAR,STAR,STAR
      WRITE(6,2001) BLANK,BLANK,BLANK
      GO TO 300
  220 CONTINUE
      FATAL = .TRUE.
      WRITE(PGMFIL,2002) STAR,STAR,STAR
      WRITE(6,2002) BLANK,BLANK,BLANK
C
  300 CONTINUE
      RETURN
C
 2000 FORMAT(1H*/32H*        MODULE SEQUENCE IS LEGAL)
 2001 FORMAT(A1/A1,33H------ WARNING
     $          A1,31H        MODULE SEQUENCE IS NULL    )
```

```
C
C  GLOBAL VARIABLES
C
      INTEGER EONFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFIL,PARAM,CO
     $        SLASH,LPAREN,RPAREN,BLANK,AMP,E,ZERO,ONE,TWO,THREE
     $        MODNAM,DMTYPE,VALID,LEVEL,DOT,NULL,NRMODS,
     $        NUDCOL,STAR,DOLLAR,HEADER,DIGIT,LETTER,MODATA,DASH
      LOGICAL FATAL
      COMMON / GLOBAL / EONFIL,GRDFIL,OPTFIL,OUTFIL,MACFIL,PGMFI
     $                  HEADER,COMMA,SLASH,LPAREN,RPAREN,BLANK,A
     $                  DOT,DOLLAR,STAR,NULL,ZERO,ONE,TWO,THREE,
     $                  MODATA,DASH,
     $                  PARAM(10,35),MODNAM(70,10),DMTYPE(70,15)
     $                  VALID(70,20),IZEROS(15),DIGIT(10),LETTER
     $                  LEVEL,NRMODS,NUDCOL,FATAL
C
      INTEGER ERRMSG,IPAR
C
      CALL ONSW(1)
      CALL REMARK(32H**********************************    )
      CALL REMARK(32HERROR-IN-ENCODED-ELLPACK-PROGRAM     )
      CALL REMARK(32H**********************************    )
      WRITE(6,2000)
      END FILE PGMFIL
C
      GO TO (10,20,30,40,50,60,70,80,90,100,110,120,130),ERRMSG
C
   10    CONTINUE
         STOP
   20    CONTINUE
         WRITE(6,2020) IPAR
         STOP
   30    CONTINUE
         WRITE(6,2030) IPAR
         STOP
   40    CONTINUE
         WRITE(6,2040) IPAR
         STOP
   50    CONTINUE
         WRITE(6,2050) IPAR
         STOP
   60    CONTINUE
         WRITE(6,2060) IPAR
         STOP
   70    CONTINUE
         WRITE(6,2070) IPAR
         STOP
   80    CONTINUE
         WRITE(6,2080) IPAR
         STOP
   90    CONTINUE
         WRITE(6,2090) IPAR
         STOP
  100    CONTINUE
         WRITE(6,2100) IPAR
         STOP
  110    CONTINUE
         WRITE(6,2110) IPAR
         STOP
  120    CONTINUE
         WRITE(6,2120) IPAR
         STOP
  130    CONTINUE
```

```
 2080 FORMAT(8X,40HINVALID MACRO NUMBER SPECIFIED FOR FILE ,I3)
 2090 FORMAT(8X,42HINVALID OPTIONS NUMBER SPECIFIED FOR FILE ,I3
 2100 FORMAT(8X,39HINVALID GRID NUMBER SPECIFIED FOR FILE ,I3)
 2110 FORMAT(8X,49HINVALID OUTPUT SEGMENT NUMBER SPECIFIED FOR F
 2120 FORMAT(8X,41HINVALID RECORD NUMBER SPECIFIED FOR FILE ,I3)
 2130 FORMAT(8X,48HNO PROBLEM NUMBERED SPECIFIED IN ENCODED PROC
C
      END
*PREFIX,$
          IDENT  SKIP
*
*         AUTHOR:  JOHN F. BROPHY
*                  DEPARTMENT OF MATHEMATICS
*                  PURDUE UNIVERSITY
*                  NOVEMBER 1, 1979
*
*            THIS CDC ASSEMBLY LANGUAGE (COMPASS) ROUTINE PROVII
*         AUTOMATIC SKIPPING OF FILES AND RECORDS. THE CALLING S
*         ARE GIVEN BY:
*
*            CALL SKIPF(NSKIPS,FILE),  OR  SKIPR(NSKIPS,FILE)
*
*         WHERE NSKIPS IS THE NUMBER OF FILES OR RECORDS RESPECT
*         TO BE SKIPPED IN FILE.  SEE THE DISCUSSION OF SKIPPIN
*         AND RECORDS IN THE COMMENTS IN GENPGM.
*
          ENTRY  SKIPR,SKIPF
 SKIPR    PS
          SB2    -B2
          RJ     =XGETBA
          SA1    B1
          ZR     X1,RDR
          SKIPF  X2,X1,R
 RDR      READ   X2,R
          EQ     SKIPR
 SKIPF    PS
          SB2    -B2
          RJ     =XGETBA
          SA1    B1
          ZR     X1,RDF
          SKIPFF X2,X1,R
 RDF      READ   X2,R
          EQ     SKIPF
          END
*
*            THE FOLLOWING FORTRAN ROUTINES ARE PROVIDED AS SUBS
*         FOR THE COMPASS ROUTINE SKIP FOR INSTALLATIONS WHERE
*         FILE AND RECORD SKIPPING CANNOT BE IMPLEMENTED.
*
*       SUBROUTINE SKIPF(NSKIPS,FILE)
*C
*C        THIS ROUTINE SKIPS NSKIPS FILES IN FILE BY SCANNING FI
*C     END-OF-FILE MARKER 'EOF IN A4 FORMAT STARTING IN COLUMN
*C
*       INTEGER FILE,EOF,EOFCHEK
*       DATA EOF/4H'EOF/
*       NSKIPED=0
*C
*100    READ(FILE,300)EOFCHEK
*C
*       IF (EOFCHEK .EQ. 4H$   ) CALL ERREND(5,FILE)
*       IF (EOFCHEK .NE. EOF) GO TO 100
*       NSKIPED=NSKIPED+1
```

```
*C
*       NSKIPED=0
*       MNSKIPS=NSKIPS
*       IF (FILE .EQ. EOMFIL) MNSKIPS=MNSKIPS+1
*C
*100    READ(FILE,300)EORCHEK
*C
*       IF (EORCHEK .EQ. 4HS    ) CALL ERREND(12,FILE)
*       IF (EORCHEK .EQ. EOF) CALL ERREND(12,FILE)
*       IF (EORCHEK .NE. EOR) GO TO 100
*       NSKIPED=NSKIPED+1
*       IF (NSKIPED .LT. MNSKIPS) GO TO 100
*       RETURN
*C
*300    FORMAT(A4)
*C
*       END
```

EDNFIL CONTAINS THE INFORMATION NEEDED TO GENERATE
BOUNDARY AND FORTRAN SEGMENTS OF AN ELLPACK PROGRAM.

EDNFIL IS DIVIDED UP INTO (SUB) FILES WHICH ARE IN
UP INTO RECORDS. THE FILES ARE NUMBERED 0, 1, 2,...E
RECORDS WITHIN EACH FILE ARE SIMILARLY NUMBERED 0, 1,
THE END-OF-FILE AND END-OF-RECORD ARE DESIGNATED BY *E
RESPECTIVELY ('EOF AND 'EOF RESPECTIVELY ON NON-CDC IN

FILE 0 CONTAINS A DESCRIPTION OF THE PURPOSE AND OR
EDNFIL. FILES 1, 2, 3,... CONTAIN THE INFORMATION NEE
ATE THE ELLPACK PROGRAM CORRESPONDING TO PROBLEMS 1, 2
RESPECTIVELY OF THE ELLPACK PDE POPULATION. AS MENTION
FILES ARE DIVIDED INTO RECORDS WHICH HAVE THE FOLLOWIN

RECORD 0 CONTAINS THE ELLPACK PDE POPULATION PRO
RECORDS 1, 2,... MAY TAKE ONE OF THE FOLLOWING FORM

ALTERNATIVE 1:

LINE 1: LINE 1 IS EITHER BLANK OR CONTAINS DESCRIP
TION SOME OF WHICH BECOMES A COMMENT IN THE GENERAT
PROGRAM. IF NON-BLANK, LINE 1 CONTAINS THE PARAMET
IN I3 FORMAT STARTING IN COLUMN 16. FOLLOWED BY THE
OF THE PROBLEM IN FREE FORMAT STARTING IN COLUMN 19

LINE 2: LINE 2 CONTAINS THE COMPLEXITY MEASURES OF
PROBLEM FEATURES IN FREE FORMAT STARTING IN COLUMN
INFORMATATION BECOMES A COMMENT IN THE GENERATED EL
GRAM.

LINE 3: LINE 3 CONTAINS THE PROBLEM TYPE INFORMATI
IN COLUMN 11 WHICH IS USED TO TEST THE COMPATIBILIY
BLEM WITH THE ELLPACK ROUTINES SELECTED IN THE ENCO
(SEE ROUTINE COMPAT). THIS INFORMATION BECOMES A C
GENERATED ELLPACK PROGRAM.

COMPAT USES THE PROBLEM TYPE INFO (PTYPE) AND DISRE
MODULE INFO (DMTYPE) TO DETERMINE WHETHER MODULE DI
COMPATIBLE WITH THE GIVEN PDE PROBLEM. THE NRTYPE (
ITEMS IN THE ARRAY PTYPE AND IN EACH ROW OF THE TAB
HAVE THE FOLLOWING MEANINGS:

VALUE	PTYPE MEANING	DMTYPE ME
0	ITEM NOT PRESENT	ITEM MUST NOT
1	ALWAYS MATCHES	ALWAYS MATCHES
2	ITEM PRESENT	ITEM MUST BE P

THE 14 ITEMS CURRENTLY CHECKED FOR COMPATIBILITY AR

CONCERNING THE OPERATOR
1 TWO DIMENSIONAL
2 THREE DIMENSIONAL
3 POISSON EQUATION
4 LAPLACE EQUATION
5 UX OR UY TERMS
6 CONSTANT COEFFICIENTS
7 SELF-ADJOINT FORM
8 HOMOGENEOUS

REMAINING LINES: THE REMAINING LINES OF THE RECOR
INFORMATION COPIED INTO THE EQUATION, BOUNDARY AND
MENTS OF THE GENERATED ELLPACK PROGRAM IN THE ABOV
THE EQUATION, BOUNDARY AND FORTRAN CARDS ARE MARKE
OR 3 IN COLUMN 1 RESPECTIVELY. NOTE THAT FORTRAN
BEGIN IN COLUMN 7 AS USUAL AND THAT A FORTRAN COMM
WRITTEN BY TYPING A C IN COLUMN 2. FOR PORTABILIT
THE END-OF-RECORD IS DETECTED BY THE PRESENCE OF A
IN COLUMN 1.

ALTERNATIVE 2:

LINE 1: LINE 1 CONTAINS THE PARAMETER SET NUMBER
STARTING IN COLUMN 16, FOLLOWED BY THE PARAMETERS
BLEM IN FREE FORMAT STARTING IN COLUMN 19.

LINE 2: LINE 2 CONTAINS THE COMPLEXITY MEASURES O
PROBLEM FEATURES IN FREE FORMAT STARTING IN COLUMN
INFORMATION BECOMES A COMMENT IN THE GENERATED ELL
GRAM.

LINE 3: LINE 3 IS OF THE FORM EXPAND N/N1,N2,....
IN COLUMN 1. THIS SPECIFIES THAT MACRO N (THE NTH
MACFIL) SHOULD BE EXPANDED WITH ACTUAL PARAMETERS
REPLACING THE DUMMIES &A,&B,&C,...ETC. THE PARAME
CONTAIN ANY CHARACTERS EXCEPT A BLANK, COMMA OR SL
MAY BE NO LONGER THAN 30 CHARACTERS.

RELATED FILES: GENPGM, MACFIL, OPTFIL, GRDFIL, OUTFI

```
'EOR
'EOF
**************
* PROBLEM   1 *
**************
'EOR
*PARAMETER SET   1(A=0.0)
*          000.04    006.05    000.00    006.05
*          2000002002002
1          TWO DIMENSIONS $ SELF-ADJOINT
1          EXP(X*Y)UXX$ + EXP(-X*Y)UYY$ - 1./(1.+X+Y)U$ =
2          DIRICHLET $ HOMOGENEOUS
2          X=0. , U=0.
2          X=1. , U=0.
2          Y=0. , U=0.
2          Y=1. , U=0.
3      FUNCTION F(X,Y)
3      COMMON /CONCOM/ PI
3      DATA PI/3.14159265358979/
3      PX = PI*X
3      PY = PI*Y
3      SPX = SIN(PX)
3      SPY = SIN(PY)
3      EXY = EXP(X*Y)
3      F = .75*(EXY*EXY*SPY*((2.*Y*Y-PI*PI)*SPX+3.*PI*Y*C
3     $  + PI*SPX*(X*COS(PY)-PI*SPY)-EXY*SPX*SPY/(1.+X+Y)
3      RETURN
3      END
3      FUNCTION TRUE(X,Y)
3      COMMON /CONCOM/ PI
3      TRUE = .75*EXP(X*Y)*SIN(PI*X)*SIN(PI*Y)
3      RETURN
```

```
EXPAND 1/0.1/
'EOR
*PARAMETER SET   3(A=1.0)
*          000.04    006.05    000.00    006.05
EXPAND 1/1.0/
'EOR
*PARAMETER SET   4(A=10.)
*          000.04    006.05    000.00    006.05
EXPAND 1/10./
```

```
'EOR
'EOF
***************
* PROBLEM   2 *
***************
'EOR
*
*          000.04    000.00    004.05    010.02
*          2000200000020
1          TWO DIMENSIONS
1          UXX$ + (1.+Y*Y)UYY$ - UX$ - (1.+Y*Y)UY$ = F(X,Y
2          MIXED
2          X=0. , MIXED = (1.)U + ( 1.)UX  =  0.27*EXP(Y)
2          X=1. , MIXED = (1.)U + (-1.)UX  =  0.
2          Y=0. , MIXED = (1.)U + ( 1.)UY  =  0.27*EXP(X)
2          Y=1. , MIXED = (1.)U + (-1.)UY  =  0.135*(ALOG(2
3      FUNCTION TRUE(X,Y)
3      TRUE = 0.135*(EXP(X+Y)+(X*X-X)**2*ALOG(1.+Y*Y))
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      F = 0.135*( (-4.*X*X*X+18.*X*X-14.*X+2.)*ALOG(1.+Y*Y
3     $     - 2.*((X*X-X)**2)*(Y*Y+Y**3+Y-1.)/(1.+Y*Y) )
3      RETURN
3      END
```

```
'EOR
'EOF
***************
* PROBLEM   3 *
***************
'EOR
*PARAMETER SET   1(A=1.5)
*          000.43    090.60    000.00    070.40
EXPAND 3/1.5/
'EOR
*PARAMETER SET   2(A=2.5)
*          000.35    080.50    000.00    060.20
EXPAND 3/2.5/
'EOR
*PARAMETER SET   3(A=3.5)
*          000.28    070.30    000.00    050.15
EXPAND 3/3.5/
'EOR
*PARAMETER SET   4(A=4.5)
*          000.23    055.20    000.00    040.20
EXPAND 3/4.5/
```

```
'EOR
'EOF
***************
* PROBLEM   4 *
***************
```

```
3     RETURN
3     END
-
'EOR
*PARAMETER SET   2(A=0.1)
*          000.02    004.00    000.00    006.00
EXPAND 4/0.1/
'EOR
*PARAMETER SET   3(A=1.0)
*          000.02    004.00    000.00    006.00
EXPAND 4/1.0/
'EOR
*PARAMETER SET   4(A=10.)
*          000.02    004.00    000.00    006.00
EXPAND 4/10./
-------------------------------------------------------------------
'EOR
'EOF
**************
* PROBLEM   5 *
**************
'EOR
*PARAMETER SET   1(A=0.0)
*          000.01    002.00    000.00    006.00
EXPAND 5/0./
'EOR
*PARAMETER SET   2(A=5.0)
*          000.01    002.00    000.00    006.00
EXPAND 5/5./
'EOR
*PARAMETER SET   3(A=8.0)
*          000.01    002.00    000.00    006.00
EXPAND 5/8./
'EOR
*PARAMETER SET   4(A=10.0)
*          000.03    010.00    000.00    006.00
EXPAND 5/10./
'EOR
*PARAMETER SET   5(A=20.0)
*          000.04    015.00    000.00    006.05
EXPAND 5/20./
'EOR
*PARAMETER SET   6(A=64.0)
*          000.06    030.00    000.00    006.00
EXPAND 5/64./
'EOR
*PARAMETER SET   7(A=100.0)
*          000.08    040.00    000.00    006.00
EXPAND 5/100./
-------------------------------------------------------------------
'EOR
'EOF
**************
* PROBLEM   6 *
**************
'EOR
*
*          008.16    040.20    000.00    008.30
*          2000001002002
1          TWO DIMENSIONS
1          UXX$ + UYY$ - COEFU(X,Y)U$ = F(X,Y)
2          DIRICHLET $ HOMOGENEOUS
2          X=0. , U=0.
```

```
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      COMMON /CONCOM/ PI, FOURPI
3      CFPX = COS(FOURPI*X)
3      T1   = 5.4 - CFPX
3      T1X  = FOURPI*SIN(FOURPI*X)
3      T1XX = FOURPI**2*CFPX
3      T2   = SIN(PI*X)
3      T2X  = PI*COS(PI*X)
3      T2XX = -PI**2*T2
3      T3   = Y**2 - Y
3      T3Y  = 2.*Y - 1.
3      T3YY = 2.
3      CFPY = COS(FOURPI*Y)
3      T4 = 5.4 - CFPY
3      T4Y  = FOURPI*SIN(FOURPI*Y)
3      T4YY = FOURPI**2*CFPY
3      PHI  = 4.*((X-0.5)**2 + (Y-0.5)**2)
3      PHI2 = PHI**2
3      PHI3 = PHI2*PHI
3      PHI4 = PHI3*PHI
3      PHIX = 8.*(X-0.5)
3      PHIXX= 8.
3      PHIY = 8.*(Y-0.5)
3      PHIYY= 8.
3      TMP  = 1./(1.+PHI4)
3      TMP2 = TMP**2
3      TMP3 = TMP2*TMP
3      T5   = TMP - 0.5
3      T5X  = -4.*PHI3*PHIX*TMP2
3      T5XX = 32.*(PHI3*PHIX)**2*TMP3 - 12.*PHI2*PHIX**2*
3     $       - 4.*PHI3*PHIXX*TMP2
3      T5Y  = -4.*PHI3*PHIY*TMP2
3      T5YY = 32.*(PHI3*PHIY)**2*TMP3 - 12.*PHI2*PHIY**2*
3     $       - 4.*PHI3*PHIYY*TMP2
3      U    = T1 * T2 * T3 * T4 * T5
3      UXX  = T3 * T4 * (   T1XX*T2*T5 +
3     $                   2.*T1X*T2X*T5 +
3     $                   2.*T1X*T2*T5X +
3     $                      T1*T2XX*T5 +
3     $                   2.*T1*T2X*T5X +
3     $                      T1*T2*T5XX)
3      UYY  = T1 * T2 * (   T3YY*T4*T5 +
3     $                   2.*T3Y*T4Y*T5 +
3     $                   2.*T3Y*T4*T5Y +
3     $                      T3*T4YY*T5 +
3     $                   2.*T3*T4Y*T5Y +
3     $                      T3*T4*T5YY)
3      F    = -0.31 * ( UXX + UYY - COEFU(X,Y)*U )
3      RETURN
3      END
3      FUNCTION COEFU(X,Y)
3      COMMON /CONCOM/ PI, FOURPI
3      COEFU = 100. + COS(2.*PI*X) + SIN(3.*PI*Y)
3      RETURN
3      END
```

```
'EOR
'EOF
**************
* PROBLEM   7 *
**************
```

```
3      DATA N/16/, NZB/4/, NA/2/,
3     $     XB(1),YB(1), XB(2),YB(2), XB(3),YB(3), XB(4),
3     $     /  0.,    0.,    1.,    0.,    1.,    1.,    0.,
3     $     AB / -2.356, -0.785,  0.785,  2.356/
3      DATA C
3     $     / .1469642607553E+00, .2000000125856E+01, .10
3     $      -.2983663620768E-01,-.9430879889679E-03, .43
3     $       .2394115460351E-05, .3183099062144E+00,-.31
3     $       .3183099062144E+00,-.3183099062144E+00/
3      DATA  PI/3.14159265358979/, TWOPI/6.28318530717959
3      QABS(X,Y) = SQRT(X*X + Y*Y)
3      RCST = X - 0.5
3      RSNT = Y - 0.5
3      R = QABS(RCST, RSNT)
3      SUM = R*R/4.
3      L = 1
3      SUM = SUM + C(L)
3      RKCSKT = 1
3      RKSNKT = 0
3      DO 1 K=1,N
3         TEMP   = RCST*RKCSKT - RSNT*RKSNKT
3         RKSNKT = RSNT*RKCSKT + RCST*RKSNKT
3         RKCSKT = TEMP
3         IF ((K.GT.2) .AND. (MOD(K,4).NE.0))  GO TO 1
3         L = L+1
3         SUM = SUM + C(L)*RKCSKT
3    1 CONTINUE
3      DO 2 I=1,NZB
3         ZR = X - XB(I)
3         ZI = Y - YB(I)
3         WR = QABS(ZR, ZI)
3         IF (WR .NE. 0.)  WR = ALOG(WR)
3         WI = QARG(ZR, ZI)
3         IF (WI .GT. AB(I))  WI = WI - TWOPI
3         TR = ZR*WR - ZI*WI
3         TI = ZR*WI + ZI*WR
3         L = L+1
3         SUM = SUM + C(L)*(ZR*TI + ZI*TR)
3    2 CONTINUE
3      TRUE = SUM
3      RETURN
3      END
3      REAL FUNCTION  QARG(X, Y)
3      QARG = 0
3      IF ((X.NE.0.) .OR. (Y.NE.0.))  QARG = ATAN2(Y,X)
3      RETURN
3      END
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM   8 *
***************
'EOR
*PARAMETER SET   1(A=0.1)
*         000.32    070.30    000.00    050.40
EXPAND 8/0.1/
'EOR
*PARAMETER SET   2(A=0.15)
*         000.38    080.40    000.00    060.50
EXPAND 8/0.15/
'EOR
*PARAMETER SET   3(A=0.35)
```

```
*PARAMETER SET   1(A=10.0)
*          002.18    040.00    000.00    006.60
EXPAND 9/10./
'EOR
*PARAMETER SET   2(A=20.0)
*          002.26    040.45    000.00    006.65
EXPAND 9/20./
'EOR
*PARAMETER SET   3(A=50.0)
*          002.27    040.45    000.00    006.70
EXPAND 9/50./
'EOR
*PARAMETER SET   4(A=100.0)
*          002.29    040.45    000.00    006.80
EXPAND 9/100./
```

```
'EOR
'EOF
**************
* PROBLEM  10 *
**************
'EOR
*PARAMETER SET   1(A=10.0, B=0.5)
*          000.04    005.10    000.00    005.05
EXPAND 10/10.,.5/
'EOR
*PARAMETER SET   2(A=50.0, B=0.5)
*          000.16    025.40    000.00    008.25
EXPAND 10/50.,.5/
'EOR
*PARAMETER SET   3(A=100.0, B=0.5)
*          000.23    040.60    000.00    010.30
EXPAND 10/100.,.5/
'EOR
*PARAMETER SET   4(A=500.0, B=0.5)
*          000.32    065.70    000.00    020.35
EXPAND 10/500.,.5/
'EOR
*PARAMETER SET   5(A=1000.0, B=0.5)
*          000.38    080.75    000.00    035.40
EXPAND 10/1000.,.5/
'EOR
*PARAMETER SET   6(A=10.0, B=0.117)
*          000.04    005.10    000.00    005.05
EXPAND 10/10.,.117/
'EOR
*PARAMETER SET   7(A=50.0, B=0.117)
*          000.16    025.40    000.00    008.25
EXPAND 10/50.,.117/
'EOR
*PARAMETER SET   8(A=100.0, B=0.117)
*          000.23    040.60    000.00    010.30
EXPAND 10/100.,.117/
'EOR
*PARAMETER SET   9(A=500.0, B=0.117)
*          000.32    065.70    000.00    020.35
EXPAND 10/500.,.117/
'EOR
*PARAMETER SET  10(A=1000.0, B=0.117)
*          000.38    080.75    000.00    035.40
EXPAND 10/1000.,.117/
```

```
'EOR
```

```
*PARAMETER SET   3(A=3*PI)
*          000.15    006.40    000.00    006.40
EXPAND 11/3.*PI/
'EOR
*PARAMETER SET   4(A=5*PI)
*          000.22    006.60    000.00    006.60
EXPAND 11/5.*PI/
'EOR
*PARAMETER SET   5(A=10*PI)
*          000.27    006.75    000.00    006.75
EXPAND 11/10.*PI/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  12 *
***************
'EOR
*PARAMETER SET   1(A=PI, B=PI)
*          000.05    006.10    000.00    006.10
EXPAND 12/3.14159265358979,3.14159265358979/
'EOR
*PARAMETER SET   2(A=PI, B=10.0)
*          000.12    006.30    000.00    006.30
EXPAND 12/3.14159265358979,10./
'EOR
*PARAMETER SET   3(A=10.0, B=PI)
*          000.12    006.30    000.00    006.30
EXPAND 12/10.,3.14159265358979/
'EOR
*PARAMETER SET   4(A=20.0, B=PI)
*          000.22    006.60    000.00    006.60
EXPAND 12/20.,3.14159265358979/
'EOR
*PARAMETER SET   5(A=10.0, B=10.0)
*          000.12    006.30    000.00    006.30
EXPAND 12/10.,10./
'EOR
*PARAMETER SET   6(A=10.0, B=20.0)
*          000.22    006.60    000.00    006.60
EXPAND 12/10.,20./
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  13 *
***************
'EOR
*
*          000.48    090.90    000.00    080.25
*          2000002002000
1          TWO DIMENSIONS $ SELF-ADJOINT
1          COEF1(X,Y)UXX$ + UYY$ = F(X,Y)
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3      FUNCTION COEF1(X,Y)
3      COEF1 = 2.
3      IF ( X.LT..4 ) COEF1 =1.
3      RETURN
3      END
```

```
3      RETURN
3      END
3      FUNCTION TRUE(X,Y)
3      FX2 = .7 + .5*(X-.4) + (X-.4)**2/(1+X*X)
3      TRUE =AMIN1(X+.3,FX2)*(1.+(Y-1.)**2*EXP(-Y))
3      RETURN
3      END
3      FUNCTION CDXU(X,Y)
3      CDXU = 0.
3      RETURN
3      END
3      FUNCTION CDYU(X,Y)
3      CDYU = 0.
3      RETURN
3      END
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  14 *
***************
'EOR
*PARAMETER SET   1(A=0.01)
*          000.28    045.05    060.05    045.05
EXPAND 14/0.01/
'EOR
*PARAMETER SET   2(A=0.1)
*          000.34    060.05    070.05    060.05
EXPAND 14/0.1/
'EOR
*PARAMETER SET   3(A=1.0)
*          000.39    065.10    075.10    065.10
EXPAND 14/1.0/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  15 *
***************
'EOR
*PARAMETER SET   1(A=0.2, B=1.5, C=0.1)
*          000.28    095.00    000.00    070.00
EXPAND 15/.2,1.5,0.1/
'EOR
*PARAMETER SET   2(A=1.0, B=2.5, C=0.1)
*          000.17    050.00    000.00    050.00
EXPAND 15/1.,2.5,0.1/
'EOR
*PARAMETER SET   3(A=0.2, B=1.5, C=0.04)
*          000.27    090.00    000.00    070.00
EXPAND 15/.2,1.5,0.04/
'EOR
*PARAMETER SET   4(A=0.2, B=2.5, C=0.04)
*          000.15    040.00    000.00    050.00
EXPAND 15/.2,2.5,0.04/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  16 *
***************
'EOR
*PARAMETER SET   1(B=1.)
```

```
*PARAMETER SET   1(A=1.0, B=2.0)
*          000.08    002.20    000.00    010.15
EXPAND 17/1..2./
'EOR
*PARAMETER SET   2(A=5.0, B=3.0)
*          000.20    010.50    000.00    020.40
EXPAND 17/5..3./
'EOR
*PARAMETER SET   3(A=8.0, B=5.0)
*          000.45    040.80    000.00    080.70
EXPAND 17/8..5./
-------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  18 *
**************
'EOR
*PARAMETER SET   1(A=1.0, B=2.0)
*          000.07    010.10    000.00    010.10
EXPAND 18/1..2./
'EOR
*PARAMETER SET   2(A=5.0, B=3.0)
*          000.11    010.20    000.00    020.15
EXPAND 18/5..3./
'EOR
*PARAMETER SET   3(A=8.0, B=5.0)
*          000.32    010.70    000.00    070.40
EXPAND 18/8..5./
-------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  19 *
**************
'EOR
*PARAMETER SET   1(A=0.15)
*          008.15    070.10    000.00    006.05
EXPAND 19/0.15/
'EOR
*PARAMETER SET   2(A=0.25)
*          008.17    080.10    000.00    006.05
EXPAND 19/.25/
-------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  20 *
**************
'EOR
*PARAMETER SET   1(A=0.0)
*          007.23    040.60    000.00    010.30
EXPAND 20/0./
'EOR
*PARAMETER SET   2(A=10.)
*          007.23    040.60    000.00    010.30
EXPAND 20/10./
-------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  21 *
**************
```

```
3       RETURN
3       END
3       FUNCTION B(X,Y)
3       B = -2.*TRUEX(X,Y)*TRUEY(X,Y)
3       RETURN
3       END
3       FUNCTION C(X,Y)
3       C = 1. + TRUEY(X,Y)**2
3       RETURN
3       END
3       FUNCTION TRUEX(X,Y)
3       TRUEX = EXP(X+Y)
3       RETURN
3       END
3       FUNCTION TRUEY(X,Y)
3       TRUEY = EXP(X+Y)
3       RETURN
3       END
3       FUNCTION F(X,Y)
3       TUXX = EXP(X+Y)
3       TUXY = TUXX
3       TUYY = TUXY
3       F = A(X,Y)*TUXX + B(X,Y)*TUXY + C(X,Y)*TUYY
3       RETURN
3       END
3       FUNCTION TRUE(X,Y)
3       TRUE = EXP(X+Y)
3       RETURN
3       END
```

```
'EOR
'EOF
***************
* PROBLEM  22 *
***************
'EOR
*
*           008.31     090.85     000.00     004.05
*           2000200002000
1           TWO DIMENSIONS
1           W(X,Y)UXX$ + W(X,Y)UYY$  + WX(X,Y)UX$ +
1.          WY(X,Y)UY$ = F(X,Y)
2           DIRICHLET
2           X=0. , U=TRUE(X,Y)
2           X=1. , U=TRUE(X,Y)
2           Y=0. , U=TRUE(X,Y)
2           Y=1. , U=TRUE(X,Y)
3       FUNCTION W(X,Y)
3       COMMON /CONCOM/ C1,C2
3       DATA C1, C2/ 17.06, 3.62/
3       F1 = (X*X-1.)*(Y*Y-1.)
3       DXF1 = 2.*X*(Y*Y-1.)
3       DYF1 = (X*X-1.)*2.*Y
3       UX = C1*DXF1 + C2*DXF1*(X*X+Y*Y) + C2*F1*2.*X
3       UY = C1*DYF1 + C2*DYF1*(X*X+Y*Y) + C2*F1*2.*Y
3       A = SQRT(UX*UX + UY*UY)
3       W = 7996.
3       IF (A .GT. .0025) W = 19.4/A + 236.
3       RETURN
3       END
3       FUNCTION WX(X,Y)
3       COMMON /CONCOM/ C1,C2
3       F1 = (X*X-1.)*(Y*Y-1.)
```

```
3      RETURN
3      END
3      FUNCTION WY(X,Y)
3      WY = WX(Y,X)
3      RETURN
3      END
3      FUNCTION TRUE(X,Y)
3      COMMON /CONCOM/ C1,C2
3      F1 = (X*X-1.)*(Y*Y-1.)
3      TRUE = C1*F1 + C2*F1*(X*X+Y*Y)
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      COMMON /CONCOM/ C1,C2
3      F1 = (X*X-1.)*(Y*Y-1.)
3      DXF1 = 2.*X*(Y*Y-1.)
3      DYF1 = (X*X-1.)*2.*Y
3      UX = C1*DXF1 + C2*DXF1*(X*X+Y*Y) + C2*F1*2.*X
3      UY = C1*DYF1 + C2*DYF1*(X*X+Y*Y) + C2*F1*2.*Y
3      DXXF1 = 2.*(Y*Y-1.)
3      DYYF1 = (X*X-1.)*2.
3      UXX = C1*DXXF1 + C2*DXXF1*(X*X+Y*Y) + C2*DXF1*2.*X
3     $       C2*DXF1*2.*X + C2*F1*2.
3      UYY = C1*DYYF1 + C2*DYYF1*(X*X+Y*Y) + C2*DYF1*2.*Y
3     $       C2*DYF1*2.*Y + C2*F1*2.
3      F = W(X,Y)*(UXX+UYY) + WX(X,Y)*UX + WY(X,Y)*UY
3      RETURN
3      END
-------------------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  23 *
**************
'EOR
*PARAMETER SET   1(A=387.75, B=50.0, C=1, D=0.10)
*          009.47    060.70    020.00    070.60
EXPAND 23/387.75,50.,,1,0.10/
'EOR
*PARAMETER SET   2(A=554.5, B=0.554, C=1, D=0.10)
*          009.50    070.80    020.00    070.60
EXPAND 23/554.5,.544,1,0.10/
'EOR
*PARAMETER SET   3(A=387.75, B=50.0, C=2, D=0.15)
*          009.52    080.30    020.00    070.60
EXPAND 23/387.75,50.,,2,0.15/
'EOR
*PARAMETER SET   4(A=554.5, B=0.554, C=2, D=0.15)
*          009.52    080.80    020.00    070.60
EXPAND 23/554.5,.544,2,0.15/
'EOR
*PARAMETER SET   5(A=387.75, B=50.0, C=3, D=0.20)
*          009.53    085.80    020.00    070.60
EXPAND 23/387.75,50.,,3,0.20/
'EOR
*PARAMETER SET   6(A=554.5, B=0.554, C=3, D=0.20)
*          009.53    085.85    020.00    070.60
EXPAND 23/554.5,.544,3,0.20/
-------------------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  24 *
```

```
'EOR
*PARAMETER SET   4(A=0.50, B=10., C=-0.10)
*          007.07    015.10    005.00    010.00
EXPAND 24/0.50,10.,-0.10,4/
'EOR
*PARAMETER SET   5(A=1.00, B=1., C=-0.10)
*          007.05    015.10    005.00    000.00
EXPAND 24/1.00,1.,-0.10,5/
'EOR
*PARAMETER SET   6(A=1.00, B=10., C=-0.10)
*          007.07    015.10    005.00    010.00
EXPAND 24/1.00,10.,-0.10,6/
'EOR
*PARAMETER SET   7(A=1.00, B=100., C=-1.00)
*          007.14    015.10    005.00    015.37
EXPAND 24/1.00,100.,-1.00,7/
'EOR
*PARAMETER SET   8(A=1.00, B=1000., C=-1.00)
*          007.10    015.10    005.00    010.20
EXPAND 24/1.00,1000.,-1.00,8/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  25 *
***************
'EOR
*PARAMETER SET   1(A=1.5)
*          000.19    065.40    000.00    006.05
EXPAND 25/1.5/
'EOR
*PARAMETER SET   2(A=2.5)
*          000.14    055.20    000.00    006.05
EXPAND 25/2.5/
'EOR
*PARAMETER SET   3(A=3.5)
*          000.12    045.15    000.00    006.05
EXPAND 25/3.5/
'EOR
*PARAMETER SET   4(A=4.5)
*          000.11    035.20    000.00    006.05
EXPAND 25/4.5/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  26 *
***************
'EOR
*PARAMETER SET   1(A=1)
*          007.03    010.00    000.00    005.02
EXPAND 26/1.,1/
'EOR
*PARAMETER SET   2(A=5)
*          007.04    010.00    000.00    010.02
EXPAND 26/5.,2/
'EOR
*PARAMETER SET   3(A=10)
*          008.04    010.00    000.00    013.02
EXPAND 26/10.,3/
-------------------------------------------------------------
'EOR
'EOF
```

```
2          Y=1. , U=0.
3       FUNCTION TRUE(X,Y)
3       REAL DERUSL(6), GRID(20), TABLE(20,20),
3      $     T1B1(100), T1B2(100), T1B3(100), T1B4(100)
3       EQUIVALENCE (TABLE(1, 1) , T1B1(1)),
3      $            (TABLE(1, 6) , T1B2(1)),
3      $            (TABLE(1,11) , T1B3(1)),
3      $            (TABLE(1,16) , T1B4(1))
3       DATA NGRID, NGRDD/20, 20/
3       DATA  GRID/0.1000000, 0.1473684, 0.1947368, 0.2
3      $0.2894737, 0.3368421, 0.3842105, 0.4315789, 0.4
3      $0.5263158, 0.5736842, 0.6210526, 0.6684211, 0.7
3      $0.7631579, 0.8105263, 0.8578948, 0.9052632, 0.9
3      $1.0000000/
3C
3C      APPROXIMATE SOLUTION OF PROBLEM USING
3C      5-POINT STAR (20 X 20 GRID)
3C
3       DATA T1B1/  .000000,  .000000,  .000000,  .0000
3      $  .000000,  .000000,  .000000,  .000000,  .0000
3      $  .000000,  .000000,  .000000,  .000000,  .0000
3      $  .000000,  .000000,  .000000,  .000000,  .8818
3      $ 2.331187, 3.089675, 3.848034, 4.576355, 5.2441
3      $ 6.280833, 6.595470, 6.741719, 6.698443, 6.4475
3      $ 5.267140, 4.318752, 3.124975, 1.683172,  .0000
3      $  .212529,  .384810,  .561946,  .745076,  .9283
3      $ 1.266651, 1.407139, 1.519366, 1.596998, 1.6342
3      $ 1.567710, 1.455809, 1.287426, 1.060624,  .7745
3      $  .000000,  .000000,  .552663, 1.000754, 1.4610
3      $ 2.411748, 2.868264, 3.286824, 3.648825, 3.9365
3      $ 4.225089, 4.197475, 4.039408, 3.741501, 3.2965
3      $ 1.948547, 1.045257,  .000000,  .000000,  .4732
3      $ 1.251265, 1.658857, 2.066712, 2.458812, 2.8187
3      $ 3.379439, 3.550801, 3.631853, 3.611042, 3.4783
3      $ 2.846377, 2.336002, 1.691798,  .912854,  .0000
3C
3       DATA T1B2/  .000000,  .464187,  .840562, 1.2276
3      $ 2.028654, 2.414265, 2.768664, 3.076216, 3.3220
3      $ 3.573599, 3.555061, 3.426466, 3.179470, 2.8074
3      $ 1.671219,  .902587,  .000000,  .000000,  .4558
3      $ 1.205983, 1.599694, 1.994266, 2.374329, 2.7241
3      $ 3.272186, 3.441889, 3.524660, 3.508960, 3.3846
3      $ 2.777809, 2.283302, 1.656372,  .895336,  .0000
3      $  .445506,  .806807, 1.178974, 1.564531, 1.9513
3      $ 2.668757, 2.968850, 3.210428, 3.379829, 3.4643
3      $ 3.333606, 3.099227, 2.742008, 2.256430, 1.6387
3      $  .000000,  .000000,  .432198,  .782752, 1.1442
3      $ 1.896254, 2.260549, 2.597361, 2.892076, 3.1305
3      $ 3.386145, 3.378907, 3.267262, 3.042015, 2.6954
3      $ 1.615767,  .875612,  .000000,  .000000,  .4154
3      $ 1.100502, 1.462205, 1.826402, 2.179265, 2.5065
3      $ 3.028509, 3.196449, 3.285531, 3.284077, 3.1813
3      $ 2.634821, 2.175900, 1.585775,  .861033,  .0000
3C
3       DATA T1B3/  .000000,  .394736,  .714991, 1.0464
3      $ 1.739828, 2.078272, 2.393379, 2.671785, 2.9004
3      $ 3.158241, 3.163557, 3.071640, 2.872382, 2.5567
3      $ 1.546922,  .842118,  .000000,  .000000,  .3698
3      $  .981139, 1.305999, 1.634778, 1.955363, 2.2551
3      $ 2.742733, 2.906054, 2.999906, 3.012788, 2.9335
3      $ 2.457377, 2.041654, 1.497066,  .817794,  .0000
3      $  .340371,  .616556,  .903818, 1.204466, 1.5097
3      $ 2.089478, 2.341033, 2.551814, 2.710515, 2.8058
```

```
3     $  .465007,  .623554,  .787267,  .950892, 1.108899,
3     $ 1.385363, 1.492081, 1.569477, 1.610682, 1.607997,
3     $ 1.433466, 1.237622,  .948177,  .544009,  .000000,
3     $  .120825,  .218604,  .322383,  .433139,  .548090,
3     $  .776212,  .881823,  .976602, 1.056454, 1.116997,
3     $ 1.160107, 1.130482, 1.056126,  .925841,  .723818,
3     $  .000000,  .000000,  .062599,  .113172,  .167151,
3     $  .285452,  .346599,  .406606,  .463546,  .515437,
3     $  .595592,  .619121,  .627881,  .618299,  .585695,
3     $  .420924,  .259428,  .000000,  .000000,  .000000,
3     $  .000000,  .000000,  .000000,  .000000,  .000000,
3     $  .000000,  .000000,  .000000,  .000000,  .000000,
3     $  .000000,  .000000,  .000000,  .000000,  .000000/
3C
3C     INTERPOLATE NUMERICAL SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRDD,
3C
3      TRUE = DERUSL(6)
3C
3      RETURN
3      END
3      FUNCTION E(X,Y)
3      E = 1./(TAN(Y)**3*X**2)
3      RETURN
3      END
-------------------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  28 *
**************
'EOR
*PARAMETER SET   1(A=1.)
*          008.03    000.00    005.00    007.03
EXPAND 28/1..1/
'EOR
*PARAMETER SET   2(A=10.)
*          008.05    000.00    005.00    010.15
EXPAND 28/10..2/
'EOR
*PARAMETER SET   3(A=100.)
*          008.06    000.00    005.00    010.22
EXPAND 28/100..3/
-------------------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  29 *
**************
'EOR
*PARAMETER SET   1(A=-3.)
*          008.18    090.05    000.02    010.02
EXPAND 29/-3..1/
'EOR
*PARAMETER SET   2(A=-1.)
*          008.18    090.05    000.00    010.00
EXPAND 29/-1..2/
'EOR
*PARAMETER SET   3(A=1.)
*          008.19    090.05    000.00    010.10
EXPAND 29/1..3/
'EOR
```

```
'EOR
•PARAMETER SET   1(A=1.0, B=8.0, C=-1.0)
•          000.16    010.40    000.00    010.35
EXPAND 30/1.,8.,-1./
'EOR
•PARAMETER SET   2(A=1.0, B=2.0, C=0.5)
•          000.06    006.10    000.00    006.15
EXPAND 30/1.,2.,5./
'EOR
•PARAMETER SET   3(A=23.0, B=2.0, C=1.0)
•          000.19    030.30    000.00    030.25
EXPAND 30/23.,2.,1./
'EOR
•PARAMETER SET   4(A=10.0, B=11.0, C=0.0)
•          000.13    010.30    000.00    015.25
EXPAND 30/10.,11.,0./
'EOR
•PARAMETER SET   5(A=100.0, B=2.0, C=2.0)
•          000.19    030.30    000.00    030.25
EXPAND 30/100.,2.,2./
'EOR
•PARAMETER SET   6(A=10.0, B=4.0, C=1.0)
•          000.13    010.30    000.00    015.25
EXPAND 30/10.,4.,1./
'EOR
•PARAMETER SET   7(A=4.0, B=5.0, C=-0.5)
•          000.10    006.25    000.00    010.20
EXPAND 30/4.,5.,-0.5/
'EOR
•PARAMETER SET   8(A=3.0, B=6.0, C=2.0)
•          000.10    006.25    000.00    006.25
EXPAND 30/3.,6.,2./
'EOR
•PARAMETER SET   9(A=0.5, B=3.0, C=10.0)
•          000.09    006.20    000.00    006.20
EXPAND 30/.5,3.,10./
-----------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  31 *
***************
'EOR
•
•          009.04    002.00    004.08    006.05
•          2020021000020
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS $ POISS
1          UXX$ + UYY$ = -1.
2          MIXED
2          X=-1. , MIXED = (1.)U + (-1.)UX = TRUE(X,Y) -
2          X= 1. , MIXED = (1.)U + ( 1.)UX = TRUE(X,Y) +
2          Y=-1. , MIXED = (1.)U + (-1.)UY = TRUE(X,Y) -
2          Y= 1. , MIXED = (1.)U + ( 1.)UY = TRUE(X,Y) +
3      FUNCTION TRUE(X,Y)
3      COMMON /CONCOM/ A0,A2,A4,A6
3      DATA A0,A2,A4,A6 /.821564,-.0144,.0000493,-.000000
3      X2 = X*X
3      X4 = X2*X2
3      X6 = X4*X2
3      X8 = X6*X2
3      X10= X8*X2
3      X12= X10*X2
3      Y2 = Y*Y
```

```
3       X2 = X*X
3       X3 = X*X2
3       X5 = X3*X2
3       X7 = X5*X2
3       X9 = X7*X2
3       X11= X9*X2
3       Y2 = Y*Y
3       Y4 = Y2*Y2
3       Y6 = Y4*Y2
3       Y8 = Y6*Y2
3       Y10= Y8*Y2
3       DUX = -.5*X + A2*(4.*X3-12.*X*Y2) +
3      $       A4*(8.*X7-168.*X5*Y2+280.*X3*Y4-56.*X*Y6) +
3      $       A6*(12.*X11-660.*X9*Y2+3960.*X7*Y4-5544.*X5*
3      $           1980.*X3*Y8-132.*X*Y10)
3       RETURN
3       END
3       FUNCTION DUY(X,Y)
3       DUY = DUX(Y,X)
3       RETURN
3       END
```

```
'EOR
'EOF
***************
* PROBLEM  32 *
***************
'EOR
*
*           009.07    010.05    000.00    010.15
*           2000200002002
1           TWO DIMENSIONS
1           UXX$ + UYY$ + 3./(5.-Y)UY$ = F(X,Y)
2           DIRICHLET $ HOMOGENEOUS
2           X=-.5 , U=0.
2           X= .5 , U=0.
2           Y=-1. , U=0.
2           Y= 1. , U=0.
3       FUNCTION TRUE(X,Y)
3       DATA A,B /.0010185,.0004838/
3       TRUE = (1.-Y*Y)*(1.-4.*X*X)*(5.-Y)**3*(A+B*Y)
3       RETURN
3       END
3       FUNCTION F(X,Y)
3       DATA A,B /.0010185,.0004838/
3       Z = A-A*Y**2+B*Y-B*Y**3
3       Q = 1.-4.*X**2
3       T = 5.-Y
3       T2 = T*T
3       T3 = T2*T
3       UXX = -8.*Z*T3
3       UY  = Q*((B-2.*A*Y-3.*B*Y**2)*T3 -3.*Z*T2)
3       UYY = Q*((-2.*A-6.*B*Y)*T3 - 6.*(B-2.*A*Y-3.*B*Y**
3      $        + 6.*Z*T)
3       F   = UXX + UYY + 3./(5.-Y)*UY
3       RETURN
3       END
```

```
'EOR
'EOF
***************
* PROBLEM  33 *
***************
```

```
3      K1 = SQRT(14.+S133)
3      K2 = SQRT(14.-S133)
3      A = (-7.+S133)/(2.*S133)
3      B = (-7.-S133)*A/16.
3      EK1X = EXP(K1*X)
3      EK2X = EXP(K2*X)
3      EDIFF = EK1X - EK2X
3      F1 = A*EDIFF + EK2X
3      F2 = B*EDIFF
3      Y21 = 1.-Y*Y
3      TRUE = Y21*(F1 + Y21*F2)
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      S133 = SQRT(133.)
3      K1 = SQRT(14.+S133)
3      K2 = SQRT(14.-S133)
3      A = (-7.+S133)/(2.*S133)
3      B = (-7.-S133)*A/16.
3      EK1X = EXP(K1*X)
3      EK2X = EXP(K2*X)
3      EDIFF = EK1X-EK2X
3      F1 = A*EDIFF + EK2X
3      F2 = B*EDIFF
3      Y21 = 1.-Y*Y
3      DEK1X = K1*K1*EK1X
3      DEK2X = K2*K2*EK2X
3      DEDIFF = DEK1X - DEK2X
3      DDF1 = A*DEDIFF + DEK2X
3      DDF2 = B*DEDIFF
3      UXX = Y21*(DDF1 + Y21*DDF2)
3      UYY = -2.*(F1 + 2.*(1.-3.*Y*Y)*F2)
3      F = UXX + UYY
3      RETURN
3      END
```

```
'EOR
'EOF
**************
* PROBLEM  34 *
**************
'EOR
*
*          008.03    002.01    000.01    006.05
*          2020021002000
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS $ POISS
1           UXX$ + UYY$ = -1.
2          DIRICHLET
2          X=-1. , U=TRUE(X,Y)
2          X= 1. , U=TRUE(X,Y)
2          Y=-1. , U=TRUE(X,Y)
2          Y= 1. , U=TRUE(X,Y)
3      FUNCTION TRUE(X,Y)
3      X2 = X*X
3      X4 = X2*X2
3      X6 = X4*X2
3      X8 = X6*X2
3      Y2 = Y*Y
3      Y4 = Y2*Y2
3      Y6 = Y4*Y2
3      Y8 = Y6*Y2
3      TRUE = 0.295776 - .25*(X2+Y2) +
3     $        (-14476.*(X4-6.*X2*Y2+Y4) +
```

```
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS $ POISSO
1          LAPLACE
1          UXX$ + UYY$ = 0.
2          DIRICHLET
2          X=-1. , U=TRUE(X,Y)
2          X= 1. , U=TRUE(X,Y)
2          Y=-1. , U=TRUE(X,Y)
2          Y= 1. , U=TRUE(X,Y)
3      FUNCTION TRUE(X,Y)
3      X2 = X*X
3      X4 = X2*X2
3      X6 = X4*X2
3      X8 = X6*X2
3      Y2 = Y*Y
3      Y4 = Y2*Y2
3      Y6 = Y4*Y2
3      Y8 = Y6*Y2
3      U1 = X4 - 6.*X2*Y2 + Y4
3      U2 = X8 - 28.*X6*Y2 + 70.*X4*Y4 - 28.*X2*Y6 + Y8
3      TRUE = 1.1786 - .1801*U1 + .006*U2
3      RETURN
3      END
-
'EOR
*PARAMETER SET    2(A=0.1)
*          009.11     000.00     000.10     006.50
EXPAND 35/0.1/
'EOR
*PARAMETER SET    3(A=1.0)
*          009.11     000.00     000.10     006.50
EXPAND 35/1.0/
'EOR
*PARAMETER SET    4(A=10.)
*          009.11     000.00     000.10     006.50
EXPAND 35/10./
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  36 *
***************
'EOR
*PARAMETER SET     (A=0.0, B=0.0)
*          008.28     098.60     000.00     006.05
EXPAND 36/0.0,0.0/
'EOR
*PARAMETER SET    2(A=0.1, B=-0.5)
*          006.23     090.25     000.00     010.15
EXPAND 36/0.1,-0.5/
'EOR
*PARAMETER SET    3(A=0.0, B=1.0)
*          007.49     100.45     000.00     100.50
EXPAND 36/0.0,1.0/
'EOR
*PARAMETER SET    4(A=0.5, B=0.0)
*          006.20     090.25     000.00     005.00
EXPAND 36/0.5,0.0/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  37 *
***************
```

```
3      RETURN
3      END
3      FUNCTION B(X,Y)
3      B = -2.*TRUEX(X,Y)*TRUEY(X,Y)
3      RETURN
3      END
3      FUNCTION C(X,Y)
3      C = 1. + TRUEY(X,Y)**2
3      RETURN
3      END
3      FUNCTION TRUEX(X,Y)
3      TRUEX = (2.*(X-3.*Y) + (X-3.*Y)**2)*EXP(X-Y)
3      RETURN
3      END
3      FUNCTION TRUEY(X,Y)
3      TRUEY = (-6.*(X-3.*Y) - (X-3.*Y)**2)*EXP(X-Y)
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      EXMY= EXP(X-Y)
3      XM3Y= X-3.*Y
3      UXX = (2. + 4.*XM3Y + XM3Y**2)*EXMY
3      UXY = (-6. - 8.*XM3Y - XM3Y**2)*EXMY
3      UYY = (18. + 12.*XM3Y + XM3Y**2)*EXMY
3      F = A(X,Y)*UXX+B(X,Y)*UXY+C(X,Y)*UYY
3      RETURN
3      END
3      FUNCTION TRUE(X,Y)
3      TRUE = (X-3.*Y)**2*EXP(X-Y)
3      RETURN
3      END
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  38 *
***************
'EOR
*PARAMETER SET   1(A=1.0)
*          000.09    000.00    000.20    008.25
EXPAND 38/1.0/
'EOR
*PARAMETER SET   2(A=3.0)
*          000.13    000.00    000.30    010.40
EXPAND 38/3.0/
'EOR
*PARAMETER SET   3(A=7.0)
*          000.26    000.00    000.60    015.80
EXPAND 38/7.0/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  39 *
***************
'EOR
*PARAMETER SET   1(B=0.50, H(X)=1/X)
*          005.23    090.10    000.00    010.25
EXPAND 39/1,0.50/
'EOR
*PARAMETER SET   2(B=1.00, H(X)=1/X)
*          005.22    090.10    000.00    010.20
EXPAND 39/2,1.00/
```

```
'EOR
'EOF
**************
* PROBLEM  40 *
**************
'EOR
*PARAMETER SET   1(A=0.5, B=0.5)
*          000.10    010.08    010.08    015.10
EXPAND 40/0.5,0.5/
'EOR
*PARAMETER SET   2(A=0.15, B=0.85)
*          000.13    010.08    025.10    015.10
EXPAND 40/0.15,0.85/
'EOR
*PARAMETER SET   3(A=0.85, B=0.15)
*          000.09    010.08    005.08    015.10
EXPAND 40/0.85,0.15/
-------------------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  41 *
**************
'EOR
*PARAMETER SET   1(A=-10.0, B=5)
*          000.02    002.02    000.00    006.04
EXPAND 41/-10.0,5/
'EOR
*PARAMETER SET   2(A=1.0, B=10)
*          000.02    002.02    000.00    006.04
EXPAND 41/1.0,10/
'EOR
*PARAMETER SET   3(A=-10.0, B=25)
*          000.02    002.02    000.00    006.04
EXPAND 41/-10.0,25/
-------------------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  42 *
**************
'EOR
*PARAMETER SET   1(A=0.0, B=1.0, C=1.0)
*          000.04    000.00    000.02    010.10
EXPAND 42/0.0,1.0,1.0/
'EOR
*PARAMETER SET   2(A=-1.0, B=2.0, C=2.0)
*          000.05    000.00    000.05    010.15
EXPAND 42/-1.0,2.0,2.0/
'EOR
*PARAMETER SET   3(A=-1.0, B=2.0, C=5.0)
*          000.08    000.00    000.10    010.30
EXPAND 42/-1.0,2.0,5.0/
-------------------------------------------------------------
'EOR
'EOF
**************
* PROBLEM  43 *
**************
'EOR
*
*          000.02    000.00    000.00    006.04
*          2000220200020
```

```
3       EXPX = EXP(X)
3       SINH = 0.5*(EXPX-1./EXPX)
3       RETURN
3       END
3       FUNCTION G(Y)
3       G = SIN(Y) - COS(Y)
3       RETURN
3       END
```

```
'EOR
'EOF
**************
* PROBLEM  44 *
**************
'EOR
*PARAMETER SET   1(A=1.425, B=1, C=.50, D=2)
*          008.03    010.03    000.00    005.02
EXPAND 44/1.425,1,.50,2,1/
'EOR
*PARAMETER SET   2(A=10.0, B=1, C=.50, D=2)
*          008.03    010.03    000.00    005.02
EXPAND 44/10.0,1,.50,2,2/
'EOR
*PARAMETER SET   3(A=1.425, B=2, C=.04, D=25)
*          008.02    010.03    000.00    000.00
EXPAND 44/1.425,2,.04,25,3/
'EOR
*PARAMETER SET   4(A=1.425, B=2, C=.50, D=2)
*          008.02    010.03    000.00    000.00
EXPAND 44/1.425,2,.50,2,4/
```

```
'EOR
'EOF
**************
* PROBLEM  45 *
**************
'EOR
*PARAMETER SET   1(A=2, B=1)
*          008.03    008.03    005.00    000.00
EXPAND 45/2.,1,1/
'EOR
*PARAMETER SET   2(A=1000, B=1)
*          008.14    008.03    005.00    010.60
EXPAND 45/1000.,1,2/
'EOR
*PARAMETER SET   3(A=2, B=2)
*          008.03    008.03    005.00    000.00
EXPAND 45/2.,2,3/
```

```
'EOR
'EOF
**************
* PROBLEM  46 *
**************
'EOR
*PARAMETER SET   1(A=1., B=2.)
*          008.05    000.00    000.00    010.20
EXPAND 46/1.,2.,1/
'EOR
*PARAMETER SET   2(A=4., B=2.)
*          008.06    000.00    000.00    010.25
EXPAND 46/4.,2.,2/
'EOR
```

```
'EOR
*PARAMETER SET   1(A=3.0)
*          000.44    090.60    000.10    070.35
EXPAND 47/3./
'EOR
*PARAMETER SET   2(A=5.0)
*          000.34    080.50    000.00    060.15
EXPAND 47/5./
'EOR
*PARAMETER SET   3(A=7.0)
*          000.27    070.30    000.00    050.10
EXPAND 47/7./
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  48 *
***************
'EOR
*PARAMETER SET   1(A=2, B=.04, C=1)
*          008.03    010.03    000.00    005.02
EXPAND 48/2,.04,1,1/
'EOR
*PARAMETER SET   2(A=25, B=.04, C=1)
*          008.04    010.03    000.00    005.04
EXPAND 48/25,0.04,1,2/
'EOR
*PARAMETER SET   3(A=2, B=.04, C=2)
*          008.04    010.03    000.00    005.04
EXPAND 48/2,.04,2,3/
'EOR
*PARAMETER SET   4(A=25, B=.04, C=2)
*          008.04    010.03    000.00    005.04
EXPAND 48/25,.04,2,4/
'EOR
*PARAMETER SET   5(A=2, B=.50, C=2)
*          008.04    010.03    000.00    005.04
EXPAND 48/2,.50,2,5/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  49 *
***************
'EOR
*PARAMETER SET   1(A=1, B=.50, C=2)
*          008.04    010.03    000.00    005.04
EXPAND 49/1,.50,2,1/
'EOR
*PARAMETER SET   2(A=1, B=.50, C=25)
*          008.14    010.03    000.00    020.50
EXPAND 49/1,.50,25,2/
'EOR
*PARAMETER SET   3(A=2, B=.04, C=2)
*          008.02    010.03    000.00    000.00
EXPAND 49/2,.04,2,3/
'EOR
*PARAMETER SET   4(A=2, B=.50, C=2)
*          008.02    010.03    000.00    000.00
EXPAND 49/2,.50,2,4/
-------------------------------------------------------------
'EOR
'EOF
```

```
2          Y=0. ,                  U=0.75*SIN(X)-SIN(3.*X)
2          Y=1. ,                  U=0.
3      FUNCTION TRUE(X,Y)
3      DATA PI/3.14159265358979/
3      T1 = 3.*SINH(1.-Y)*SIN(X)/(4.*SINH(1.))
3      T2 = SINH(3.*(1.-Y))*SIN(3.*X)/SINH(3.)
3      T3 = SINH(PI*(PI-X))*SIN(PI*Y)/SINH(PI*PI)
3      TRUE = T1 - T2 + T3
3      RETURN
3      END
3      FUNCTION SINH(X)
3      EXPX = EXP(X)
3      SINH = 0.5*(EXPX-1./EXPX)
3      RETURN
3      END
-------------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  51 *
***************
'EOR
*PARAMETER SET   1(A=0.3)
*          009.38    100.05    090.05    010.15
EXPAND 51/0.3,1/
'EOR
*PARAMETER SET   2(A=0.5)
*          009.37    100.05    090.05    010.10
EXPAND 51/0.5,2/
'EOR
*PARAMETER SET   3(A=0.6)
*          009.36    100.05    090.05    010.05
EXPAND 51/0.6,3/
'EOR
*PARAMETER SET   4(A=0.7)
*          009.36    100.05    090.05    010.05
EXPAND 51/0.7,4/
-------------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  52 *
***************
'EOR
*PARAMETER SET   1(A=2.)
*          008.06    010.05    005.00    010.05
EXPAND 52/2.,1/
'EOR
*PARAMETER SET   2(A=4.)
*          008.07    010.07    005.00    015.07
EXPAND 52/4.,2/
'EOR
*PARAMETER SET   3(A=49.)
*          008.15    010.25    005.00    015.35
EXPAND 52/49.,3/
-------------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  53 *
***************
'EOR,
*PARAMETER SET   1(A=30.0, B=20.0)
```

```
EXPAND 53/30..10./
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  54 *
***************
'EOR
*PARAMETER SET   1(A=0.5)
*          000.09    010.05    000.10    010.20
EXPAND 54/.5/
'EOR
*PARAMETER SET   2(A=0.9)
*          000.17    010.20    000.25    015.30
EXPAND 54/0.9/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  55 *
***************
'EOR
*PARAMETER SET   1(A=1., B=3., C=1)
*          008.13    000.00    005.20    020.35
EXPAND 55/1.,3.,1,1/
'EOR
*PARAMETER SET   2(A=3., B=2., C=1)
*          008.12    000.00    005.20    013.35
EXPAND 55/3.,2.,1,2/
'EOR
*PARAMETER SET   3(A=1., B=3., C=2)
*          008.15    000.00    005.20    027.40
EXPAND 55/1.,3.,2,3/
'EOR
*PARAMETER SET   4(A=6., B=2., C=2)
*          008.07    000.00    005.20    005.10
EXPAND 55/6.,2.,2,4/
-------------------------------------------------------------
'EOR
'EOF
***************
* PROBLEM  56 *
***************
'EOR
*PARAMETER SET   1(A=0, B=1)
*          050.53    100.80    030.35    050.25
EXPAND 56/0,1/
'EOR
*PARAMETER SET   2(A=0, B=4)
*          050.58    100.80    030.45    050.40
EXPAND 56/0,4/
'EOR
*PARAMETER SET   3(A=3, B=1)
*          050.53    100.80    030.35    050.25
EXPAND 56/3,1/
'EOR
*PARAMETER SET   4(A=3, B=4)
*          050.58    100.80    030.45    050.40
EXPAND 56/3,4/
'EOR
*PARAMETER SET   5(A=3, B=8)
*          050.58    100.80    030.45    050.45
EXPAND 56/3,8/
```

MACFIL IS DIVIDED UP INTO RECORDS WHICH ARE NUMBERE
ETC. THE END-OF-RECORD IS DESIGNATED BY *EOR ('EOR ON
INSTALLATIONS).

RECORD 0 CONTAINS A DESCRIPTION OF THE PURPOSE AND
OF MACFIL. RECORDS 1, 2, 3... CONTAIN THE INFORMATION
GENERATE THE ELLPACK PROGRAMS CORRESPONDING TO PROBLEM
OF THE ELLPACK PDE POPULATION. ALTHOUGH SOME OF THE P
NOT PARAMETERIZED AND HENCE HAVE NO MACRO CALLS, YET,
DUMMY RECORDS FOR THESE PROBLEMS ON MACFIL SO THAT THE
MACRO NUMBERS MAY BE IN A ONE-TO-ONE CORRESPONDENCE.
1, 2, 3... HAVE THE FOLLOWING FORMAT:

LINES 1-3: LINES 1 THROUGH 3 CONTAIN THE MACRO NUM

LINE 4: LINE 4 CONTAINS THE PROBLEM TYPE INFORMATI
IN COLUMN 11 WHICH IS USED TO TEST THE COMPATIBILIY
BLEM WITH THE ELLPACK ROUTINES SELECTED IN THE ENCO
(SEE ROUTINE COMPAT). THIS INFORMATION BECOMES A C
GENERATED ELLPACK PROGRAM.

COMPAT USES THE PROBLEM TYPE INFO (PTYPE) AND DISRE
MODULE INFO (DMTYPE) TO DETERMINE WHETHER MODULE DI
COMPATIBLE WITH THE GIVEN PDE PROBLEM. THE NRTYPE (
ITEMS IN THE ARRAY PTYPE AND IN EACH ROW OF THE TAB
HAVE THE FOLLOWING MEANINGS:

VALUE	PTYPE MEANING	DMTYPE ME
0	ITEM NOT PRESENT	ITEM MUST NOT
1	ALWAYS MATCHES	ALWAYS MATCHES
2	ITEM PRESENT	ITEM MUST BE P

THE 14 ITEMS CURRENTLY CHECKED FOR COMPATIBILITY AR

CONCERNING THE OPERATOR
1 TWO DIMENSIONAL
2 THREE DIMENSIONAL
3 POISSON EQUATION
4 LAPLACE EQUATION
5 UX OR UY TERMS
6 CONSTANT COEFFICIENTS
7 SELF-ADJOINT FORM
8 HOMOGENEOUS
9 UXY TERM

CONCERNING THE BOUNDARY CONDITIONS
10 DIRICHLET PROBLEM
11 SOME NORMAL DERIVATIVE CONDITIONS
12 SOME MIXED CONDITIONS
13 HOMOGENEOUS

CONCERNING THE GRID
14 HX=HY(=HZ)
15 UNIFORM GRID

REMAINING LINES: THE REMAINING LINES OF THE RECORD
INFORMATION COPIED INTO THE EQUATION, BOUNDARY AND
MENTS OF THE GENERATED ELLPACK PROGRAM IN THE ABOVE
THE EQUATION, BOUNDARY AND FORTRAN CARDS ARE MARKED
OR 3 IN COLUMN 1 RESPECTIVELY. NOTE THAT FORTRAN C
BEGIN IN COLUMN 7 AS USUAL AND THAT A FORTRAN COMME

```
1          EXP(X*Y)UXX$ + EXP(-X*Y)UYY$ - 1./(1.+X+Y)U$ =
2          MIXED
2          X=0. , MIXED = (1.)U + (-(&A))UX = GX0(Y)
2          X=1. , MIXED = (1.)U +  ( &A )UX = GX1(Y)
2          Y=0. , MIXED = (1.)U + (-(&A))UY = GY0(X)
2          Y=1. , MIXED = (1.)U +  ( &A )UY = GY1(X)
3      FUNCTION TRUE(X,Y)
3      COMMON /CONCOM/ PI
3      DATA PI/3.14159265358979/
3      TRUE = .75*EXP(X*Y)*SIN(PI*X)*SIN(PI*Y)
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      COMMON /CONCOM/ PI
3      PX = PI*X
3      PY = PI*Y
3      SPX = SIN(PX)
3      SPY = SIN(PY)
3      EXY = EXP(X*Y)
3      F = .75*(EXY*EXY*SPY*((2.*Y*Y-PI*PI)*SPX+3.*PI*Y*C
3     $    PI*SPX*(X*COS(PY)-PI*SPY)-EXY*SPX*SPY/(1.+X+Y)
3      RETURN
3      END
3      FUNCTION GX0(Y)
3      COMMON /CONCOM/ PI
3      GX0 = -(&A)*PI*.75*SIN(PI*Y)
3      RETURN
3      END
3      FUNCTION GX1(Y)
3      COMMON /CONCOM/ PI
3      GX1 = -(&A)*PI*.75*EXP(Y)*SIN(PI*Y)
3      RETURN
3      END
3      FUNCTION GY0(X)
3      COMMON /CONCOM/ PI
3      GY0 = -(&A)*PI*.75*SIN(PI*X)
3      RETURN
3      END
3      FUNCTION GY1(X)
3      COMMON /CONCOM/ PI
3      GY1 = -(&A)*PI*.75*EXP(X)*SIN(PI*X)
3      RETURN
3      END
3      FUNCTION CDXU(X,Y)
3      CDXU = Y*EXP(X*Y)
3      RETURN
3      END
3      FUNCTION CDYU(X,Y)
3      CDYU = -X*EXP(-X*Y)
3      RETURN
3      END
```

```
'EOR
*************
* MACRO   2 *
*************
```

```
'EOR
*************
* MACRO   3 *
*************
*          2020021002002
1          TWO DIMENSIONS  $  CONSTANT COEFFICIENTS  $  P
```

```
3         AM2 = &A-2.
3         INITL = 0
3   10 CONTINUE
3      TRUE = C*(X**(&A)-X)*(Y**(&A)-Y)
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      COMMON /CONCOM/ C,CAA1,AM2,INITL
3      IF (INITL .EQ. 0)  GO TO 10
3         C = 1./((&A)**((&A)/(1.-(&A)))
3     $           -(&A)**(1./(1.-(&A))))**2
3         CAA1 = C*(&A)*((&A)-1.)
3         AM2  = &A-2.
3         INITL = 0
3   10 CONTINUE
3      IF ( X.EQ.0. .OR. Y.EQ.0. ) GO TO 20
3         F = CAA1*(X**AM2*(Y**(&A)-Y) +
3     $               Y**AM2*(X**(&A)-X)    )
3      RETURN
3   20 F = 0.
3      RETURN
3      END
```

```
'EOR
*************
* MACRO   4 *
*************
*         202002!000020
1         TWO DIMENSIONS $ CONSTANT COEFFICIENTS $ POISS
1         UXX$ + UYY$ = 6.*X*Y*EXP(X+Y)*(X*Y+X+Y-3.)
2         MIXED
2         X=0. , MIXED = (1.)U + (-(&A)*(Y-Y*Y))UX = G(Y
2         X=1. , U=0.
2         Y=0. , U=0.
2         Y=1. , U=0.
3      FUNCTION TRUE(X,Y)
3      TRUE = 3.*EXP(X+Y)*X*Y*(1.-X)*(1.-Y)
3      RETURN
3      END
3      FUNCTION G(Y)
3      TEMP = (Y-Y*Y)**2
3      G  = -(&A)*TEMP*3.*EXP(Y)
3      RETURN
3      END
```

```
'EOR
*************
* MACRO   5 *
*************
*         2000021002002
1         TWO DIMENSIONS $ CONSTANT COEFFICIENTS
1         4.UXX$ + UYY$ - (&A)U$ = F(X,Y)
2         DIRICHLET $ HOMOGENEOUS
2         X=0. , U=0.
2         X=1. , U=0.
2         Y=0. , U=0.
2         Y=1. , U=0.
3      FUNCTION TRUE(X,Y)
3      COMMON /CONCOM/ TWOPI
3      DATA TWOPI/6.28318530717958/
3      TRUE = 2.*X*(X-1.)*(COS(TWOPI*Y)-1.)
3      RETURN
3      END
```

```
'EOR
************
* MACRO   7 *
************
```

```
'EOR
************
* MACRO   8 *
************
*          2020021002000
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS $ POISS
1          UXX$ + UYY$ = D2P(X)*P(Y)+P(X)*D2P(Y)
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3      FUNCTION TRUE(X,Y)
3      TRUE = P(X)*P(Y)
3      RETURN
3      END
3      FUNCTION P(X)
3      COMMON /CONCOM/ A,B
3      DATA A, B/1., 0./
3      X1 = &A
3      X2 = 1. - (&A)
3      IF (X .LT. X1) GO TO 1
3      IF (X .GT. X2) GO TO 2
3      DPHI = B - A
3      DX = X2 - X1
3      P = A + DPHI*(X-X1)**3/(DX**3)-3.*DPHI*(X-X1)**3*(
3     $     /DX**4 + 6.*DPHI*(X-X1)**3*(X-X2)**2/DX**5
3      RETURN
3    1 P = A
3      RETURN
3    2 P = B
3      RETURN
3      END
3      FUNCTION D2P(X)
3      COMMON /CONCOM/ A,B
3      X1 = &A
3      X2 = 1. - (&A)
3      IF (X .LT. X1) GO TO 1
3      IF (X .GT. X2) GO TO 1
3      DPHI = B - A
3      DX = X2 - X1
3      C3 = DPHI/DX**3
3      C4 = -3.*DPHI/DX**4
3      C5 = 6.*DPHI/DX**5
3      D2P = 6.*C3*(X-X1)+6.*C4*(X-X1)*(X-X2)+
3     $      6.*C4*(X-X1)**2+6.*C5*(X-X1)*(X-X2)**2+
3     $      12.*C5*(X-X1)**2*(X-X2)+2.*C5*(X-X1)**3
3      RETURN
3    1 D2P = 0.
3      RETURN
3      END
```

```
'EOR
************
* MACRO   9 *
************
*          2000020002000
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS
```

```
3     RETURN
3     END
```

```
'EOR
*************
* MACRO  10 *
*************
*           2020021002002
1           TWO DIMENSIONS  $  CONSTANT COEFFICIENTS  $  PI
1           UXX$ + UYY$ = F(X,Y)
2           DIRICHLET $ HOMOGENEOUS
2           X=0. , U=0.
2           X=1. , U=0.
2           Y=0. , U=0.
2           Y=1. , U=0.
3     FUNCTION TRUE(X,Y)
3     U = -&A*((X-.5)**2+(Y-(&B))**2)
3     TRUE = EXP(U)*X*(X-1.)*Y*(Y-1.)
3     RETURN
3     END
3     FUNCTION F(X,Y)
3     TEMP = EXP(-&A*((X-.5)**2+(Y-(&B))**2))
3     UXX = TEMP*Y*(Y-1.)*(-2.*&A*(X-.5)*
3    $       (-2.*&A*X**3+3.*&A*X**2+(2.-&A)*X-1.)+
3    $       (-6.*&A*X**2+6.*&A*X+(2.-&A)))
3     UYY = TEMP*X*(X-1.)*(-2.*&A*(Y-&B)*
3    $       (-2*&A*Y**3+2.*&A*(1.+&B)*Y**2+
3    $       2.*(1.-&A*&B)*Y-1.)+(-6.*&A*Y**2+
3    $       4.*&A*(1.+&B)*Y+2.*(1.-&A*&B)))
3     F = UXX + UYY
3     RETURN
3     END
```

```
'EOR
*************
* MACRO  11 *
*************
*           2020021002000
1           TWO DIMENSIONS  $  CONSTANT COEFFICIENTS  $  PI
1           UXX$ + UYY$ = F(X,Y)
2           DIRICHLET
2           X=0. , U=TRUE(X,Y)
2           X=1. , U=TRUE(X,Y)
2           Y=0. , U=TRUE(X,Y)
2           Y=1. , U=TRUE(X,Y)
3     FUNCTION TRUE(X,Y)
3     COMMON /CONCOM/ PI
3     DATA PI/3.1415926535879/
3     TEMP = X-Y+2.
3     TEMP4 = TEMP**4
3     TRUE = SIN(&A*TEMP4*TEMP/(1.+TEMP4))
3     RETURN
3     END
3     FUNCTION F(X,Y)
3     COMMON /CONCOM/ PI
3     W = X-Y+2.
3     R = W**5
3     S = 1.+W**4
3     U = R/S
3     AU = &A*U
3     RX = 5.*W**4
3     RXX = 20.*W**3
3     SX = 4.*W**3
```

```
'EOR
************
* MACRO  12 *
************
*          2000200002000
1          TWO DIMENSIONS
1          UXX$ + UYY$ + (1.+SIN(&A*X))UX$
1.         - COS(&A*Y)U$ = F(X,Y)
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3      FUNCTION TRUE(X,Y)
3      TRUE = COS(&B*Y)+SIN(&B*(X-Y))
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      BXMY = &B*(X-Y)
3      UX = &B*COS(BXMY)
3      B2 = (&B)*(&B)
3      UXX = -B2*SIN(BXMY)
3      U = COS(&B*Y)+SIN(BXMY)
3      UYY = -B2*U
3      F = UXX + UYY + (1.+SIN(&A*X))*UX
3     $     - COS(&A*Y)*U
3      RETURN
3      END
------------------------------------------------------------
'EOR
************
* MACRO  13 *
************
------------------------------------------------------------
'EOR
************
* MACRO  14 *
************
*          2000220002000
1          CONSTANT COEFFICIENTS
1          UXX$ + 2.UYY$ + 3.UX$ - 4.UY$ - U$ = F(X,Y)
2          DIRICHLET
2          X=0. , U=Y
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=0.
2          Y=1. , U=1. - 0.8*(&A) + (&A)*ABS(X-0.8)
3      FUNCTION TRUE(X,Y)
3      TRUE = Y*(1.-.8*(&A)**(2.-Y)+(&A)*ABS(X-.8)**(2.-Y
3     $        X*Y*EXP(-X*Y)
3     $        *(Y-1.)
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      Q = (2.-Y)*ALOG(ABS(X-.8))
3      P = 1.-.8*(&A)**(2.-Y) + (&A)*EXP(Q)
3      DXQ = (2.-Y)*G(X)/ABS(X-.8)
3      DXXQ = -(2.-Y)*G(X)**2/(X-.8)**2
3      DYQ = -ALOG(ABS(X-.8))
3      DYYQ = 0.
3      DXP = &A*DXQ*EXP(Q)
3      DXXP = (&A)*DXXQ*EXP(Q)+(&A)*DXQ**2*EXP(Q)
3      DYP = .8*ALOG(&A)*EXP((2.-Y)*ALOG(&A)) +
3     $       (&A)*DYQ*EXP(Q)
3      DYYP = -.8*ALOG(&A)**2*EXP((2.-Y)*ALOG(&A))+
```

```
3       RETURN
3       END
```

```
'EOR
*************
* MACRO  15 *
*************
*           2000200002000
1           TWO DIMENSIONS
1           UXX$ + UYY$ + (&A)/(Y+(&C))UY$ = F(X,Y)
2           DIRICHLET
2           X=0. , U=TRUE(X,Y)
2           X=1. , U=TRUE(X,Y)
2           Y=0. , U=TRUE(X,Y)
2           Y=1. , U=TRUE(X,Y)
3       FUNCTION TRUE(X,Y)
3       TRUE = (Y**(&B) + COS(X*Y*Y)-1.)*X*X*(X-1.)**2
3       RETURN
3       END
3       FUNCTION F(X,Y)
3       F1 = Y**&B + COS(X*Y*Y) - 1.
3       DXF1 = -Y*Y*SIN(X*Y*Y)
3       DXXF1 = -Y**4*COS(X*Y*Y)
3       F2 = X*X*(X-1.)*(X-1.)
3       DYF2 = 4.*X**3 - 6.*X*X + 2.*X
3       DXXF2 = 12.*X*X - 12.*X + 2.
3       UX = DXF1*F2 + F1*DXF2
3       UXX = DXXF1*F2 + DXF1*DXF2 + DXF1*DXF2 + F1*DXXF2
3       DYF1 = &B*Y**(&B-1.)-2.0*X*Y*SIN(X*Y*Y)
3       DYYF1 = &B*(&B-1.)*Y**(&B-2.)-2.0*X*SIN(X*Y*Y)-
3      $          (2.0*X*Y)**2*COS(X*Y*Y)
3       UY = DYF1*F2
3       UYY = DYYF1*F2
3       F = UXX + UYY + (&A)/(Y+(&C))*UY
3       RETURN
3       END
```

```
'EOR
*************
* MACRO  16 *
*************
*           2000200002002
1           TWO DIMENSIONS
1           UXX$ + UYY$ + (500./(250.*Y-1.))UY$ = -1./&B**
2           DIRICHLET $ HOMOGENEOUS
2           X=0. , U=0.
2           X=&B , U=0.
2           Y=0. , U=0.
2           Y=&B , U=0.
3       FUNCTION TRUE(X,Y)
3C
3C      ***************************************************
3C      *
3C      *                  MACRO 16 PARAMETERS
3C      *
3C      ***************************************************
3C      *
3C      *                        A  I   B
3C      *                       ----I------
3C      *                        1  I   1
3C      *                           I
3C      *                       10  I   2
3C      *
```

```
3     $0.7368421, 0.7894737, 0.8421053, 0.8947368, 0.9
3     $1.0000000/
3      DATA GRID2/0.0000000, 0.5163158, 1.0526316, 1.5
3     $2.1052632, 2.6315789, 3.1578947, 3.6842105, 4.2
3     $4.7368421, 5.2631579, 5.7894737, 6.3157895, 6.8
3     $7.3684211, 7.8947368, 8.4210526, 8.9473684, 9.4
3     $10.000000/
3C
3C     APPROXIMATE SOLUTION OF PROBLEM USING
3C     P3-C1 COLLOCATION (8 X 8 GRID)
3C
3      DATA T1B1/  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .4373
3     $  .845004,  .667101,  .268576, -.282318, -.8651
3     $-1.614187,-1.614187,-1.349772, -.865116, -.2823
3     $  .667101,  .845004,  .752189,  .437323,  .0000
3     $  .137048,  .236861,  .270609,  .224243,  .1124
3     $ -.212208, -.351906, -.428204, -.428204, -.3519
3     $ -.044762,  .112455,  .224243,  .270609,  .2368
3     $  .000000,  .000000, -.003242, -.003557,  .0037
3     $  .046046,  .076174,  .106132,  .130279,  .1433
3     $  .130279,  .106132,  .076174,  .046046,  .0209
3     $ -.003557, -.003242,  .000000,  .000000,  .0098
3     $  .030544,  .044457,  .060368,  .077176,  .0927
3     $  .111355,  .111355,  .104903,  .092789,  .0771
3     $  .044457,  .030544,  .019390,  .009896,  .0000
3C
3      DATA T1B2/  .000000,  .013971,  .026502,  .0387
3     $  .063996,  .076176,  .086842,  .094823,  .0990
3     $  .094823,  .086842,  .076176,  .063996,  .0513
3     $  .026502,  .013971,  .000000,  .000000,  .0158
3     $  .042343,  .054096,  .064900,  .074556,  .0825
3     $  .091329,  .091329,  .088339,  .082564,  .0745
3     $  .054096,  .042343,  .029741,  .015859,  .0000
3     $  .016673,  .031077,  .043666,  .054754,  .0643
3     $  .078918,  .083398,  .085685,  .085685,  .0833
3     $  .072490,  .064370,  .054754,  .043666,  .0310
3     $  .000000,  .000000,  .016733,  .031075,  .0433
3     $  .062755,  .069986,  .075547,  .079335,  .0812
3     $  .079335,  .075547,  .069986,  .062755,  .0539
3     $  .031075,  .016733,  .000000,  .000000,  .0164
3     $  .042367,  .052332,  .060529,  .067076,  .0720
3     $  .076961,  .076961,  .075305,  .072004,  .0670
3     $  .052332,  .042367,  .030497,  .016475,  .0000
3C
3      DATA T1B3/  .000000,  .015968,  .029464,  .0407
3     $  .057737,  .063711,  .068146,  .071081,  .0725
3     $  .071081,  .068146,  .063711,  .057737,  .0501
3     $  .029464,  .015968,  .000000,  .000000,  .0152
3     $  .038703,  .047416,  .054403,  .059840,  .0638
3     $  .067764,  .067764,  .066459,  .063836,  .0598
3     $  .047416,  .038703,  .028075,  .015271,  .0000
3     $  .014413,  .026379,  .036211,  .044187,  .0505
3     $  .058956,  .061275,  .062425,  .062425,  .0612
3     $  .055398,  .050518,  .044187,  .036211,  .0263
3     $  .000000,  .000000,  .013380,  .024349,  .0332
3     $  .046011,  .050300,  .053406,  .055418,  .0564
3     $  .055418,  .053406,  .050300,  .046011,  .0403
3     $  .024349,  .013380,  .000000,  .000000,  .0121
3     $   029782,  .035988,  .040806,  .044458,  .0470
3     $  .049614,  .049614,  .048778,  .047085,  .0444
```

```
3      $  .010637,  .011357,  .011865,  .012187,  .0123
3      $  .012187,  .011865,  .011357,  .010637,  .0096
3      $  .006573,  .004061,  .000000,  .000000,  .0000
3      $  .000000,  .000000,  .000000,  .000000,  .0000
3      $  .000000,  .000000,  .000000,  .000000,  .0000
3      $  .000000,  .000000,  .000000,  .000000,  .0000
3C
3       DATA T2B1/  .000000,  .000000,  .000000,  .0000
3      $  .000000,  .000000,  .000000,  .000000,  .0000
3      $  .000000,  .000000,  .000000,  .000000,  .0000
3      $  .000000,  .000000,  .000000,  .000000, 1.3350
3      $ 3.841175, 4.942613, 5.905559, 6.704662, 7.3209
3      $ 7.952477, 7.952477, 7.740846, 7.320925, 6.7046
3      $ 4.942613, 3.841175, 2.627660, 1.335077,  .0000
3      $  .461828,  .907873, 1.325933, 1.704934, 2.0359
3      $ 2.522111, 2.666250, 2.738883, 2.738883, 2.6662
3      $ 2.310499, 2.035971, 1.704934, 1.325933,  .9078
3      $  .000000,  .000000,  .004144,  .006349,  .0072
3      $  .006650,  .005866,  .005079,  .004448,  .0041
3      $  .004448,  .005079,  .005866,  .006650,  .0072
3      $  .006349,  .004144,  .000000,  .000000,  .0097
3      $  .022919,  .027114,  .030140,  .032266,  .0336
3      $  .034958,  .034958,  .034547,  .033690,  .0322
3      $  .027114,  .022919,  .017273,  .009788,  .0000
3C
3       DATA T2B2/  .000000,  .012342,  .022290,  .0302
3      $  .041151,  .044680,  .047169,  .048744,  .0495
3      $  .048744,  .047169,  .044680,  .041151,  .0364
3      $  .022290,  .012342,  .000000,  .000000,  .0138
3      $  .034447,  .041830,  .047590,  .051948,  .0550
3      $  .058058,  .058058,  .057071,  .055069,  .0519
3      $  .041830,  .034447,  .025208,  .013829,  .0000
3      $  .014005,  .026819,  .036834,  .044885,  .0512
3      $  .059531,  .061777,  .062886,  .062886,  .0617
3      $  .056050,  .051221,  .044885,  .036834,  .0268
3      $  .000000,  .000000,  .015010,  .027527,  .0378
3      $  .052736,  .057767,  .061402,  .063752,  .0649
3      $  .063752,  .061402,  .057767,  .052736,  .0461
3      $  .027527,  .015010,  .000000,  .000000,  .0150
3      $  .037925,  .046287,  .052894,  .057949,  .0616
3      $  .065137,  .065137,  .063968,  .061504,  .0579
3      $  .046287,  .037925,  .027595,  .015044,  .0000
3C
3       DATA T2B3/  .000000,  .014816,  .027148,  .0372
3      $  .051915,  .056847,  .060411,  .062715,  .0638
3      $  .062715,  .060411,  .056847,  .051915,  .0454
3      $  .027148,  .014816,  .000000,  .000000,  .0143
3      $  .035987,  .043815,  .049968,  .054657,  .0580
3      $  .061298,  .061298,  .060220,  .058039,  .0546
3      $  .043815,  .035987,  .026261,  .014364,  .0000
3      $  .013709,  .024977,  .034123,  .041441,  .0471
3      $  .054625,  .056634,  .057626,  .057626,  .0566
3      $  .051503,  .047161,  .041441,  .034123,  .0249
3      $  .000000,  .000000,  .012843,  .023282,  .0316
3      $  .043491,  .047391,  .050184,  .051976,  .0528
3      $  .051976,  .050184,  .047391,  .043491,  .0383
3      $  .023282,  .012843,  .000000,  .000000,  .0117
3      $  .028607,  .034455,  .038948,  .042319,  .0447
3      $  .047016,  .047016,  .046259,  .044723,  .0423
3      $  .034455,  .028607,  .021157,  .011754,  .0000
3C
3       DATA T2B4/  .000000,  .010412,  .018546,  .0248
3      $  .033474,  .036241,  .038203,  .039452,  .0400
```

```
3     $  .000000,  .000000,  .000000,  .000000,  .000000,
3     $  .000000,  .000000,  .000000,  .000000,  .000000/
3C
3C     INTERPOLATE NUMERICAL SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID&A,GRID&A,NGRID,NGRID,NG
3     $            DERUSL)
3C
3      TRUE = DERUSL(6)
3C
3      RETURN
3      END
------------------------------------------------------------
'EOR
*************
* MACRO  17 *
*************
*          2020021002000
1          TWO DIMENSIONS  $  CONSTANT COEFFICIENTS  $  P
1          UXX$ + UYY$ = F(X,Y)
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3      FUNCTION TRUE (X,Y)
3      COMMON /CONCOM/ A,B
3      DATA A,B /&A,&B/
3      BX3 = (B*X)**3
3      TRUE = SIN(X-Y+.5) + EXP(-Y*Y-(A*BX3/(1.+BX3))**2)
3      RETURN
3      END
3      FUNCTION F (X,Y)
3      COMMON /CONCOM/ A,B
3      BX  = (B*X)**3
3      DDH = 6.*X*B**3
3      DH = DDH*X/2.
3      H = 1. + DH*X/3.
3      G = A*(H-1.)
3      DG = A*DH
3      FE = G/H
3      DDG = A*DDH
3      DFE = DG/H - G*DH/(H**2)
3      DDFE = (H*DDG - 2.*DG*DH - G*DDH + 2.*G*DH*DH/H)/H
3      S = FE*FE
3      DS = 2.*FE*DFE
3      DDS = 2.*(FE*DDFE + DFE*DFE)
3      SINXY = SIN(X-Y+.5)
3      EE = EXP(-Y*Y-S)
3      UXX = -SINXY - EE*(DDS-DS*DS)
3      UYY = -SINXY - EE*2.*(1.-2.*Y*Y)
3      F = UXX + UYY
3      RETURN
3      END
------------------------------------------------------------
'EOR
*************
* MACRO  18 *
*************
*          2000200002000
1          TWO DIMENSIONS
1          UXX$ + (1.+X*Y)UYY$ + COS(X)UX$ - EXP(-X)UY$ +
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
```

```
3      DDH = 6.*X*B**3
3      DH = DDH*X/2.
3      H = 1. + DH*X/3.
3      G = A**(H-1.)
3      DG = A*DH
3      DDG = A*DDH
3      FE = G/H
3      DFE = DG/H - G*DH/(H**2)
3      DDFE = (H*DDG - 2.*DG*DH - G*DDH + 2.*G*DH*DH/H)/H
3      S = FE*FE
3      DS = 2.*FE*DFE
3      DDS = 2.*(FE*DDFE + DFE*DFE)
3      SINXY = SIN(X-Y+.5)
3      EE = EXP(-Y*Y-S)
3      UXX = -SINXY - EE*(DDS-DS*DS)
3      UYY = -SINXY - EE*2.*(1.-2.*Y*Y)
3      UX = -2.*DFE*FE*EE + COS(X-Y+.5)
3      UY = -2.*Y*EE - COS(X-Y+.5)
3      U  = TRUE(X,Y)
3      F = UXX + (1.+X*Y)*UYY + COS(X)*UX-EXP(-X)*UY+3.*U
3      RETURN
3      END
```

```
*EOR
**********
* MACRO  19 *
**********
*          2000002000202
1          TWO DIMENSIONS $ SELF-ADJOINT
1          W(X,Y)UXX$ + W(X,Y)UYY$ = F(X,Y)
2          HOMOGENEOUS
2          X=0.5 , UX=0.
2          X=1.0 , U =0.
2          Y=0.5 , UY=0.
2          Y=1.0 , U =0.
3      FUNCTION W(X,Y)
3      COMMON /CONCOM/ PI
3      DATA PI/3.14159265358979/
3      W = ((PI*COS(PI*X)*SIN(PI*Y))**2 +
3     $     (PI*SIN(PI*X)*COS(PI*Y))**2)**&A
3      RETURN
3      END
3      FUNCTION TRUE(X,Y)
3      COMMON /CONCOM/ PI
3      TRUE = SIN(PI*X)*SIN(PI*Y)
3      RETURN
3      END
3      FUNCTION CDXU(X,Y)
3      COMMON /CONCOM/ PI
3      CDXU = (&A)*W(X,Y)**(1.-1./(&A))*PI*PI*SIN(2.*PI*X
3     $        COS(2.*PI*Y)
3      RETURN
3      END
3      FUNCTION CDYU(X,Y)
3      COMMON /CONCOM/ PI
3      CDYU = CDXU(Y,X)
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      COMMON /CONCOM/ PI
3      PI2 = PI * PI
3      SINPIX = SIN(PI*X)
3      SINPIY = SIN(PI*Y)
```

```
* MACRO  20 *
*************
*          2000000002000
1          TWO DIMENSIONS
1          UXX$ + UYY$ - EXP(TRUE(X,Y))U$ = F(X,Y)
2          DIRICHLET
2          X=0.00 , U=TRUE(X,Y)
2          X=0.50 , U=TRUE(X,Y)
2          Y=0.00 , U=TRUE(X,Y)
2          Y=0.75 , U=TRUE(X,Y)
3      FUNCTION TRUE(X,Y)
3      TRUE = 10.*PHI(X)*PHI(Y) + (&A)
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      U = 10.*PHI(X)*PHI(Y) + (&A)
3      W = EXP(U)
3      UXX = 10.*DDPHI(X)*PHI(Y)
3      UYY = 10.*PHI(X)*DDPHI(Y)
3      F = UXX + UYY - W*U
3      RETURN
3      END
3      FUNCTION PHI(Z)
3      PHI = (Z-1.)*Z*EXP(-100.*(Z-.5)**2)
3      RETURN
3      END
3      FUNCTION DDPHI(Z)
3      DH = -200.*(Z-.5)
3      DG = 2.*Z-1.
3      H = (Z-.5)*DH/2.
3      G = Z*(Z-1.)
3      DDPHI = (-200.*G + 2.*DH*DG + DH*DH*G + 2.)*EXP(H)
3      RETURN
3      END
-------------------------------------------------------------------
'EOR
*************
* MACRO  21 *
*************
-------------------------------------------------------------------
'EOR
*************
* MACRO  22 *
*************
-------------------------------------------------------------------
'EOR
*************
* MACRO  23 *
*************
*          2000200000200
1          TWO DIMENSIONS
1          W&C(X,Y)UXX$ + W&C(X,Y)UYY$ + DXW&C(X,Y)UX$ +
1.         DYW&C(X,Y)UY$ = F(X,Y)
2          X=0. , UX=0.
2          X=1. , UX=0.
2          Y=0. , U =TRUE(X,Y)
2          Y=1. , U =TRUE(X,Y)
3      FUNCTION TRUE(X,Y)
3      COMMON /CONCOM/ PI
3      DATA PI/3.14159265358979/
3      TRUE = COS(PI*X)*(P(Y)+1.)
3      RETURN
3      END
```

```
3      UXX = -PI*PI*COS(PI*X)*(P(Y)+1.)
3      UXY = -PI*SIN(PI*X)*D1P(Y)
3      HX = (UX*UXX+UY*UXY)/H
3      DXW1 = -(&B)*HX/(&A+(&B)*H)**2
3      RETURN
3      END
3      FUNCTION DYW1(X,Y)
3      COMMON /CONCOM/ PI
3      UX = -PI*SIN(PI*X)*(P(Y)+1.)
3      UY = COS(PI*X)*D1P(Y)
3      H = SQRT(UX*UX + UY*UY)
3      UYY = COS(PI*X)*D2P(Y)
3      UXY = -PI*SIN(PI*X)*D1P(Y)
3      HY = (UX*UXY+UY*UYY)/H
3      DYW1 = -(&B)*HY/(&A+(&B)*H)**2
3      RETURN
3      END
3      FUNCTION W2(X,Y)
3      COMMON /CONCOM/ PI
3      UX = -PI*SIN(PI*X)*(P(Y)+1.)
3      UY = COS(PI*X)*D1P(Y)
3      H = SQRT(UX*UX + UY*UY)
3      W2 = EXP(H/(&A+(&B)*H))/H
3      RETURN
3      END
3      FUNCTION DXW2(X,Y)
3      COMMON /CONCOM/ PI
3      UX = -PI*SIN(PI*X)*(P(Y)+1.)
3      UY = COS(PI*X)*D1P(Y)
3      H = SQRT(UX*UX + UY*UY)
3      UXX = -PI*PI*COS(PI*X)*(P(Y)+1.)
3      UXY = -PI*SIN(PI*X)*D1P(Y)
3      HX = (UX*UXX+UY*UXY)/H
3      DXW2 = -HX/H**2*EXP(H/(&A+(&B)*H))+
3     $        (&A)*(HX/(&A+(&B)*H)**2)/H
3     $        *EXP(H/(&A+(&B)*H))
3      RETURN
3      END
3      FUNCTION DYW2(X,Y)
3      COMMON /CONCOM/ PI
3      UX = -PI*SIN(PI*X)*(P(Y)+1.)
3      UY = COS(PI*X)*D1P(Y)
3      H = SQRT(UX*UX + UY*UY)
3      UYY = COS(PI*X)*D2P(Y)
3      UXY = -PI*SIN(PI*X)*D1P(Y)
3      HY = (UX*UXY+UY*UYY)/H
3      DYW2 = -HY/H**2*EXP(H/(&A+(&B)*H))+
3     $        (&A)*(HY/(&A+(&B)*H)**2)/H
3     $        *EXP(H/(&A+(&B)*H))
3      RETURN
3      END
3      FUNCTION W3(X,Y)
3      COMMON /CONCOM/ PI
3      UX = -PI*SIN(PI*X)*(P(Y)+1.)
3      UY = COS(PI*X)*D1P(Y)
3      H = SQRT(UX*UX + UY*UY)
3      W3 = &A/H*TANH(&B*H)
3      RETURN
3      END
3      FUNCTION SECH(X)
3      EXPX = EXP(X)
3      SECH = 2./(EXPX+1./EXPX)
3      RETURN
```

```
3      FUNCTION DYW3(X,Y)
3      COMMON /CONCOM/ PI
3      UX = -PI*SIN(PI*X)*(P(Y)+1.)
3      UY = COS(PI*X)*D1P(Y)
3      H = SQRT(UX*UX + UY*UY)
3      UYY = COS(PI*X)*D2P(Y)
3      UXY = -PI*SIN(PI*X)*D1P(Y)
3      HY = (UX*UXY+UY*UYY)/H
3     $        *EXP(H/(&A+(&B)*H))
3      DYW3 = (&A)*(&B)*SECH((&B)*H)**2*HY/H
3     $       -(&A)*TANH((&B)*H)*HY/H**2
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      COMMON /CONCOM/ PI
3      UX = -PI*SIN(PI*X)*(P(Y)+1.)
3      UXX = -PI*PI*COS(PI*X)*(P(Y)+1.)
3      UY  = COS(PI*X)*D1P(Y)
3      UYY = COS(PI*X)*D2P(Y)
3      F = W&C(X,Y)*(UXX+UYY)+DXW&C(X,Y)*UX+DYW&C(X,
3      RETURN
3      END
3      FUNCTION P(X)
3      DATA A, B, C /1.0, 0.0, &D/
3      X1 = .5 - C
3      X2 = .5 + C
3      IF (X .LT. X1) GO TO 1
3      IF (X .GT. X2) GO TO 2
3      DPHI = B - A
3      DX = X2 - X1
3      P = A + DPHI*(X-X1)**3/(DX**3)-3.*DPHI*(X-X1)
3     $    /DX**4 + 6.*DPHI*(X-X1)**3*(X-X2)**2/DX**
3      RETURN
3    1 P = A
3      RETURN
3    2 P = B
3      RETURN
3      END
3      FUNCTION D1P(X)
3      DATA A, B, C /1.0, 0.0, &D/
3      X1 = .5 - C
3      X2 = .5 + C
3      IF (X .LT. X1) GO TO 1
3      IF (X .GT. X2) GO TO 1
3      DPHI = B - A
3      DX = X2 - X1
3      D1P =  3.*DPHI*(X-X1)**2/(DX**3)-9.*DPHI*(X-X
3     $       /DX**4 +18.*DPHI*(X-X1)**2*(X-X2)**2/D
3     $       -3.*DPHI*(X-X1)**3/DX**4
3     $       +12.*DPHI*(X-X1)**3*(X-X2)/DX**5
3      RETURN
3    1 D1P = 0.
3      RETURN
3      END
3      FUNCTION D2P(X)
3      DATA A, B, C /1.0, 0.0, &D/
3      X1 = .5 - C
3      X2 = .5 + C
3      IF (X .LT. X1) GO TO 1
3      IF (X .GT. X2) GO TO 1
3      DPHI = B - A
3      DX = X2 - X1
3      C3 = DPHI/DX**3
```

```
* MACRO  24 *
************
*          2000200000022
1          TWO DIMENSIONS
1          UXX$ + UYY$ + W(X,Y)UX$ = F(X,Y)
2          MIXED $ HOMOGENEOUS
2          X=0.1 , MIXED = (&B)U + (1.)UX + (1.)UY = 0.
2          X=1.0 , MIXED = (&B)U + (1.)UX + (1.)UY = 0.
2          Y=0.1 , MIXED = (&B)U + (1.)UX + (1.)UY = 0.
2          Y=1.0 , MIXED = (&B)U + (1.)UX + (1.)UY = 0.
3      FUNCTION TRUE(X,Y)
3C
3C     ***************************************************
3C     *
3C     *                 MACRO 24 PARAMETERS
3C     *
3C     ***************************************************
3C     *
3C     *            A   I    B   I    C     I  D
3C     *         -------I-------I--------I-----
3C     *           0.25  I   100  I  -0.10  I  1
3C     *                 I      I         I
3C     *           0.25  I  1000  I  -0.10  I  2
3C     *                 I      I         I
3C     *           0.50  I     1  I  -0.01  I  3
3C     *                 I      I         I
3C     *           0.50  I    10  I  -0.10  I  4
3C     *                 I      I         I
3C     *           1.00  I     1  I  -0.10  I  5
3C     *                 I      I         I
3C     *           1.00  I    10  I  -0.10  I  6
3C     *                 I      I         I
3C     *           1.00  I   100  I  -1.00  I  7
3C     *                 I      I         I
3C     *           1.00  I  1000  I  -1.00  I  8
3C     *
3C     ***************************************************
3C
3      REAL DERUSL(6), GRID(20), TABLE(20,20),
3     $     T1B1(100), T1B2(100), T1B3(100), T1B4(100),
3     $     T2B1(100), T2B2(100), T2B3(100), T2B4(100),
3     $     T3B1(100), T3B2(100), T3B3(100), T3B4(100),
3     $     T4B1(100), T4B2(100), T4B3(100), T4B4(100),
3     $     T5B1(100), T5B2(100), T5B3(100), T5B4(100),
3     $     T6B1(100), T6B2(100), T6B3(100), T6B4(100),
3     $     T7B1(100), T7B2(100), T7B3(100), T7B4(100),
3     $     T8B1(100), T8B2(100), T8B3(100), T8B4(100)
3      EQUIVALENCE (TABLE(1, 1) , T&DB1(1)),
3     $            (TABLE(1, 6) , T&DB2(1)),
3     $            (TABLE(1,11) , T&DB3(1)),
3     $            (TABLE(1,16) , T&DB4(1))
3      DATA NGRID, NGRDD/20, 20/
3      DATA  GRID/0.1000000, 0.1473684, 0.1947368, 0.2421
3     $0.2894737, 0.3368421, 0.3842105, 0.4315789, 0.4789
3     $0.5263158, 0.5736842, 0.6210526, 0.6684211, 0.7157
3     $0.7631579, 0.8105263, 0.8578948, 0.9052632, 0.9526
3     $1.0000000/
3C
3C     APPROXIMATE SOLUTION OF PROBLEM USING
3C     P3-C1 COLLOCATION (8 X 8 GRID)
3C
3      DATA T1B1/ 5.947140,-1.985312,-1.841566, -.643261,
3     $ -.275850, -.190552, -.135905, -.104104, -.079619,
```

```
3     $  .442757,  .357037,  .288794,  .234286,  .1876
3     $  .116168,  .085766,  .058902,  .032230,  .0055
3C
3      DATA T1B2/-1.393252, 2.129313, 2.004093, 1.4875
3     $ 1.031383,  .836326,  .678080,  .549958,  .4471
3     $  .295747,  .239752,  .192711,  .152808,  .1179
3     $  .058438,  .031497,  .005473,-1.134656, 1.8818
3     $ 1.321198, 1.145069,  .956455,  .785785,  .6459
3     $  .434900,  .356585,  .291702,  .236866,  .1907
3     $  .116267,  .085189,  .056817,  .030344,  .0052
3     $ 1.569241, 1.500241, 1.165376, 1.027406,  .8713
3     $  .603959,  .501659,  .413736,  .341264,  .2803
3     $  .184114,  .145758,  .111953,  .081748,  .0542
3     $  .004961, -.782273, 1.220587, 1.250749, 1.0233
3     $  .783558,  .662149,  .556095,  .464370,  .3864
3     $  .263930,  .215689,  .174105,  .137822,  .1057
3     $  .050981,  .026971,  .004638, -.661002, 1.1318
3     $  .892689,  .805670,  .697903,  .592875,  .5021
3     $  .353462,  .294906,  .244300,  .199721,  .1618
3     $  .098153,  .071621,  .047066,  .024826,  .0042
3C
3      DATA T1B3/ -.563133,  .933199,  .944760,  .7789
3     $  .617529,  .529291,  .451081,  .382281,  .3209
3     $  .223126,  .183079,  .148431,  .117609,  .0901
3     $  .043236,  .022787,  .003928, -.482987,  .7835
3     $  .677625,  .619581,  .542300,  .467650,  .4004
3     $  .287247,  .241166,  .200784,  .165144,  .1340
3     $  .081582,  .059429,  .039094,  .020596,  .0035
3     $  .712754,  .711772,  .586039,  .537103,  .4720
3     $  .350091,  .299381,  .252503,  .212863,  .1777
3     $  .119083,  .094390,  .072406,  .052927,  .0346
3     $  .003178, -.358920,  .582193,  .600844,  .5034
3     $  .406022,  .351899,  .302912,  .258938,  .2195
3     $  .154740,  .127701,  .103946,  .082605,  .0634
3     $  .030488,  .016082,  .002794, -.308659,  .5156
3     $  .426427,  .390566,  .343038,  .296757,  .2556
3     $  .185361,  .156696,  .131194,  .108120,  .0883
3     $  .053866,  .039470,  .025829,  .013633,  .0024
3C
3      DATA T1B4/ -.264752,  .442604,  .435370,  .3527
3     $  .281741,  .243068,  .209160,  .179290,  .1515
3     $  .107475,  .088549,  .072451,  .057527,  .0442
3     $  .021248,  .011241,  .002007, -.221997,  .3430
3     $  .279588,  .253203,  .220467,  .190078,  .1631
3     $  .118131,  .099710,  .083473,  .069067,  .0563
3     $  .034636,  .025352,  .016809,  .008944,  .0015
3     $  .300253,  .270576,  .203343,  .182382,  .1574
3     $  .114500,  .098110,  .082209,  .069674,  .0584
3     $  .039449,  .031228,  .024017,  .017882,  .0115
3     $  .001172, -.136533,  .199651,  .169111,  .1194
3     $  .090391,  .076217,  .064822,  .055357,  .0462
3     $  .032812,  .026864,  .022154,  .017549,  .0135
3     $  .006575,  .003565,  .000723, -.056951,  .0334
3     $  .023803,  .020102,  .017100,  .014334,  .0120
3     $  .008609,  .007239,  .006054,  .005017,  .0041
3     $  .002573,  .001919,  .001322,  .000767,  .0002
3C
3      DATA T2B1/  .020503, -.198403, -.110755, -.0481
3     $ -.024519, -.017436, -.012868, -.009768, -.0075
3     $ -.004679, -.003745, -.003018, -.002435, -.0019
3     $ -.001163, -.000755, -.000041, -.241794, 1.4466
3     $ 1.537994, 1.182641,  .808284,  .629514,  .4739
3     $  .280418,  .214742,  .166834,  .139473,  .1076
```

```
3     $ 1.177533,  .945746,  .761053,  .613970,  .4966
3     $  .325604,  .262651,  .209936,  .165240,  .1262
3     $  .059598,  .029516,  .000602, -.090438, 2.3256
3     $ 1.555952, 1.310861, 1.081989,  .880512,  .7185
3     $  .478747,  .390803,  .318292,  .257099,  .2059
3     $  .123185,  .088788,  .057337,  .028129,  .0005
3     $ 1.938513, 1.752303, 1.362960, 1.168458,  .9790
3     $  .667323,  .551478,  .452412,  .371492,  .3038
3     $  .197311,  .154988,  .117755,  .084574,  .0543
3     $  .000533, -.062830, 1.549239, 1.469093, 1.1911
3     $  .875821,  .733206,  .611304,  .507666,  .4203
3     $  .284415,  .231223,  .185514,  .145717,  .1106
3     $  .050811,  .024698,  .000495, -.053121, 1.3929
3     $ 1.033154,  .906942,  .776062,  .653020,  .5490
3     $  .382441,  .317626,  .261907,  .212944,  .1715
3     $  .102078,  .073270,  .046615,  .022573,  .0004
3C
3      DATA T2B3/ -.045262, 1.159650, 1.096960,  .8977
3     $  .683635,  .580390,  .491027,  .413897,  .3456
3     $  .238070,  .194287,  .156568,  .123078,  .0933
3     $  .042637,  .020636,  .000413, -.038786,  .9786
3     $  .777727,  .691978,  .597660,  .510473,  .4339
3     $  .307925,  .257305,  .213201,  .174415,  .1407
3     $  .084041,  .060210,  .038372,  .018564,  .0003
3     $  .872088,  .818037,  .669181,  .596446,  .5175
3     $  .377311,  .320947,  .269212,  .225894,  .1877
3     $  .124335,  .097762,  .074171,  .053339,  .0337
3     $  .000330, -.028625,  .721378,  .690619,  .5721
3     $  .442722,  .379956,  .324617,  .275935,  .2327
3     $  .162463,  .133350,  .107882,  .085059,  .0646
3     $  .029587,  .014328,  .000287, -.024485,  .6290
3     $  .481266,  .428403,  .371283,  .317960,  .2717
3     $  .194950,  .164031,  .136686,  .112012,  .0909
3     $  .054426,  .039238,  .024845,  .012022,  .0002
3C
3      DATA T2B4/ -.020705,  .533350,  .490835,  .3944
3     $  .301810,  .257700,  .220058,  .187633,  .1577
3     $  .110796,  .090760,  .073832,  .058145,  .0442
3     $  .020211,  .009795,  .000201, -.017017,  .4154
3     $  .308098,  .270828,  .232401,  .198336,  .1688
3     $  .120975,  .101621,  .084664,  .069677,  .0564
3     $  .034084,  .024533,  .015757,  .007689,  .0001
3     $  .344010,  .290702,  .217058,  .188425,  .1607
3     $  .114743,  .097923,  .081482,  .068782,  .0574
3     $  .038327,  .030064,  .022849,  .016768,  .0104
3     $  .000109, -.008694,  .213308,  .167012,  .1171
3     $  .084763,  .070630,  .059631,  .050778,  .0420
3     $  .029629,  .024060,  .019766,  .015495,  .0117
3     $  .005416,  .002669,  .000059, -.000470,  .0051
3     $  .002640,  .002238,  .001853,  .001543,  .0012
3     $  .000911,  .000761,  .000633,  .000521,  .0004
3     $  .000258,  .000187,  .000123,  .000063,  .0000
3C
3      DATA T3B1/ 6.100690, 5.995871, 5.871823, 5.8040
3     $ 5.702606, 5.661853, 5.619429, 5.577221, 5.5363
3     $ 5.451070, 5.407568, 5.363039, 5.317831, 5.2717
3     $ 5.177497, 5.129447, 5.080941, 5.840706, 5.7705
3     $ 5.590085, 5.537480, 5.483309, 5.442325, 5.4010
3     $ 5.321777, 5.281188, 5.240684, 5.200820, 5.1582
3     $ 5.073308, 5.028373, 4.984751, 4.938880, 4.8921
3     $ 5.512315, 5.427096, 5.357731, 5.308021, 5.2622
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881. -.002944. -.000841. -.198926.  .339504.  .353729.
531.  .281064.  .251962.  .222401.  .194991.  .169480.
824.  .122585.  .101292.  .080508.  .060595.  .040544.
884.  .001546. -.013779. -.018868. -.003537/

7B4/ -.170206.  .290999.  .298050.  .250600.  .232711.
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898. -.045554. -.011774. -.142107.  .224905.  .236045.
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606.  .076119.  .060939.  .045589.  .029387.  .011614.
874. -.037326. -.078986. -.099355. -.022593. -.114640.
626.  .184444.  .144734.  .132090.  .116396.  .100996.
590.  .075464.  .062939.  .051859.  .040568.  .028478.
859.  .000628. -.017080. -.042533. -.138151. -.282689.
865. -.085889.  .128752.  .114429.  .084707.  .076481.
462.  .057101.  .049136.  .042006.  .034644.  .028103.
819.  .013858.  .006062. -.004365. -.019614. -.022363.
769. -.522002. -.193543. -.035473.  .021226.  .023300.
626.  .014315.  .012416.  .010604.  .009026.  .007662.
841.  .005053.  .004104.  .002340.  .001152.  .001070.
897.  .001837. -.030882. -.166443. -.110505/

3B1/  .012823. -.124150. -.069442. -.030288. -.022530.
630. -.011090. -.008222. -.006270. -.004844. -.003813.
044. -.002445. -.001976. -.001597. -.001280. -.001003.
757. -.000487. -.000026. -.151312.  .909453. 1.126745.
667.  .749165.  .514700.  .402488.  .304620.  .218520.
829.  .140518.  .109859.  .092141.  .071490.  .058595.
008.  .034555.  .026744.  .015776.  .000364. -.140881.
702. 1.925371. 1.305422.  .994210.  .735677.  .564774.
639.  .334347.  .268243.  .212850.  .170390.  .138702.
785.  .088939.  .069897.  .052101.  .036603.  .019780.
439. -.105809. 2.517989. 2.014119. 1.331698. 1.037765.
608.  .627527.  .492816.  .391457.  .313779.  .252546.
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275.  .021223.  .000447. -.086277. 2.172311. 1.803325.
178. 1.016208.  .813013.  .647477.  .518001.  .417269.
640.  .274768.  .223570.  .182123.  .146418.  .116730.
407.  .066103.  .044243.  .022378.  .000460/

3B2/ -.070019. 1.702433. 1.520918. 1.148748.  .957298.
```

```
3     $  .578200,  .559233,  .471265,  .428024,  .379095,
3     $  .288192,  .249820,  .213563,  .181836,  .152738,
3     $  .100940,  .077790,  .056495,  .037645,  .021343,
3     $  .000155, -.018770,  .480933,  .475888,  .406673,
3     $  .328115,  .287828,  .251099,  .217590,  .186667,
3     $  .132653,  .108286,  .085488,  .063739,  .043229,
3     $  .009812,  .000840, -.000069, -.016065,  .420251,
3     $  .344744,  .313372,  .277815,  .243416,  .212485,
3     $  .157668,  .133718,  .111141,  .089303,  .068671,
3     $  .027627,  .008697, -.006648, -.012317, -.000461/
3C
3      DATA T8B4/ -.013563,  .356172,  .340725,  .284037,
3     $  .227350,  .198706,  .173215,  .150157,  .127881,
3     $  .088771,  .069761,  .051311,  .032011,  .010900,
3     $ -.030504, -.035025, -.001270, -.011102,  .276859,
3     $  .222391,  .199815,  .175645,  .153331,  .133165,
3     $  .097603,  .081535,  .065998,  .050515,  .034431,
3     $ -.002308, -.029346, -.067240, -.079523, -.002858,
3     $  .227875,  .201050,  .156672,  .139058,  .121411,
3     $  .090364,  .077929,  .065074,  .054000,  .042863,
3     $  .018915,  .004595, -.011847, -.033173, -.126577,
3     $ -.012511, -.005571,  .139887,  .114633,  .084224,
3     $  .063663,  .054263,  .046549,  .039878,  .032893,
3     $  .020740,  .013835,  .006983, -.002251, -.017543,
3     $ -.163935, -.520170, -.027095, -.000299,  .003324,
3     $  .001880,  .001623,  .001377,  .001170,  .000993,
3     $  .000691,  .000551,  .000413,  .000247,  .000093,
3     $ -.000745,  .000224, -.006217, -.024134, -.002506/
3C
3C     INTERPOLATE NUMERICAL SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRDD,
3C
3      TRUE = DERVSL(6)
3C
3      RETURN
3      END
3      FUNCTION W(X,Y)
3      DATA PI/3.141592656/
3      T = &A*PI*Y
3      W = 3.*T*COS(T*X)/SIN(T*X)
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      DATA PI/3.141592656/
3      T = &A*PI*Y
3      F = &C*T*COS(T*X)/SIN(T*X)**3
3      RETURN
3      END
```

```
'EOR
************
* MACRO  25 *
************
*          2000200002002
1          TWO DIMENSIONS
1          (-X**&A)UXX$ - (Y**&A)UYY$ - (&A*X**(&A-1.))U
1          (&A*Y**(&A-1.))UY$ + ((X*Y)**&A)U$ = F(X,Y)
2          DIRICHLET $ HOMOGENEOUS
2          X=0. , U=0.
2          X=1. , U=0.
2          Y=0. , U=0.
2          Y=1. , U=0.
```

```
3      TUY = EXEY3*(1. - Y - Y*Y)*XMXX
3      TUYY = EXEY3*(-3*Y-Y*Y)*XMXX
3      F = -X**&A *TUXX - Y**&A * TUYY - &A * X**(&A-1.)*
3     $      &A * Y**(&A-1.) * TUY + (X*Y)**&A *TU
3      RETURN
3      END
```

```
'EOR
*************
* MACRO  26 *
*************
*          2000200002002
:          TWO DIMENSIONS
1          UXX$ + UYY$ + A(X)UX$ = F(X)
2          DIRICHLET $ HOMOGENEOUS
2          X=0. , U=0.
2          X=&A , U=0.
2          Y=0. , U=0.
2          Y=&A , U=0.
3      FUNCTION TRUE(X,Y)
3C
3C     ***************************************************
3C     *
3C     *                  MACRO 26 PARAMETERS
3C     *
3C     ***************************************************
3C     *
3C     *                        A  I  B
3C     *                      -----I-----
3C     *                        1  I  1
3C     *                           I
3C     *                        5  I  2
3C     *                           I
3C     *                       10  I  3
3C     *
3C     ***************************************************
3C
3      REAL DERUSL(6), TABLE(20,20), GRID1(20),
3     $     GRID2(20), GRID3(20),
3     $     T1B1(100), T1B2(100), T1B3(100), T1B4(100),
3     $     T2B1(100), T2B2(100), T2B3(100), T2B4(100),
3     $     T3B1(100), T3B2(100), T3B3(100), T3B4(100)
3      EQUIVALENCE (TABLE(1, 1) , T&BB1(1)),
3     $            (TABLE(1, 6) , T&BB2(1)),
3     $            (TABLE(1,11) , T&BB3(1)),
3     $            (TABLE(1,16) , T&BB4(1))
3      DATA NGRID, NGRDD/20, 20/
3      DATA GRID1/0.0000000, 0.0526316, 0.1052632, 0.1578
3     $0.2105263, 0.2631579, 0.3157895, 0.3684211, 0.4210
3     $0.4736842, 0.5263158, 0.5789474, 0.6315789, 0.6842
3     $0.7368421, 0.7894737, 0.8421053, 0.8947368, 0.9473
3     $1.0000000/
3      DATA GRID2/0.0000000, 0.2631579, 0.5263158, 0.7894
3     $1.0526316, 1.3157895, 1.5789474, 1.8421053, 2.1052
3     $2.3684211, 2.6315789, 2.8947368, 3.1578947, 3.4210
3     $3.6842105, 3.9473684, 4.2105263, 4.4736842, 4.7368
3     $5.0000000/
3      DATA GRID3/0.0000000, 0.5163158, 1.0526316, 1.5789
3     $2.1052632, 2.6315789, 3.1578947, 3.6842105, 4.2105
3     $4.7368421, 5.2631579, 5.7894737, 6.3157895, 6.8421
3     $7.3684211, 7.8947368, 8.4210526, 8.9473684, 9.4736
3     $10.000000/
3C
```

```
3     $  .000000,  .000000,  .106618,  .208464,  .3010
3     $  .444612,  .491633,  .521213,  .534100,  .5312
3     $  .486031,  .447462,  .400612,  .347072,  .2879
3     $  .154880,  .080594,  .000000,  .000000,  .1299
3     $  .366840,  .463675,  .541872,  .599148,  .6351
3     $  .646884,  .626105,  .590636,  .542855,  .4848
3     $  .345776,  .267031,  .183171,  .094230,  .0000
3C
3      DATA T1B2/  .000000,  .148361,  .290094,  .4189
3     $  .618776,  .684134,  .725081,  .742592,  .7379
3     $  .672691,  .617395,  .550375,  .473971,  .3900
3     $  .204389,  .104343,  .000000,  .000000,  .1625
3     $  .458858,  .579991,  .677772,  .749306,  .7940
3     $  .807676,  .780758,  .735204,  .674014,  .5999
3     $  .423187,  .324204,  .220118,  .111816,  .0000
3     $  .172777,  .337838,  .487852,  .616634,  .7205
3     $  .844024,  .864032,  .858098,  .829142,  .7802
3     $  .635516,  .545414,  .446823,  .341541,  .2312
3     $  .000000,  .000000,  .179440,  .350867,  .5066
3     $  .748331,  .827217,  .876422,  .897075,  .8907
3     $  .809402,  .741012,  .658381,  .564512,  .4619
3     $  .238331,  .120447,  .000000,  .000000,  .1827
3     $  .515980,  .652177,  .762075,  .842388,  .8924
3     $  .906873,  .875887,  .823785,  .753977,  .6696
3     $  .469371,  .358024,  .241813,  .122095,  .0000
3C
3      DATA T1B3/  .000000,  .182740,  .357318,  .5159
3     $  .762075,  .842388,  .892457,  .913425,  .9068
3     $  .823785,  .753977,  .669666,  .573930,  .4693
3     $  .241813,  .122095,  .000000,  .000000,  .1794
3     $  .506665,  .640407,  .748331,  .827217,  .8764
3     $  .890731,  .860421,  .809402,  .741012,  .6583
3     $  .461928,  .352586,  .238331,  .120447,  .0000
3     $  .172777,  .337838,  .487852,  .616634,  .7205
3     $  .844024,  .864032,  .858098,  .829142,  .7802
3     $  .635516,  .545414,  .446823,  .341541,  .2312
3     $  .000000,  .000000,  .162509,  .317759,  .4588
3     $  .677772,  .749306,  .794040,  .813019,  .8076
3     $  .735204,  .674014,  .599943,  .515620,  .4231
3     $  .220118,  .111816,  .000000,  .000000,  .1483
3     $  .413904,  .529491,  .618776,  .684134,  .7250
3     $  .737993,  .713799,  .672691,  .617395,  .5503
3     $  .390028,  .299795,  .204389,  .104343,  .0000
3C
3      DATA T1B4/  .000000,  .129926,  .254044,  .3668
3     $  .541872,  .599148,  .635105,  .690622,  .6468
3     $  .590636,  .542855,  .484873,  .418680,  .3457
3     $  .183171,  .094230,  .000000,  .000000,  .1066
3     $  .301010,  .380454,  .444612,  .491633,  .5212
3     $  .531294,  .514625,  .486031,  .447462,  .4006
3     $  .287969,  .223801,  .154880,  .080594,  .0000
3     $  .077817,  .152142,  .219668,  .277620,  .3244
3     $  .380349,  .389856,  .388002,  .376143,  .3557
3     $  .294592,  .256281,  .213945,  .167776,  .1178
3     $  .000000,  .000000,  .042634,  .083348,  .1203
3     $  .177656,  .196430,  .208277,  .213527,  .2126
3     $  .195348,  .180582,  .162657,  .142198,  .1196
3     $  .068245,  .038037,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3C
3      DATA T2B1/  .000000,  .000000,  .000000,  .0000
```

```
3     $  .695443,  .811300,  .862805,  .857289,  .8090
3     $  .643302,  .549094,  .457293,  .372718,  .2964
3     $  .170743,  .119718,  .075076,  .035566,  .0000
3C
3      DATA T2B2/  .000000,  .299704,  .566987,  .7744
3     $  .968026,  .965795,  .915086,  .832302,  .7329
3     $  .523721,  .427567,  .340413,  .263309,  .1959
3     $  .085726,  .040429,  .000000,  .000000,  .3210
3     $  .832154,  .976754, 1.045908, 1.046674,  .9947
3     $  .801115,  .687293,  .574760,  .469854,  .3743
3     $  .215416,  .150580,  .093951,  .044183,  .0000
3     $  .335930,  .636759,  .872549, 1.026008, 1.1008
3     $ 1.051508,  .961191,  .850211,  .730602,  .6117
3     $  .399131,  .308845,  .229647,  .160405,  .0999
3     $  .000000,  .000000,  .345240,  .654738,  .8979
3     $ 1.135704, 1.140559, 1.087806,  .995738,  .8818
3     $  .635824,  .520632,  .415256,  .321371,  .2389
3     $  .103868,  .048706,  .000000,  .000000,  .3497
3     $  .910398, 1.072339, 1.152813, 1.158518, 1.1056
3     $  .897483,  .772463,  .647716,  .530542,  .4232
3     $  .243531,  .169990,  .105809,  .049592,  .0000
3C
3      DATA T2B3/  .000000,  .349799,  .663544,  .9103
3     $ 1.152813, 1.158518, 1.105685, 1.012777,  .8974
3     $  .647716,  .530542,  .423247,  .327581,  .2435
3     $  .105809,  .049592,  .000000,  .000000,  .3452
3     $  .897948, 1.057089, 1.135704, 1.140559, 1.0878
3     $  .881856,  .758614,  .635824,  .520632,  .4152
3     $  .238928,  .166812,  .103868,  .048706,  .0000
3     $  .335930,  .636759,  .872549, 1.026008, 1.1008
3     $ 1.051508,  .961191,  .850211,  .730602,  .6117
3     $  .399131,  .308845,  .229647,  .160405,  .0999
3     $  .000000,  .000000,  .321059,  .608088,  .8321
3     $ 1.045908, 1.046674,  .994722,  .907380,  .8011
3     $  .574760,  .469854,  .374373,  .289632,  .2154
3     $  .093951,  .044183,  .000000,  .000000,  .2997
3     $  .774418,  .906644,  .968026,  .965795,  .9150
3     $  .732935,  .627401,  .523721,  .427567,  .3404
3     $  .195928,  .137123,  .085726,  .040429,  .0000
3C
3      DATA T2B4/  .000000,  .270292,  .510525,  .6954
3     $  .862805,  .857289,  .809007,  .732986,  .6433
3     $  .457293,  .372718,  .296468,  .229286,  .1707
3     $  .075076,  .035566,  .000000,  .000000,  .2300
3     $  .588658,  .683554,  .723241,  .714863,  .6711
3     $  .528963,  .449927,  .373668,  .303966,  .2415
3     $  .139261,  .097908,  .061685,  .029424,  .0000
3     $  .176184,  .331016,  .447048,  .515726,  .5418
3     $  .495973,  .444483,  .386486,  .327343,  .2709
3     $  .174553,  .135020,  .100848,  .071197,  .0452
3     $  .000000,  .000000,  .102525,  .191764,  .2571
3     $  .306095,  .297702,  .275360,  .244927,  .2116
3     $  .147093,  .119082,  .094408,  .073060,  .0547
3     $  .025009,  .012448,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3C
3      DATA T3B1/  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .1367
3     $  .300166,  .309204,  .286664,  .248177,  .2049
```

```
3      DATA T3B2/  .000000,  .356408,  .649089,  .8327
3     $  .871521,  .786942,  .673828,  .553568,  .4434
3     $  .267728,  .203343,  .151932,  .111112,  .0786
3     $  .031790,  .014504,  .000000,  .000000,  .3767
3     $  .885147,  .957941,  .935238,  .848686,  .7300
3     $  .484381,  .380883,  .294030,  .223721,  .1673
3     $  .086661,  .058095,  .034918,  .015888,  .0000
3     $  .390468,  .713611,  .920844,  .999654,  .9793
3     $  .769750,  .637142,  .513691,  .404848,  .3130
3     $  .178587,  .130709,  .092514,  .061978,  .0372
3     $  .000000,  .000000,  .398828,  .729526,  .9427
3     $ 1.006835,  .918904,  .794843,  .659279,  .5324
3     $  .325452,  .248203,  .185921,  .136116,  .0963
3     $  .038706,  .017565,  .000000,  .000000,  .4028
3     $  .953417, 1.038004, 1.020257,  .932162,  .8071
3     $  .541764,  .427941,  .331563,  .252981,  .1895
3     $  .098238,  .065777,  .039450,  .017894,  .0000
3C
3      DATA T3B3/  .000000,  .402888,  .737259,  .9534
3     $ 1.020257,  .932162,  .807155,  .670167,  .5417
3     $  .331563,  .252981,  .189557,  .138799,  .0982
3     $  .039450,  .017894,  .000000,  .000000,  .3988
3     $  .942756, 1.025437, 1.006835,  .918904,  .7948
3     $  .532496,  .420307,  .325452,  .248203,  .1859
3     $  .096339,  .064516,  .038706,  .017565,  .0000
3     $  .390468,  .713611,  .920844,  .999654,  .9793
3     $  .769750,  .637142,  .513691,  .404848,  .3130
3     $  .178587,  .130709,  .092514,  .061978,  .0372
3     $  .000000,  .000000,  .376737,  .687549,  .8851
3     $  .935238,  .848686,  .730092,  .602415,  .4843
3     $  .294030,  .223721,  .167338,  .122430,  .0866
3     $  .034918,  .015888,  .000000,  .000000,  .3564
3     $  .832755,  .897181,  .871521,  .786942,  .6738
3     $  .443463,  .347646,  .267728,  .203343,  .1519
3     $  .078670,  .052794,  .031790,  .014504,  .0000
3C
3      DATA T3B4/  .000000,  .327257,  .594199,  .7585
3     $  .783379,  .702581,  .597871,  .488376,  .3893
3     $  .233482,  .176946,  .132044,  .096533,  .0683
3     $  .027761,  .012719,  .000000,  .000000,  .2850
3     $  .653481,  .693679,  .663031,  .589513,  .4977
3     $  .320122,  .248860,  .190431,  .143963,  .1072
3     $  .055604,  .037472,  .022729,  .010482,  .0000
3     $  .225332,  .404604,  .507648,  .532131,  .5020
3     $  .368813,  .296628,  .233567,  .180624,  .1376
3     $  .077267,  .056478,  .040114,  .027141,  .0165
3     $  .000000,  .000000,  .136769,  .242995,  .3001
3     $  .286664,  .248177,  .204969,  .163207,  .1275
3     $  .074440,  .055969,  .041610,  .030429,  .0215
3     $  .009126,  .004383,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3C
3C     INTERPOLATE NUMERICAL SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID&B,GRID&B,NGRID,NGRID
3     $             DERVSL)
3C
3      TRUE = DERVSL(6)
3C
3      RETURN
3      END
```

```
*EOR
************
* MACRO  28 *
************
*          2000002002002
1          TWO DIMENSIONS $ SELF-ADJOINT
1          W(X,Y)UXX$ + W(X,Y)UYY$ * 1.0
2          DIRICHLET $ HOMOGENEOUS
2          X=-1. , U=0.
2          X= 1. , U=0.
2          Y=-1. , U=0.
2          Y= 1. , U=0.
3      FUNCTION TRUE(X,Y)
3C
3C     ***************************************************
3C     *
3C     *                  MACRO 28 PARAMETERS
3C     *
3C     ***************************************************
3C     *
3C     *                        A   I  B
3C     *                      ------I-----
3C     *                          1 I  1
3C     *                            I
3C     *                         10 I  2
3C     *                            I
3C     *                       1000 I  3
3C     *
3C     ***************************************************
3C
3      REAL DERUSL(6), GRID(20), TABLE(20,20),
3     $     T1B1(100), T1B2(100), T1B3(100), T1B4(100),
3     $     T2B1(100), T2B2(100), T2B3(100), T2B4(100),
3     $     T3B1(100), T3B2(100), T3B3(100), T3B4(100)
3      EQUIVALENCE (TABLE(1, 1) , T&BB1(1)),
3     $            (TABLE(1, 6) , T&BB2(1)),
3     $            (TABLE(1,11) , T&BB3(1)),
3     $            (TABLE(1,16) , T&BB4(1))
3      DATA NGRID, NGRDD, GRID /20, 20, -1.000000, -.894
3     $-.7894737, -.6842105, -.5789474, -.4736842, -.368
3     $-.2631579, -.1578947, -.0526316, 0.0526316, 0.157
3     $0.2631579, 0.3684211, 0.4736842, 0.5789474, 0.684
3     $0.7894737, 0.8947368, 1.0000000/
3C
3C     APPROXIMATE SOLUTION OF PROBLEM USING
3C     IYAKANOU-CG4  ITMAX=11, DEMAND=6, 21 X 21 GRID)
3C
3      DATA T1B1/  .000000, -.000125, -.000145, -.000140
3     $ -.000125, -.000119, -.000113, -.000109, -.000106
3     $ -.000109, -.000113, -.000119, -.000125, -.000133
3     $ -.000145, -.000125,  .000000, -.000125, -.018881
3     $ -.041981, -.049498, -.055241, -.059547, -.062627
3     $ -.065587, -.065587, -.064613, -.062627, -.059547
3     $ -.049498, -.041981, -.032143, -.018881, -.000125
3     $ -.032143, -.056504, -.075177, -.089670, -.100830
3     $ -.115263, -.119156, -.121068, -.121068, -.119156
3     $ -.109235, -.100830, -.089670, -.075177, -.056504
3     $ -.000145, -.000140, -.041981, -.075177, -.101358
3     $ -.138030, -.150180, -.158919, -.164577, -.167357
3     $ -.164577, -.158919, -.150180, -.138030, -.121994
3     $ -.075177, -.041981, -.000140, -.000133, -.049498
3     $ -.121994, -.147801, -.168029, -.183444, -.194570
```

```
3     $ -.000113, -.000109, -.064613, -.119156, -.1645
3     $ -.231560, -.254578, -.271360, -.282331, -.2877
3     $ -.282331, -.271360, -.254578, -.231560, -.2017
3     $ -.119156, -.064613, -.000109, -.000106, -.0655
3     $ -.167357, -.205343, -.235770, -.259321, -.2765
3     $ -.293291, -.293291, -.287744, -.276505, -.2593
3     $ -.205343, -.167357, -.121068, -.065587, -.0001
3C
3      DATA T1B3/ -.000106, -.065587, -.121068, -.1673
3     $ -.235770, -.259321, -.276505, -.287744, -.2932
3     $ -.287744, -.276505, -.259321, -.235770, -.2053
3     $ -.121068, -.065587, -.000106, -.000109, -.0646
3     $ -.164577, -.201792, -.231560, -.254578, -.2713
3     $ -.287744, -.287744, -.282331, -.271360, -.2545
3     $ -.201792, -.164577, -.119156, -.064613, -.0001
3     $ -.062627, -.115263, -.158919, -.194570, -.2230
3     $ -.260928, -.271360, -.276505, -.276505, -.2713
3     $ -.244953, -.223009, -.194570, -.158919, -.1152
3     $ -.000113, -.000119, -.059547, -.109235, -.1501
3     $ -.209866, -.230191, -.244953, -.254578, -.2593
3     $ -.254578, -.244953, -.230191, -.209866, -.1834
3     $ -.109235, -.059547, -.000119, -.000125, -.0552
3     $ -.138030, -.168029, -.191719, -.209866, -.2230
3     $ -.235770, -.235770, -.231560, -.223009, -.2098
3     $ -.168029, -.138030, -.100830, -.055241, -.0001
3C
3      DATA T1B4/ -.000133, -.049498, -.089670, -.1219
3     $ -.168029, -.183444, -.194570, -.201792, -.2053
3     $ -.201792, -.194570, -.183444, -.168029, -.1478
3     $ -.089670, -.049498, -.000133, -.000140, -.0419
3     $ -.101358, -.121994, -.138030, -.150180, -.1589
3     $ -.167357, -.167357, -.164577, -.158919, -.1501
3     $ -.121994, -.101358, -.075177, -.041981, -.0001
3     $ -.032143, -.056504, -.075177, -.089670, -.1008
3     $ -.115263, -.119156, -.121068, -.121068, -.1191
3     $ -.109235, -.100830, -.089670, -.075177, -.0565
3     $ -.000145, -.000125, -.018881, -.032143, -.0419
3     $ -.055241, -.059547, -.062627, -.064613, -.0655
3     $ -.064613, -.062627, -.059547, -.055241, -.0494
3     $ -.032143, -.018881, -.000125,  .000000, -.0001
3     $ -.000140, -.000133, -.000125, -.000119, -.0001
3     $ -.000106, -.000106, -.000109, -.000113, -.0001
3     $ -.000133, -.000140, -.000145, -.000125,  .0000
3C
3      DATA T2B1/  .000000, -.000125, -.000145, -.0001
3     $ -.000119, -.000107, -.000093, -.000082, -.0000
3     $ -.000058, -.000060, -.000074, -.000085, -.0001
3     $ -.000137, -.000122,  .000000, -.000125, -.0170
3     $ -.036556, -.042288, -.046292, -.048949, -.0505
3     $ -.051238, -.050636, -.049519, -.047872, -.0456
3     $ -.038827, -.033565, -.026339, -.015913, -.0001
3     $ -.028522, -.049259, -.064310, -.075204, -.0828
3     $ -.090836, -.092091, -.091967, -.090696, -.0884
3     $ -.080933, -.075283, -.067888, -.057987, -.0446
3     $ -.000155, -.000139, -.036556, -.064310, -.0850
3     $ -.110858, -.117815, -.121746, -.123233, -.1227
3     $ -.117273, -.112551, -.106441, -.098505, -.0882
3     $ -.056777, -.032559, -.000164, -.000130, -.0422
3     $ -.100207, -.118629, -.131484, -.139680, -.1440
3     $ -.143985, -.140850, -.136228, -.130168, -.1225
3     $ -.100705, -.084718, -.063891, -.036279, -.0001
3C
3      DATA T2B2/ -.000119, -.046292, -.082843, -.1108
```

```
3     $ -.122758, -.143985, -.156029, -.158805, -.1521
3     $ -.107540, -.088509, -.081210, -.074288, -.0670
3     $ -.050246, -.040189, -.028846, -.015610,  .0008
3C
3      DATA T2B3/ -.000052, -.050636, -.090696, -.1206
3     $ -.151472, -.152242, -.142678, -.119047, -.0885
3     $ -.075672, -.069153, -.062155, -.054503, -.0459
3     $ -.025711, -.013567,  .000020, -.000058, -.0495
3     $ -.117273, -.136228, -.145501, -.144827, -.1337
3     $ -.081210, -.075672, -.070241, -.064254, -.0577
3     $ -.042493, -.033556, -.023554, -.012376,  .0000
3     $ -.047872, -.085218, -.112551, -.130168, -.1382
3     $ -.125190, -.101763, -.074288, -.069153, -.0642
3     $ -.052799, -.046170, -.038815, -.030619, -.0214
3     $ -.000004, -.000074, -.045660, -.080933, -.1064
3     $ -.129650, -.127526, -.116073, -.093424, -.0670
3     $ -.057704, -.052799, -.047408, -.041462, -.0348
3     $ -.019333, -.010185, -.000008, -.000085, -.0427
3     $ -.098505, -.112947, -.118977, -.116542, -.1055
3     $ -.059065, -.054503, -.050495, -.046170, -.0414
3     $ -.030565, -.024182, -.017038, -.009015, -.0000
3C
3      DATA T2B4/ -.000106, -.038827, -.067888, -.0882
3     $ -.105695, -.103198, -.093104, -.073394, -.0502
3     $ -.042493, -.038815, -.034869, -.030565, -.0258
3     $ -.014511, -.007728, -.000013, -.000120, -.0335
3     $ -.074708, -.084718, -.088567, -.086231, -.0775
3     $ -.040189, -.036459, -.033556, -.030619, -.0275
3     $ -.020492, -.016359, -.011671, -.006277, -.0000
3     $ -.026339, -.044642, -.056777, -.063891, -.0665
3     $ -.058265, -.045064, -.028846, -.025711, -.0235
3     $ -.019333, -.017038, -.014511, -.011671, -.0084
3     $ -.000014, -.000122, -.015913, -.026074, -.0325
3     $ -.037621, -.036566, -.032930, -.025293, -.0156
3     $ -.012376, -.011285, -.010185, -.009015, -.0077
3     $ -.004599, -.002581, -.000012,  .000000, -.0001
3     $ -.000164, -.000172, -.000200, -.000224, -.0003
3     $  .000806,  .000020,  .000005, -.000004, -.0000
3     $ -.000013, -.000014, -.000014, -.000012,  .0000
3C
3      DATA T3B1/  .000000, -.000125, -.000145, -.0001
3     $ -.000116, -.000102, -.000083, -.000071, -.0000
3     $ -.000042, -.000041, -.000061, -.000073, -.0000
3     $ -.000135, -.000121,  .000000, -.000125, -.0164
3     $ -.034550, -.039624, -.042993, -.045057, -.0461
3     $ -.046095, -.045345, -.044253, -.042800, -.0409
3     $ -.035322, -.030832, -.024474, -.014964, -.0001
3     $ -.027183, -.046578, -.060289, -.069855, -.0762
3     $ -.081890, -.082262, -.081508, -.079325, -.0777
3     $ -.071370, -.066789, -.060756, -.052425, -.0408
3     $ -.000157, -.000138, -.034550, -.060289, -.0789
3     $ -.100810, -.105876, -.108096, -.108177, -.1066
3     $ -.100741, -.096635, -.091710, -.085432, -.0772
3     $ -.050947, -.029587, -.000170, -.000129, -.0396
3     $ -.092144, -.107819, -.117934, -.123467, -.1253
3     $ -.121752, -.117849, -.113267, -.108086, -.1021
3     $ -.085526, -.072880, -.055817, -.032159, -.0001
3C
3      DATA T3B2/ -.000116, -.042993, -.076201, -.1008
3     $ -.128566, -.133662, -.134274, -.131584, -.1269
3     $ -.115296, -.109297, -.102919, -.095325, -.0858
3     $ -.056022, -.032230, -.000219, -.000102, -.0450
3     $ -.105876, -.123467, -.133662, -.137289, -.1354
```

```
3      DATA T3B3/ -.000031, -.045345, -.079925, -.104105,
3     $ -.121374, -.113508, -.093592, -.054330, -.010127,
3     $ -.009306, -.008394, -.007465, -.006489, -.005436,
3     $ -.003005, -.001581,  .000003, -.000042, -.004253,
3     $ -.100741, -.113267, -.115296, -.106166, -.085096,
3     $ -.008866, -.009306, -.008554, -.007747, -.006892,
3     $ -.004996, -.003920, -.002735, -.001430,  .000001,
3     $ -.042800, -.074890, -.096635, -.108086, -.109297,
3     $ -.079162, -.044400, -.007917, -.008394, -.007747,
3     $ -.006264, -.005435, -.004536, -.003555, -.002477,
3     $ -.000000, -.000061, -.040961, -.071370, -.091710,
3     $ -.102919, -.093617, -.074048, -.041352, -.006992,
3     $ -.006892, -.006264, -.005584, -.004848, -.004049,
3     $ -.002215, -.001160, -.000001, -.000073, -.038541,
3     $ -.085432, -.094851, -.095325, -.086596, -.068520,
3     $ -.006014, -.006489, -.005981, -.005435, -.004848,
3     $ -.003525, -.002770, -.001938, -.001019, -.000001/
3C
3      DATA T3B4/ -.000099, -.035322, -.060756, -.077292,
3     $ -.085872, -.078089, -.062040, -.034576, -.005051,
3     $ -.004996, -.004536, -.004049, -.003525, -.002955,
3     $ -.001639, -.000867, -.000001, -.000114, -.030832,
3     $ -.066158, -.072880, -.073105, -.066603, -.053199,
3     $ -.003883, -.004283, -.003920, -.003555, -.003175,
3     $ -.002330, -.001847, -.001308, -.000698, -.000001,
3     $ -.024474, -.040846, -.050947, -.055817, -.056022,
3     $ -.041601, -.023489, -.002798, -.003005, -.002735,
3     $ -.002215, -.001938, -.001639, -.001308, -.000936,
3     $ -.000001, -.000121, -.014964, -.024140, -.029587,
3     $ -.032230, -.029668, -.024230, -.013647, -.001308,
3     $ -.001430, -.001294, -.001160, -.001019, -.000867,
3     $ -.000507, -.000281, -.000001,  .000000, -.000128,
3     $ -.000170, -.000179, -.000219, -.000244, -.000378,
3     $  .001546,  .000003,  .000001, -.000000, -.000001,
3     $ -.000001, -.000001, -.000001, -.000001,  .000000/
3C
3C    INTERPOLATE NUMERICAL SOLUTION BY QUADRATICS
3C
3     CALL QUADRD(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRDD,
3C
3     TRUE = DERVSL(6)
3C
3     RETURN
3     END
3     FUNCTION W(X,Y)
3     W = 1.
3     IF (X .LT. 0. .OR. Y .LT. 0.) RETURN
3     W = &A
3     RETURN
3     END
3     FUNCTION CDXU(X,Y)
3     CDXU = 0.
3     RETURN
3     END
3     FUNCTION CDYU(X,Y)
3     CDYU = 0.
3     RETURN
3     END
```

```
'EOR
**********
* MACRO  29 *
**********
```

```
3C     *
3C     ***********************************************
3C     *
3C     *                    A  I  B
3C     *                   ----I----
3C     *                   -3  I  1
3C     *                       I
3C     *                   -1  I  2
3C     *                       I
3C     *                    1  I  3
3C     *                       I
3C     *                    3  I  4
3C     *                       I
3C     *                    5  I  5
3C     *
3C     ***********************************************
3C
3      REAL DERUSL(6), GRID(20), TABLE(20,20),
3     $     T1B1(100), T1B2(100), T1B3(100), T1B4(100)
3     $     T2B1(100), T2B2(100), T2B3(100), T2B4(100)
3     $     T3B1(100), T3B2(100), T3B3(100), T3B4(100)
3     $     T4B1(100), T4B2(100), T4B3(100), T4B4(100)
3     $     T5B1(100), T5B2(100), T5B3(100), T5B4(100)
3      EQUIVALENCE (TABLE(1, 1) , T&BB1(1)),
3     $            (TABLE(1, 6) , T&BB2(1)),
3     $            (TABLE(1,11) , T&BB3(1)),
3     $            (TABLE(1,16) , T&BB4(1))
3      DATA NGRID, NGRDD, GRID /20, 20,  0.000000, 0.0
3     $0.1052632, 0.1578947, 0.2105263, 0.2631579, 0.3
3     $0.3684211, 0.4210526, 0.4736842, 0.5263158, 0.5
3     $0.6315789, 0.6842105, 0.7368421, 0.7894737, 0.8
3     $0.8947368, 0.9473684, 1.0000000/
3C
3C     APPROXIMATE SOLUTION OF PROBLEM USING
3C     P3-C1 COLLOCATION (8 X 8 GRID)
3C
3      DATA T1B1/  .000000, -.017544, -.035088, -.0526
3     $ -.087719, -.105263, -.122807, -.140351, -.1578
3     $ -.192982, -.210526, -.228070, -.245614, -.2631
3     $ -.298246, -.315789, -.333333,  .017544, -.0093
3     $ -.052066, -.069910, -.087635, -.105258, -.1228
3     $ -.157943, -.175454, -.192994, -.210590, -.2280
3     $ -.263221, -.280295, -.299006, -.315886, -.3157
3     $ -.003928, -.030358, -.050572, -.069183, -.0871
3     $ -.122592, -.140175, -.157761, -.175294, -.1928
3     $ -.227753, -.245133, -.262279, -.278698, -.2946
3     $ -.298246,  .052632,  .008113, -.022760, -.0459
3     $ -.085491, -.103763, -.121716, -.139484, -.1571
3     $ -.192134, -.209518, -.226619, -.243522, -.2597
3     $ -.286298, -.290354, -.280702,  .070175,  .0239
3     $ -.038130, -.061000, -.081863, -.101131, -.1196
3     $ -.155627, -.173151, -.190460, -.207536, -.2240
3     $ -.254583, -.266504, -.274840, -.274665, -.2631
3C
3      DATA T1B2/  .087719,  .040451,  .002569, -.0279
3     $ -.076171, -.096739, -.116130, -.134695, -.1527
3     $ -.187332, -.203870, -.219558, -.234100, -.2467
3     $ -.260683, -.257850, -.245614,  .105263,  .0573
3     $ -.015458, -.043422, -.068137, -.090047, -.1104
3     $ -.148082, -.165632, -.182446, -.198439, -.2130
3     $ -.236981, -.243769, -.246119, -.241161, -.2280
3     $  .074559,  .033319, -.001588, -.031563, -.0580
3     $ -.102829, -.122768, -.141406, -.158954, -.1754
```

```
3     $  .065868,  .032061,  .001844, -.025185, -.0494
3     $ -.090982, -.108530, -.123975, -.137237, -.1480
3     $ -.160963, -.162215, -.159438, -.152224, -.1403
3     $  .164593,  .123044,  .085589,  .051896,  .0215
3     $ -.030059, -.052101, -.071815, -.089370, -.1047
3     $ -.128458, -.136395, -.141211, -.142517, -.1401
3     $ -.122807,  .228070,  .183677,  .143248,  .1065
3     $  .043463,  .016440, -.007961, -.029942, -.0496
3     $ -.082579, -.095707, -.106387, -.114452, -.1195
3     $ -.119915, -.114558, -.105263,  .245614,  .2030
3     $  .129038,  .096869,  .067742,  .041222,  .0171
3     $ -.024183, -.041729, -.057183, -.070497, -.0815
3     $ -.095982, -.098975, -.098583, -.094947, -.0877
3C
3      DATA T1B4/  .263158,  .223164,  .186771,  .1533
3     $  .094828,  .069220,  .045823,  .024490,  .0050
3     $ -.028133, -.041853, -.053556, -.063003, -.0701
3     $ -.076179, -.074817, -.070175,  .280702,  .2442
3     $  .180002,  .151578,  .125338,  .101068,  .0786
3     $  .038764,  .021220,  .005257, -.009094, -.0217
3     $ -.041393, -.048063, -.052331, -.053958, -.0526
3     $  .265675,  .236579,  .209214,  .183535,  .1598
3     $  .116143,  .096355,  .077647,  .060104,  .0437
3     $  .014640,  .002247, -.008879, -.018628, -.0259
3     $ -.035088,  .315789,  .289347,  .265378,  .2424
3     $  .199541,  .179230,  .159725,  .140957,  .1227
3     $  .088355,  .072156,  .056530,  .041817,  .0277
3     $  .002626, -.008402, -.017544,  .333333,  .3157
3     $  .280702,  .263158,  .245614,  .228070,  .2105
3     $  .175439,  .157895,  .140351,  .122807,  .1052
3     $  .070175,  .052632,  .035088,  .017544,  .0000
3C
3      DATA T2B1/  .000000, -.052632, -.105263, -.1578
3     $ -.263158, -.315789, -.368421, -.421053, -.4736
3     $ -.578947, -.631579, -.684211, -.736842, -.7894
3     $ -.894737, -.947368,-1.000000,  .052632, -.0184
3     $ -.148470, -.203197, -.257116, -.310570, -.3636
3     $ -.469353, -.521890, -.574395, -.626929, -.6789
3     $ -.782659, -.832877, -.883945, -.928698, -.9473
3     $  .016487, -.059543, -.124162, -.183241, -.2398
3     $ -.349241, -.402782, -.455924, -.508523, -.5607
3     $ -.663414, -.713606, -.762367, -.808441, -.8510
3     $ -.894737,  .157895,  .062527, -.018256, -.0882
3     $ -.212648, -.270109, -.326005, -.380681, -.4342
3     $ -.538636, -.589469, -.638749, -.686708, -.7319
3     $ -.808486, -.832691, -.842105,  .210526,  .1140
3     $ -.045374, -.113404, -.177483, -.237384, -.2950
3     $ -.404999, -.457617, -.508870, -.558675, -.6061
3     $ -.693671, -.730167, -.761331, -.781428, -.7894
3C
3      DATA T2B2/  .263158,  .166640,  .079902,  .0017
3     $ -.136089, -.198361, -.257606, -.314339, -.3689
3     $ -.472249, -.520841, -.566837, -.609867, -.6489
3     $ -.709760, -.728217, -.736842,  .315789,  .2198
3     $  .052444, -.021066, -.089656, -.153551, -.2141
3     $ -.327039, -.379682, -.429956, -.477742, -.5222
3     $ -.600682, -.632002, -.657654, -.675148, -.6842
3     $  .273530,  .185941,  .105241,  .030469, -.0394
3     $ -.166261, -.224719, -.280129, -.332770, -.3826
3     $ -.473261, -.513414, -.549001, -.579153, -.6038
3     $ -.631579,  .421053,  .327628,  .240616,  .1596
3     $  .013880, -.052204, -.114462, -.173266, -.2289
3     $ -.331162, -.377628, -.420630, -.459811, -.4946
```

```
3     $  .122120,  .063471,  .007914, -.044743, -.0944
3     $ -.184554, -.224699, -.261348, -.294280, -.3230
3     $ -.368421,  .684211,  .602248,  .524256,  .4500
3     $  .312157,  .248120,  .187090,  .128944,  .0735
3     $ -.023548, -.076069, -.120310, -.161519, -.1995
3     $ -.265178, -.292446, -.315789,  .736842,  .6579
3     $  .511174,  .442595,  .377089,  .314366,  .2542
3     $  .141780,  .089112,  .038916, -.008753, -.0540
3     $ -.136139, -.172985, -.206238, -.236465, -.2631
3C
3      DATA T2B4/  .789474,  .714222,  .642438,  .5735
3     $  .443912,  .382753,  .323829,  .267063,  .2124
3     $  .109209,  .060791,  .014356, -.029609, -.0713
3     $ -.146704, -.180213, -.210526,  .842105,  .7711
3     $  .637482,  .574165,  .512913,  .453548,  .3959
3     $  .285832,  .233198,  .182184,  .132813,  .085
3     $ -.004764, -.046724, -.086407, -.123590, -.1578
3     $  .828229,  .764759,  .703040,  .642868,  .5843
3     $  .470982,  .416093,  .362481,  .309772,  .2582
3     $  .158715,  .111037,  .064438,  .018966, -.0240
3     $ -.105263,  .947368,  .886810,  .828426,  .7710
3     $  .658819,  .603766,  .549348,  .495547,  .4424
3     $  .337707,  .286357,  .235391,  .185350,  .1358
3     $  .039378, -.007519, -.052632, 1.000000,  .9473
3     $  .842105,  .789474,  .736842,  .684211,  .6315
3     $  .526316,  .473684,  .421053,  .368421,  .3157
3     $  .210526,  .157895,  .105263,  .052632,  .0000
3C
3      DATA T3B1/  .000000,  .052632,  .105263,  .157
3     $  .263158,  .315789,  .368421,  .421053,  .4736
3     $  .578947,  .631579,  .684211,  .736842,  .7894
3     $  .894737,  .947368, 1.000000, -.052632, -.1043
3     $ -.262116, -.272181, -.293492, -.274777, -.2544
3     $ -.182934, -.132360, -.073179, -.003713,  .0815
3     $  .273606,  .406404,  .524861,  .707392,  .9473
3     $ -.164065, -.231758, -.264326, -.273787, -.2784
3     $ -.237771, -.206132, -.161185, -.109268, -.0487
3     $  .102532,  .190674,  .293646,  .414437,  .5408
3     $  .894737, -.157895, -.201543, -.228105, -.2410
3     $ -.230304, -.209945, -.180645, -.143053, -.0980
3     $  .014856,  .082709,  .158694,  .243620,  .3381
3     $  .562403,  .694826,  .842105, -.210526, -.2433
3     $ -.266856, -.263200, -.248826, -.226531, -.1958
3     $ -.111986, -.059281,  .000565,  .067608,  .1422
3     $  .316817,  .417898,  .530785,  .653952,  .7894
3C
3      DATA T3B2/ -.263158, -.284434, -.294965, -.2959
3     $ -.270890, -.246243, -.214123, -.174897, -.1289
3     $ -.016999,  .049084,  .122232,  .202854,  .2913
3     $  .494767,  .610710,  .736842, -.315789, -.3299
3     $ -.328767, -.316784, -.296327, -.269691, -.2359
3     $ -.149164, -.096430, -.037501,  .027617,  .0991
3     $  .263226,  .355965,  .457730,  .566747,  .6842
3     $ -.375145, -.372551, -.363454, -.347586, -.3243
3     $ -.260013, -.218564, -.171763, -.119051, -.0606
3     $  .073388,  .149693,  .232192,  .321164,  .4179
3     $  .631579, -.421053, -.420442, -.413283, -.3997
3     $ -.354440, -.323089, -.286214, -.243975, -.1965
3     $ -.086080, -.023049,  .045341,  .119252,  .1988
3     $  .376217,  .474334,  .578947, -.473684, -.4687
3     $ -.438444, -.415643, -.386610, -.353484, -.3149
3     $ -.223610, -.170842, -.113410, -.051378,  .0154
3     $  .164367,  .246292,  .334647,  .427671,  .5263
```

```
3     $ -.235313, -.177215, -.116293, -.052414,  .0144
3     $  .158030,  .235011,  .315789, -.736842, -.7091
3     $ -.641629, -.603775, -.562507, -.519474, -.4736
3     $ -.375069, -.322306, -.267323, -.210193, -.1506
3     $ -.024120,  .042961,  .113444,  .186604,  .2631
3C
3      DATA T3B4/ -.789474, -.757741, -.721485, -.6831
3     $ -.598737, -.553827, -.506561, -.457198, -.4066
3     $ -.299308, -.243115, -.184998, -.124821, -.0627
3     $  .068671,  .138074,  .210526, -.842105, -.8053
3     $ -.723912, -.680122, -.634527, -.587437, -.5388
3     $ -.437570, -.384927, -.330963, -.275671, -.2189
3     $ -.101112, -.039668,  .023764,  .089453,  .1578
3     $ -.854617, -.810225, -.764943, -.718479, -.6698
3     $ -.571204, -.519912, -.468482, -.415666, -.3620
3     $ -.252345, -.195923, -.138598, -.080211, -.0199
3     $  .105263, -.947368, -.901763, -.853271, -.8043
3     $ -.704039, -.653443, -.602053, -.550034, -.4980
3     $ -.392150, -.338693, -.284712, -.230199, -.1752
3     $ -.063292, -.006182,  .052632,-1.000000, -.9473
3     $ -.842105, -.789474, -.736842, -.684211, -.6315
3     $ -.526316, -.473684, -.421053, -.368421, -.3157
3     $ -.210526, -.157895, -.105263, -.052632,  .0000
3C
3      DATA T4B1/  .000000,  .017544,  .035088,  .0526
3     $  .087719,  .105263,  .122807,  .140351,  .1578
3     $  .192982,  .210526,  .228070,  .245614,  .2631
3     $  .298246,  .315789,  .333333, -.017544,  .7790
3     $ 3.938558, 4.862859, 6.117348, 6.426946, 6.5502
3     $ 6.601650, 6.596406, 6.658000, 6.732147, 6.7139
3     $ 5.769648, 4.538145, 3.788746, 2.099877,  .3157
3     $ -.281251, -.124527, -.151831, -.209368, -.0496
3     $ -.125276, -.110463, -.105180, -.095548, -.0915
3     $  .076983,  .190508,  .231464,  .205238,  .4286
3     $  .298246, -.052632, -.521119, -.903987,-1.2771
3     $-1.788466,-1.933602,-1.998913,-2.010344,-2.0035
3     $-1.958244,-1.905362,-1.801071,-1.616921,-1.3565
3     $ -.593222, -.164227,  .280702, -.070175, -.3384
3     $ -.668069, -.823993, -.889374, -.971236,-1.0112
3     $-1.013631, -.997771, -.965932, -.914458, -.8332
3     $ -.567530, -.407092, -.152796,  .051074,  .2631
3C
3      DATA T4B2/ -.087719, -.233716, -.351070, -.4603
3     $ -.612980, -.660740, -.686680, -.694155, -.6888
3     $ -.641655, -.597507, -.535511, -.451684, -.3485
3     $ -.076052,  .079140,  .245614, -.105263, -.2043
3     $ -.352243, -.411200, -.449739, -.480297, -.4949
3     $ -.489151, -.471305, -.443364, -.404904, -.3547
3     $ -.210532, -.120470, -.009589,  .104826,  .2280
3     $ -.190775, -.241065, -.287155, -.324733, -.3476
3     $ -.372808, -.369912, -.360745, -.342965, -.3171
3     $ -.240887, -.187413, -.125739, -.056188,  .0286
3     $  .210526, -.140351, -.184985, -.222617, -.2533
3     $ -.293103, -.302121, -.304030, -.299167, -.2879
3     $ -.246532, -.216263, -.179260, -.135148, -.0838
3     $  .040710,  .113479,  .192982, -.157895, -.1921
3     $ -.237245, -.252414, -.259829, -.263839, -.2614
3     $ -.241066, -.223309, -.200490, -.172697, -.1399
3     $ -.056900, -.007999,  .049111,  .109557,  .1754
3C
3      DATA T4B3/ -.175439, -.199402, -.216444, -.2294
3     $ -.240709, -.240109, -.234741, -.224928, -.2115
3     $ -.172253, -.146551, -.116805, -.082335, -.0437
```

```
3     $ -.042360. -.014041.  .017239.  .050872.  .0877
3C
3      DATA T4B4/ -.263158. -.262081. -.257005. -.2499
3     $ -.229681. -.217448. -.203552. -.188204. -.1719
3     $ -.135557. -.115580. -.094348. -.071556. -.0473
3     $  .006752.  .037022.  .070175. -.280702. -.2755
3     $ -.257735. -.246280. -.233442. -.219533. -.2046
3     $ -.172053. -.154503. -.136110. -.116865. -.0967
3     $ -.053246. -.029640. -.004409.  .022809.  .0526
3     $ -.290171. -.278645. -.266112. -.252624. -.2376
3     $ -.206658. -.189959. -.173085. -.155460. -.1373
3     $ -.099530. -.079581. -.059033. -.037729. -.0148
3     $  .035088. -.315789. -.303547. -.288933. -.2737
3     $ -.241813. -.225459. -.208645. -.191466. -.1742
3     $ -.138829. -.120788. -.102480. -.083793. -.0648
3     $ -.025403. -.004737.  .017544. -.333333. -.3157
3     $ -.280702. -.263158. -.245614. -.228070. -.2105
3     $ -.175439. -.157895. -.140351. -.122807. -.1052
3     $ -.070175. -.052632. -.035088. -.017544.  .0000
3C
3      DATA T5B1/  .000000.  .010526.  .021053.  .0315
3     $  .052632.  .063158.  .073684.  .084211.  .0947
3     $  .115789.  .126316.  .136842.  .147368.  .1578
3     $  .178947.  .189474.  .200000. -.010526.  .6812
3     $ 3.845816. 4.555738. 5.907821. 6.453454. 7.2441
3     $ 7.859281. 7.858755. 7.914327. 7.677588. 7.0388
3     $ 5.661400. 4.360014. 3.805995. 2.127885.  .1894
3     $-1.048229.-1.487861.-2.066370.-2.628114.-2.9010
3     $-3.631497.-3.656860.-3.713229.-3.706540.-3.6244
3     $-3.155918.-2.624656.-2.168619.-1.798501.-1.0199
3     $  .178947. -.031579. -.855954.-1.378015.-1.9088
3     $-2.710588.-3.129259.-3.369029.-3.447171.-3.4884
3     $-3.412554.-3.251044.-2.976582.-2.535141.-2.1121
3     $-1.070335. -.469565.  .168421. -.042105. -.3714
3     $ -.407197. -.535131. -.474548. -.607559. -.6070
3     $ -.556805. -.542101. -.501362. -.452700. -.4108
3     $ -.169858. -.170875.  .081414.  .130547.  .1578
3C
3      DATA T5B2/ -.052632. -.225883. -.324268. -.4314
3     $ -.585268. -.662092. -.700603. -.708689. -.7138
3     $ -.676353. -.631690. -.568728. -.470424. -.3733
3     $ -.129056.  .004746.  .147368. -.063158. -.1676
3     $ -.307553. -.367719. -.406968. -.449560. -.4726
3     $ -.479430. -.468393. -.446949. -.413564. -.3684
3     $ -.236533. -.164422. -.066036.  .031173.  .1368
3     $ -.134875. -.170540. -.207148. -.238737. -.2550
3     $ -.282607. -.280775. -.277503. -.266577. -.2488
3     $ -.195461. -.154409. -.110903. -.065386. -.0012
3     $  .126316. -.084211. -.119275. -.147730. -.1705
3     $ -.200401. -.208349. -.211380. -.209573. -.2033
3     $ -.177986. -.158619. -.134516. -.105513. -.0719
3     $  .010456.  .060070.  .115789. -.094737. -.1206
3     $ -.151462. -.161565. -.165974. -.168974. -.1675
3     $ -.155206. -.144482. -.130676. -.113857. -.0941
3     $ -.043340. -.013548.  .022789.  .061583.  .1052
3C
3      DATA T5B3/ -.105263. -.123104. -.134644. -.1429
3     $ -.149109. -.148384. -.144634. -.138223. -.1300
3     $ -.106595. -.091501. -.074182. -.053871. -.0311
3     $  .024365.  .057337.  .094737. -.115789. -.1282
3     $ -.140924. -.142652. -.141579. -.138469. -.1332
3     $ -.117271. -.106718. -.094467. -.080494. -.0646
3     $ -.026622. -.003969.  .021879.  .051007.  .0842
```

```
3     $ -.082267, -.070591, -.058256, -.045046, -.030954,
3     $  .001454,  .020411,  .042105, -.168421, -.166859,
3     $ -.156940, -.149881, -.141845, -.133140, -.123867,
3     $ -.103979, -.093449, -.082536, -.071224, -.059442,
3     $ -.034062, -.020083, -.004845,  .012134,  .031579,
3     $ -.175391, -.168879, -.161370, -.153125, -.143910,
3     $ -.124808, -.114637, -.104422, -.093851, -.083056,
3     $ -.060764, -.049059, -.036978, -.024338, -.010587,
3     $  .021053, -.189474, -.182933, -.174334, -.165200,
3     $ -.145781, -.135822, -.125619, -.115234, -.104859,
3     $ -.083654, -.072916, -.062044, -.050975, -.039742,
3     $ -.016201, -.003611,  .010526, -.200000, -.189474,
3     $ -.168421, -.157895, -.147368, -.136842, -.126316,
3     $ -.105263, -.094737, -.084211, -.073684, -.063158,
3     $ -.042105, -.031579, -.021053, -.010526,  .000000/
3C
3C    INTERPOLATE NUMERICAL SOLUTION BY QUADRATICS
3C
3     CALL QUADR2(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRDD,
3C
3     TRUE = DERVSL(6)
3C
3     RETURN
3     END
3     FUNCTION W(Y)
3     W = 0.
3     IF (Y .EQ. 0.) RETURN
3     W = &A/Y
3     RETURN
3     END
```

```
'EOR
************
* MACRO  30 *
************
*          2000000002000
1          TWO DIMENSIONS
1          COEF1(X,Y)UXX$ + COEF2(X,Y)UYY$ + COEF3(X,Y)U$
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3     FUNCTION COEF1(X,Y)
3     COMMON /CONCOM/ A,B,C
3     DATA A,B,C /&A,&B,&C/
3     COEF1 = 2. + (Y-1.)*EXP(-A*Y**4)
3     RETURN
3     END
3     FUNCTION COEF2(X,Y)
3     COMMON /CONCOM/ A,B,C
3     COEF2 = 1. + 1./(1.+(2.*X)**B)
3     RETURN
3     END
3     FUNCTION COEF3(X,Y)
3     COMMON /CONCOM/ A,B,C
3     COEF3 = C*( X*(X-1.) + (Y-.3)*(Y-.7) )
3     RETURN
3     END
3     FUNCTION F(X,Y)
3     COMMON /CONCOM/ A,B,C
3     F = 0.0
3     IF ((X .EQ. 0.0) .AND. (B .LE. 3.0))  RETURN
```

```
3       S3 = SIN(X*Y)
3       XY3 = X+Y
3       U3 = C*XY3*C3
3       UXX3 = -Y*(2.*C*S3 + Y*U3)
3       UYY3 = -X*(2.*C*S3 + X*U3)
3       COE1 = 2. + (Y-1.)*E2
3       COE2 = 1. + 1./(1.+2.*X*P1)
3       COE3 = C*(X*(X-1.)+(Y-.3)*(Y-.7))
3       F = COE1*(UXX1+UXX2+UXX3) + COE2*(UYY1+UYY2+UYY3)
3       RETURN
3       END
3       FUNCTION TRUE(X,Y)
3       COMMON /CONCOM/ A,B,C
3       TRUE = (X+Y*Y)/(1.+(2.*X)**(B-1.))
3      $      + (Y-1.)*(1.+X)*EXP(-A*Y**4)
3      $      + C*(X+Y)*COS(X*Y)
3       RETURN
3       END
```

```
'EOR
*************
* MACRO  31 *
*************
```

```
'EOR
*************
* MACRO  32 *
*************
```

```
'EOR
*************
* MACRO  33 *
*************
```

```
'EOR
*************
* MACRO  34 *
*************
```

```
'EOR
*************
* MACRO  35 *
*************
*           2022021200020
1           TWO DIMENSIONS $ CONSTANT COEFFICIENTS $ HOMOG
1           POISSON $ LAPLACE
1           UXX$ + UYY$ = 0.
2           MIXED
2           X=-1. , MIXED = (1.+(&A))U + (-(&A))UX = G(X,Y
2           X= 1. , MIXED = (1.+(&A))U +    (&A)UX = G(X,Y
2           Y=-1. ,                  U             = TRUE(
2           Y= 1. ,                  U             = TRUE(
3       FUNCTION TRUE(X,Y)
3       X2 = X*X
3       X4 = X2*X2
3       X6 = X4*X2
3       X8 = X6*X2
3       Y2 = Y*Y
3       Y4 = Y2*Y2
3       Y6 = Y4*Y2
3       Y8 = Y6*Y2
3       U1 = X4 - 6.*X2*Y2 + Y4
3       U2 = X8 - 28.*X6*Y2 + 70.*X4*Y4 - 28.*X2*Y6 + Y8
```

```
'EOR
************
* MACRO  36 *
************
*          2000200002000
1          TWO DIMENSIONS
1          (1.+(&B))UXX$ + (1./(X+(&A))**2)UYY$ + (2./(X+
1.         E(X,Y)UY$ = F(X,Y)
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3      FUNCTION TRUE(X,Y)
3      TRUE = (1.-(&B))*EXP(X+Y)
3      IF (X+(&A) .NE. 0.) TRUE = TRUE + (&B)*ALOG(X+(&A)
3      RETURN
3      END
3      FUNCTION E(X,Y)
3      E = 1./((X+(&A))*TAN(Y))
3      RETURN
3      END
3      FUNCTION F(X,Y)
3      TEMP = (1.-(&B))*EXP(X+Y)
3      UX = TEMP + (&B)/(X+(&A))
3      UXX= TEMP - (&B)/(X+(&A))**2
3      UY = TEMP
3      UYY= TEMP
3      F = (1.+(&B))*UXX + 1./(X+(&A))**2*UYY +
3     $    2./(X+(&A))*UX + E(X,Y)*UY
3      RETURN
3      END
-------------------------------------------------------------
'EOR
************
* MACRO  37 *
************
-------------------------------------------------------------
'EOR
************
* MACRO  38 *
************
*          2022021200200
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS $ HOMOG
1          POISSON $ LAPLACE
1          UXX$ + UYY$ = 0.
2          X=-1.570796327 , U =TRUE(X,Y)
2          X= 1.570796327 , U =TRUE(X,Y)
2          Y= 0.          , UY=G(X)
2          Y= 1.          , U =TRUE(X,Y)
3      FUNCTION TRUE(X,Y)
3      TEMP = 2.*(&A) + 1.
3      TRUE = EXP(-SQRT(TEMP))*COS(TEMP*X)*SINH(TEMP*Y)/T
3      RETURN
3      END
3      FUNCTION G(X)
3      TEMP = 2.*(&A) + 1.
3      G = EXP(-SQRT(TEMP))*COS(TEMP*X)
3      RETURN
3      END
3      FUNCTION SINH(X)
3      EXPX = EXP(X)
3      SINH = 0.5*(EXPX - 1./EXPX)
```

```
2          Y=0. , U=1.0
2          Y=1. , U=1.0
3       FUNCTION TRUE(X,Y)
3C
3C      ************************************************
3C      *
3C      *                MACRO 39 PARAMETERS
3C      *
3C      ************************************************
3C      *
3C      *              A  I    B   I   H(X)
3C      *             ----I-------I---------
3C      *              1  I  0.50  I   1/X
3C      *                 I       I
3C      *              2  I  1.00  I   1/X
3C      *                 I       I
3C      *              3  I  0.25  I  EXP(X)
3C      *                 I       I
3C      *              4  I  0.50  I  EXP(X)
3C      *                 I       I
3C      *              5  I  1.00  I  EXP(X)
3C      *
3C      ************************************************
3C
3       REAL DERVSL(6), GRID(20), TABLE(20,20),
3      $     T1B1(100), T1B2(100), T1B3(100), T1B4(100),
3      $     T2B1(100), T2B2(100), T2B3(100), T2B4(100),
3      $     T3B1(100), T3B2(100), T3B3(100), T3B4(100),
3      $     T4B1(100), T4B2(100), T4B3(100), T4B4(100),
3      $     T5B1(100), T5B2(100), T5B3(100), T5B4(100)
3       EQUIVALENCE (TABLE(1, 1) , T&AB1(1)),
3      $            (TABLE(1, 6) , T&AB2(1)),
3      $            (TABLE(1,11) , T&AB3(1)),
3      $            (TABLE(1,16) , T&AB4(1))
3       DATA NGRID, NGRDD, GRID /20, 20, 0.0000000, 0.0526
3      $0.1052632, 0.1578947, 0.2105263, 0.2631579, 0.3157
3      $0.3684211, 0.4210526, 0.4736842, 0.5263158, 0.5789
3      $0.6315789, 0.6842105, 0.7368421, 0.7894737, 0.8421
3      $0.8947368, 0.9473684, 1.0000000/
3C
3C      APPROXIMATE SOLUTION OF LINEARIZED PROBLEM USING
3C      HODIE-ACF (METHOD=4, IORDER=41, 20 X 20 GRID)
3C
3       DATA T1B1/ 1.000000, 1.000000, 1.000000, 1.000000,
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3      $ 1.000000, 1.000000, 1.000000, 1.000000,  .614500,
3      $  .761544,  .809864,  .844607,  .870740,  .891357,
3      $  .921398,  .933918,  .944293,  .953488,  .961733,
3      $  .976105,  .982513,  .988561,  .994352, 1.000000,
3      $  .559762,  .567460,  .640266,  .696171,  .741633,
3      $  .810437,  .837105,  .860150,  .880389,  .898392,
3      $  .929362,  .942920,  .955513,  .967335,  .978571,
3      $ 1.000000, 1.000000,  .556496,  .529362,  .579361,
3      $  .673668,  .714455,  .750711,  .782923,  .811686,
3      $  .861095,  .882600,  .902424,  .920837,  .938095,
3      $  .970031,  .985148, 1.000000, 1.000000,  .551048,
3      $  .545252,  .586293,  .628471,  .669069,  .707008,
3      $  .774235,  .803898,  .831298,  .856723,  .880431,
3      $  .923671,  .943671,  .962898,  .981590, 1.000000/
3C
3       DATA T1B2/ 1.000000,  .550152,  .499893,  .526003,
3      $  .598645,  .637604,  .675565,  .711758,  .745896,
```

```
3     $  .696809,  .731899,  .765920,  .798735,  .8303
3     $  .889994,  .918371,  .946013,  .973141, 1.0000
3C
3      DATA T1B3/ 1.000000,  .551712,  .490661,  .5030
3     $  .555646,  .589295,  .624831,  .660951,  .6968
3     $  .765920,  .798734,  .830309,  .860691,  .8899
3     $  .946013,  .973141, 1.000000, 1.000000,  .5516
3     $  .504643,  .528520,  .559112,  .593351,  .6292
3     $  .701274,  .736159,  .769867,  .802292,  .8334
3     $  .892124,  .919980,  .947091,  .973682, 1.0000
3     $  .550921,  .492412,  .508229,  .534235,  .5665
3     $  .638454,  .674880,  .710489,  .744904,  .7779
3     $  .839749,  .868665,  .896434,  .923233,  .9492
3     $ 1.000000, 1.000000,  .550434,  .494926,  .5147
3     $  .579040,  .616098,  .653410,  .689916,  .7250
3     $  .790485,  .820746,  .849489,  .876857,  .9030
3     $  .952588,  .976436, 1.000000, 1.000000,  .5501
3     $  .526003,  .560375,  .598645,  .637604,  .6755
3     $  .745896,  .777960,  .808045,  .836307,  .8629
3     $  .912047,  .934972,  .957108,  .978697, 1.0000
3C
3      DATA T1B4/ 1.000000,  .551048,  .509468,  .5452
3     $  .628471,  .669069,  .707008,  .742029,  .7742
3     $  .831297,  .856723,  .880431,  .902667,  .9236
3     $  .962898,  .981590, 1.000000, 1.000000,  .5564
3     $  .579361,  .628203,  .673668,  .714454,  .7507
3     $  .811686,  .837591,  .861095,  .882600,  .9024
3     $  .938095,  .954419,  .970031,  .985148, 1.0000
3     $  .559762,  .567460,  .640266,  .696171,  .7416
3     $  .810437,  .837105,  .860158,  .880389,  .8983
3     $  .929362,  .942920,  .955513,  .967335,  .9785
3     $ 1.000000, 1.000000,  .614500,  .678566,  .7615
3     $  .844607,  .870739,  .891357,  .908065,  .9219
3     $  .944293,  .953488,  .961733,  .969219,  .9761
3     $  .988561,  .994352, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3C
3      DATA T2B1/ 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000,  .7415
3     $  .832454,  .866267,  .890710,  .909186,  .9237
3     $  .945494,  .953931,  .961256,  .967719,  .9734
3     $  .983495,  .987932,  .992108,  .996103, 1.0000
3     $  .687142,  .687418,  .737431,  .778528,  .8123
3     $  .863432,  .883135,  .900089,  .914891,  .9279
3     $  .950251,  .959905,  .968817,  .977140,  .9850
3     $ 1.000000, 1.000000,  .675673,  .650231,  .6853
3     $  .757510,  .788945,  .816743,  .841284,  .8630
3     $  .899965,  .915816,  .930298,  .943639,  .9560
3     $  .978797,  .989500, 1.000000, 1.000000,  .6691
3     $  .654969,  .686480,  .719622,  .751595,  .7813
3     $  .833483,  .856172,  .876922,  .895978,  .9135
3     $  .945208,  .959665,  .973480,  .986854, 1.0000
3C
3      DATA T2B2/ 1.000000,  .666859,  .620765,  .6371
3     $  .693772,  .724882,  .755110,  .783737,  .8105
3     $  .858492,  .879954,  .899948,  .918661,  .9362
3     $  .969075,  .984659, 1.000000, 1.000000,  .6660
3     $  .626273,  .648566,  .676340,  .706181,  .7361
3     $  .793243,  .819562,  .844297,  .867503,  .8892
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```
3      $  .654903,  .682226,  .711162,  .740446,  .7692
3      $  .823993,  .849509,  .873767,  .896840,  .9188
3      $  .960392,  .980322, 1.000000, 1.000000,  .6659
3      $  .616112,  .633757,  .658128,  .685901,  .7150
3      $  .773116,  .800823,  .827292,  .852449,  .8763
3      $  .920569,  .941262,  .961251,  .980752, 1.0000
3      $  .665755,  .612102,  .619788,  .639232,  .6649
3      $  .723193,  .752532,  .780972,  .808175,  .8339
3      $  .881458,  .903280,  .924017,  .943849,  .9629
3      $ 1.000000, 1.000000,  .666044,  .615188,  .6262
3      $  .676340,  .706181,  .736197,  .765387,  .7932
3      $  .844297,  .867503,  .889294,  .909815,  .9292
3      $  .965578,  .982914, 1.000000, 1.000000,  .6668
3      $  .637101,  .663405,  .693772,  .724882,  .7551
3      $  .810503,  .835391,  .858492,  .879954,  .8999
3      $  .936286,  .953022,  .969075,  .984659, 1.0000
3C
3       DATA T2B4/ 1.000000,  .669171,  .630817,  .6549
3      $  .719622,  .751595,  .781344,  .808614,  .8334
3      $  .876922,  .895978,  .913568,  .929909,  .9452
3      $  .973480,  .986854, 1.000000, 1.000000,  .6756
3      $  .685324,  .722473,  .757510,  .788945,  .8167
3      $  .863046,  .882481,  .899965,  .915816,  .9302
3      $  .956046,  .967705,  .978797,  .989500, 1.0000
3      $  .687142,  .687418,  .737431,  .778528,  .8123
3      $  .863432,  .883135,  .900089,  .914891,  .9279
3      $  .950251,  .959905,  .968817,  .977140,  .9850
3      $ 1.000000, 1.000000,  .741509,  .778382,  .8324
3      $  .890710,  .909186,  .923769,  .935611,  .9454
3      $  .961256,  .967719,  .973492,  .978713,  .9834
3      $  .992108,  .996103, 1.000000, 1.000000, 1.0000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3C
3       DATA T3B1/ 1.000000, 1.000000, 1.000000, 1.0000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3      $ 1.000000, 1.000000, 1.000000, 1.000000,  .9937
3      $  .981169,  .974776,  .968325,  .961847,  .9553
3      $  .942520,  .936319,  .930428,  .924962,  .9202
3      $  .915547,  .917824,  .926266,  .947896, 1.0000
3      $  .988680,  .977311,  .965896,  .954461,  .9430
3      $  .920358,  .909277,  .898541,  .888291,  .8788
3      $  .863657,  .859579,  .859567,  .866290,  .8840
3      $ 1.000000, 1.000000,  .984558,  .969098,  .9536
3      $  .922979,  .907890,  .893077,  .878694,  .8649
3      $  .840452,  .830568,  .823270,  .819846,  .8221
3      $  .858962,  .908600, 1.000000, 1.000000,  .9812
3      $  .943963,  .925501,  .907269,  .889384,  .8719
3      $  .839415,  .824843,  .811996,  .801492,  .7943
3      $  .796775,  .812068,  .843199,  .900164, 1.0000
3C
3       DATA T3B2/ 1.000000,  .978729,  .957520,  .9364
3      $  .895188,  .875226,  .855922,  .837510,  .8202
3      $  .791160,  .780512,  .773847,  .772784,  .7796
3      $  .833163,  .894928, 1.000000, 1.000000,  .9768
3      $  .930817,  .908259,  .886160,  .864690,  .8440
3      $  .806323,  .790032,  .776208,  .765637,  .7595
3      $  .768040,  .788691,  .826685,  .891622, 1.0000
3      $  .975433,  .950988,  .926797,  .903004,  .8797
3      $  .835665,  .815323,  .796584,  .779909,  .7659
3      $  .749899,  .750769,  .760494,  .782729,  .8225
```

```
3      $  .790433,  .773547,  .759540,  .749267,  .74398
3      $  .755951,  .779176,  .820182,  .888353, 1.00000
3      $  .975433,  .950988,  .926797,  .903003,  .87975
3      $  .835665,  .815323,  .796584,  .779909,  .76594
3      $  .749899,  .750769,  .760494,  .782729,  .82258
3      $ 1.000000, 1.000000,  .976808,  .953708,  .9308
3      $  .886160,  .864689,  .844035,  .824465,  .80632
3      $  .776208,  .765637,  .759520,  .759540,  .76804
3      $  .826685,  .891622, 1.000000, 1.000000,  .97872
3      $  .936459,  .915647,  .895188,  .875226,  .85592
3      $  .820298,  .804662,  .791160,  .780512,  .77384
3      $  .779605,  .797983,  .833163,  .894928, 1.00000
3C
3       DATA T3B4/ 1.000000,  .981280,  .962581,  .94396
3      $  .907269,  .889384,  .871971,  .855227,  .8394
3      $  .811995,  .801492,  .794350,  .792061,  .79673
3      $  .843199,  .900164, 1.000000, 1.000000,  .98459
3      $  .953649,  .938262,  .922979,  .907890,  .89301
3      $  .864950,  .852072,  .840452,  .830568,  .82321
3      $  .822158,  .833520,  .858962,  .908600, 1.00000
3      $  .988680,  .977311,  .965896,  .954461,  .94303
3      $  .920358,  .909277,  .898541,  .888291,  .87881
3      $  .863657,  .859579,  .859566,  .866290,  .88401
3      $ 1.000000, 1.000000,  .993784,  .987508,  .98111
3      $  .968325,  .961847,  .955346,  .948978,  .94253
3      $  .930428,  .924962,  .920279,  .916883,  .91554
3      $  .926266,  .947896, 1.000000, 1.000000, 1.00000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3C
3       DATA T4B1/ 1.000000, 1.000000, 1.000000, 1.00000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3      $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3      $ 1.000000, 1.000000, 1.000000, 1.000000,  .99600
3      $  .987907,  .983782,  .979611,  .975413,  .97127
3      $  .962937,  .958994,  .955309,  .952007,  .94937
3      $  .947416,  .949647,  .955896,  .969798, 1.00000
3      $  .992618,  .985194,  .977717,  .970195,  .96267
3      $  .947622,  .940278,  .933185,  .926480,  .92037
3      $  .911221,  .909277,  .910276,  .915861,  .92867
3      $ 1.000000, 1.000000,  .989784,  .979538,  .96927
3      $  .948707,  .938529,  .928514,  .918787,  .90957
3      $  .893260,  .886985,  .882696,  .881288,  .88407
3      $  .911265,  .944131, 1.000000, 1.000000,  .98747
3      $  .962366,  .949857,  .937439,  .925200,  .91327
3      $  .890904,  .880998,  .872412,  .865649,  .86147
3      $  .865422,  .877300,  .899736,  .937977, 1.00000
3C
3       DATA T4B2/ 1.000000,  .985609,  .971226,  .95687
3      $  .928534,  .914715,  .901305,  .888499,  .87657
3      $  .856650,  .849727,  .845852,  .846178,  .85227
3      $  .891984,  .933918, 1.000000, 1.000000,  .98417
3      $  .952662,  .937079,  .921726,  .906731,  .89227
3      $  .865801,  .854478,  .845040,  .838122,  .83467
3      $  .843105,  .859051,  .886756,  .931222, 1.00000
3      $  .983141,  .966321,  .949601,  .933061,  .91687
3      $  .885755,  .871378,  .858159,  .846489,  .83687
3      $  .826886,  .828634,  .836934,  .854118,  .88337
3      $ 1.000000, 1.000000,  .982464,  .964977,  .94767
3      $  .913616,  .897256,  .881562,  .866787,  .85327
3      $  .831729,  .824963,  .822042,  .824218,  .83317
3      $  .881239,  .928405, 1.000000, 1.000000,  .98217
```

```
3     $  .826886,  .828634,  .836934,  .854118,  .8833
3     $ 1.000000, 1.000000,  .984181,  .968385,  .9526
3     $  .921726,  .906731,  .892253,  .878514,  .8658
3     $  .845040,  .838122,  .834620,  .835741,  .8431
3     $  .886756,  .931222, 1.000000, 1.000000,  .9856
3     $  .956880,  .942625,  .928534,  .914715,  .9013
3     $  .876552,  .865785,  .856650,  .849727,  .8458
3     $  .852277,  .866476,  .891984,  .933918, 1.0000
3C
3      DATA T4B4/ 1.000000,  .987464,  .974914,  .9623
3     $  .937439,  .925200,  .913248,  .901745,  .8909
3     $  .872412,  .865649,  .861458,  .860892,  .8654
3     $  .899736,  .937977, 1.000000, 1.000000,  .9897
3     $  .969260,  .958972,  .948707,  .938529,  .9285
3     $  .909515,  .900903,  .893260,  .886985,  .8826
3     $  .884038,  .893007,  .911265,  .944131, 1.0000
3     $  .992618,  .985194,  .977717,  .970195,  .9626
3     $  .947622,  .940278,  .933185,  .925480,  .9203
3     $  .911221,  .909277,  .910276,  .915861,  .9286
3     $ 1.000000, 1.000000,  .996011,  .991983,  .9879
3     $  .979611,  .975413,  .971205,  .967027,  .9629
3     $  .955309,  .952007,  .949329,  .947623,  .9474
3     $  .955896,  .969798, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3C
3      DATA T5B1/ 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000,  .9976
3     $  .992734,  .990248,  .987731,  .985194,  .9826
3     $  .977678,  .975333,  .973173,  .971293,  .9698
3     $  .969210,  .970876,  .974928,  .983205, 1.0000
3     $  .995523,  .991017,  .986469,  .981881,  .9772
3     $  .968054,  .963553,  .959220,  .955157,  .9515
3     $  .946327,  .945492,  .946555,  .950453,  .9586
3     $ 1.000000, 1.000000,  .993750,  .987473,  .9811
3     $  .968472,  .962159,  .955939,  .949896,  .9441
3     $  .934210,  .930512,  .928155,  .927703,  .9299
3     $  .947751,  .967740, 1.000000, 1.000000,  .9922
3     $  .976750,  .968973,  .961223,  .953559,  .9460
3     $  .932034,  .925861,  .920583,  .916549,  .9142
3     $  .917771,  .925800,  .940237,  .963735, 1.0000
3C
3      DATA T5B2/ 1.000000,  .991070,  .982128,  .9731
3     $  .955396,  .946673,  .938185,  .930069,  .9225
3     $  .910056,  .905888,  .903779,  .904464,  .9089
3     $  .934995,  .960986, 1.000000, 1.000000,  .9901
3     $  .970401,  .960589,  .950878,  .941353,  .9321
3     $  .915254,  .908072,  .902163,  .897969,  .8960
3     $  .902587,  .913348,  .931351,  .959097, 1.0000
3     $  .989442,  .978888,  .968361,  .957902,  .9475
3     $  .927729,  .918506,  .910031,  .902587,  .8965
3     $  .890697,  .892310,  .898239,  .909848,  .9288
3     $ 1.000000, 1.000000,  .988989,  .977987,  .9670
3     $  .945415,  .934945,  .924864,  .915355,  .9066
3     $  .892942,  .888806,  .887279,  .889173,  .8955
3     $  .927379,  .957054, 1.000000, 1.000000,  .9887
3     $  .966360,  .955271,  .944348,  .933695,  .9234
3     $  .904992,  .897320,  .891181,  .887067,  .8856
3     $  .894206,  .906628,  .926653,  .956681, 1.0000
3C
```

```
3     $  .931351,  .959097, 1.000000, 1.000000,  .991070
3     $  .973182,  .964256,  .955396,  .946673,  .938189
3     $  .922506,  .915730,  .910056,  .905888,  .903779
3     $  .908919,  .918496,  .934995,  .960986, 1.000000
3C
3      DATA T5B4/ 1.000000,  .992271,  .984521,  .976750
3     $  .961223,  .953559,  .946057,  .938831,  .932034
3     $  .920583,  .916549,  .914251,  .914356,  .917772
3     $  .940237,  .963735, 1.000000, 1.000000,  .993750
3     $  .981160,  .974819,  .968472,  .962159,  .955939
3     $  .944149,  .938849,  .934210,  .930512,  .928155
3     $  .929942,  .936052,  .947751,  .967740, 1.000000
3     $  .995523,  .991017,  .986469,  .981881,  .977264
3     $  .968054,  .963553,  .959221,  .955157,  .951509
3     $  .946328,  .945492,  .946555,  .950453,  .958648
3     $ 1.000000, 1.000000,  .997605,  .995186,  .992734
3     $  .987731,  .985194,  .982652,  .980134,  .977678
3     $  .973174,  .971293,  .969844,  .969042,  .969210
3     $  .974928,  .983205, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3C
3C     INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRDI
3C
3      TRUE = DERVSL(6)
3C
3      RETURN
3      END
3      FUNCTION H(X)
3      GO TO(10,10,20,20,20),&A
3   10 H = 0.0
3      IF (X .NE. 0.) H = 1./X
3      RETURN
3   20 H = EXP(X)
3      RETURN
3      END
3      FUNCTION W(X,Y)
3C
3      REAL DERVSL(6), GRID(9), TAB1(9,9), TAB2(9,9),
3     $     TAB3(9,9), TAB4(9,9), TAB5(9,9)
3      DATA NGRID, NGRDD, GRID /9, 9, 0., 0.125, 0.250,
3     $0.375, 0.500, 0.625, 0.750, 0.875, 1.00/
3C
3C     APPROXIMATE SOLUTION OF NONLINEAR PROBLEM FOR NON
3C     COLLOCATION
3C
3      DATA TAB1/ 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $  .667850,  .769627,  .839584,  .891757,  .933243
3     $ 1.000000, 1.000000,  .410154,  .535076,  .650126
3     $  .822954,  .888785,  .946495, 1.000000, 1.000000
3     $  .483191,  .594221,  .696058,  .785410,  .863715
3     $ 1.000000, 1.000000,  .374994,  .469323,  .577968
3     $  .773552,  .855656,  .929943, 1.000000, 1.000000
3     $  .483191,  .594221,  .696058,  .785411,  .863715
3     $ 1.000000, 1.000000,  .410154,  .535076,  .650126
3     $  .822954,  .888785,  .946495, 1.000000, 1.000000
3     $  .667850,  .769628,  .839585,  .891758,  .933243
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
```

```
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3      DATA TAB3/ 1.000000, 1.000000, 1.000000, 1.000000,
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3     $  .938216,  .907820,  .879233,  .855269,  .842995,
3     $ 1.000000, 1.000000,  .950959,  .902895,  .857303,
3     $  .786550,  .778103,  .820581, 1.000000, 1.000000,
3     $  .884947,  .832317,  .787160,  .755922,  .751881,
3     $ 1.000000, 1.000000,  .938692,  .879478,  .824808,
3     $  .747215,  .744783,  .801431, 1.000000, 1.000000,
3     $  .884947,  .832318,  .787160,  .755922,  .751882,
3     $ 1.000000, 1.000000,  .950960,  .902895,  .857303,
3     $  .786551,  .778103,  .820582, 1.000000, 1.000000,
3     $  .938217,  .907820,  .879233,  .855269,  .842995,
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3      DATA TAB4/ 1.000000, 1.000000, 1.000000, 1.000000,
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3     $  .959246,  .938933,  .919863,  .904380,  .897858,
3     $ 1.000000, 1.000000,  .966833,  .933969,  .902427,
3     $  .854068,  .850356,  .882916, 1.000000, 1.000000,
3     $  .920374,  .883276,  .851374,  .830024,  .829479,
3     $ 1.000000, 1.000000,  .957565,  .916106,  .877340,
3     $  .822928,  .823561,  .867130, 1.000000, 1.000000,
3     $  .920374,  .883276,  .851374,  .830024,  .829479,
3     $ 1.000000, 1.000000,  .966833,  .933969,  .902427,
3     $  .854068,  .850356,  .882916, 1.000000, 1.000000,
3     $  .959246,  .938933,  .919863,  .904380,  .897858,
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3      DATA TAB5/ 1.000000, 1.000000, 1.000000, 1.000000,
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3     $  .975113,  .962598,  .950876,  .941611,  .938390,
3     $ 1.000000, 1.000000,  .979429,  .958883,  .938986,
3     $  .908740,  .907369,  .929229, 1.000000, 1.000000,
3     $  .949842,  .926125,  .905660,  .892267,  .892922,
3     $ 1.000000, 1.000000,  .973287,  .946951,  .922060,
3     $  .887281,  .888697,  .918071, 1.000000, 1.000000,
3     $  .949843,  .926125,  .905660,  .892266,  .892921,
3     $ 1.000000, 1.000000,  .979431,  .958885,  .938987,
3     $  .908738,  .907368,  .929228, 1.000000, 1.000000,
3     $  .975115,  .962599,  .950875,  .941609,  .938389,
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3C     INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TAB&A,GRID,GRID,NGRID,NGRID,NGRDD,
3C
3      W = DERVSL(6)
3C
3      RETURN
3      END
```

```
'EOR
************
* MACRO  40 *
************
*          2000200000020
1          TWO DIMENSIONS
```

```
3      END
3      FUNCTION F(X,Y)
3      DATA ALOGE10/2.3025850929940/
3      U    = (X+Y)/(2.+X-Y)
3      UX   = 2.*(1.-Y)/(2.+X-Y)**2
3      UXX  = 4.*(Y-1.)/(2.+X-Y)**3
3      UY   = 2.*(1.+X)/(2.+X-Y)**2
3      UYY  = 4.*(1.+X)/(2.+X-Y)**3
3      U1   = EXP(2.*U-2.)
3      U1X  = 2.*UX*U1
3      U1XX = 2.*(UXX + 2.*UX**2)*U1
3      U1Y  = 2.*UY*U1
3      U1YY = 2.*(UYY + 2.*UY**2)*U1
3      W    = (X+1.)/(Y+1.)
3      WX   = 1./(Y+1.)
3      WY   = -(X+1.)/(Y+1.)**2
3      WYY  = 2.*(X+1.)/(Y+1.)**3
3      U2   = ALOG10(W)
3      U2X  = (WX/W)/ALOGE10
3      U2XX = (-(WX/W)**2)/ALOGE10
3      U2Y  = (WY/W)/ALOGE10
3      U2YY = (WYY/W - (WY/W)**2)/ALOGE10
3      UX   = U1X + U2X
3      UXX  = U1XX + U2XX
3      UYY  = U1YY + U2YY
3      F = UXX + (1.+X**2)*UYY - Y*UX
3      RETURN
3      END
3      FUNCTION G(X,Y)
3      DATA ALOGE10/2.3025850929940/
3      U = (X+Y)/(2.+X-Y)
3      UX = 2.*(1.-Y)/(2.+X-Y)**2
3      U1 = EXP(2.*U-2.)
3      U1X = 2.*UX*U1
3      W = (X+1.)/(Y+1.)
3      WX = 1./(Y+1.)
3      U2 = ALOG10(W)
3      U2X = (WX/W)/ALOGE10
3      U = U1 + U2
3      UX = U1X + U2X
3      G = &A*U + (&B)*UX
3      RETURN
3      END
```

```
'EOR
*************
* MACRO  41 *
*************
*          2000021002002
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS
1          UXX$ + UYY$ + (&A)U$ = F(X,Y)
2          DIRICHLET $ HOMOGENEOUS
2          X=0. ,                  U=0.
2          X=3.14159265358979 ,  U=0.
2          Y=0. ,                  U=0.
2          Y=3.14159265358979 ,  U=0.
3      FUNCTION TRUE(X,Y)
3      DATA PI/3.14159265358979/
3      SUM = 0.
3      DO 100 K = 1,&B
3          TEMP = 2.*K - 1.
3          SUM  = SUM + SIN(TEMP*X)*COSH(TEMP*(Y-PI*.5))/
3     $                 (TEMP**3*COSH(TEMP*PI*.5))
```

```
3      END
-------------------------------------------------------------------------
'EOR
*************
* MACRO  42 *
*************
*          2000220200200
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS
1          UXX$ + UYY$ + UY$ - U$ = 0.
2          NEUMANN
2          X=&A , UX = -G1(X,Y)
2          X=&B , UX =  G1(X,Y)
2          Y=0. , UY = -G2(X,Y)
2          Y=1. , UY =  G2(X,Y)
3      FUNCTION TRUE(X,Y)
3      COMMON /CONCOM/ PI
3      DATA PI/3.1415926535897 9/
3      F0 = (&C)*PI/((&B)-(&A))
3      F1 = SQRT(5./4. +F0*F0)
3      TRUE = EXP(-Y*.5)*SINH(F1*Y)*SIN(F0*(X-(&A)))
3      RETURN
3      END
3      FUNCTION G1(X,Y)
3      COMMON /CONCOM/PI
3      F0 = (&C)*PI/((&B)-(&A))
3      F1 = SQRT(5./4. +F0*F0)
3      G1 = EXP(-Y*.5)*SINH(F1*Y)*F0*COS(F0*(X-(&A)))
3      IF (ABS(X-(&A)) .LT. 1.E-7) G1 = -1.*G1
3      RETURN
3      END
3      FUNCTION G2(X,Y)
3      COMMON /CONCOM/PI
3      F0 = (&C)*PI/((&B)-(&A))
3      F1 = SQRT(5./4. +F0*F0)
3      G2 = -.5*TRUE(X,Y) + F1*EXP(-Y*.5)*COSH(F1*Y)*
3     $     SIN(F0*(X-(&A)))
3      IF (Y .EQ. 0.) G2 = -1.*G2
3      RETURN
3      END
3      FUNCTION SINH(X)
3      EXPX = EXP(X)
3      SINH = 0.5*(EXPX - 1./EXPX)
3      RETURN
3      END
3      FUNCTION COSH(X)
3      EXPX = EXP(X)
3      COSH = 0.5*(EXPX + 1./EXPX)
3      RETURN
3      END
-------------------------------------------------------------------------
'EOR
*************
* MACRO  43 *
*************
-------------------------------------------------------------------------
'EOR
*************
* MACRO  44 *
*************
*          2000001002002
1          TWO DIMENSIONS
1          UXX$ + UYY$ + W(X,Y)U$ = W(X,Y)
2          DIRICHLET $ HOMOGENEOUS
2          X=0. , U=0.
```

```
3C    *          1.425  I  1  I  .50  I   2  I  1
3C    *                 I     I      I      I
3C    *         10.000  I  1  I  .50  I   2  I  2
3C    *                 I     I      I      I
3C    *          1.425  I  2  I  .04  I  25  I  3
3C    *                 I     I      I      I
3C    *          1.425  I  2  I  .50  I   2  I  4
3C    *
3C    ***********************************************
3C
3      REAL DERVSL(6), GRID(20), TABLE(20,20),
3     $     T1B1(100), T1B2(100), T1B3(100), T1B4(100)
3     $     T2B1(100), T2B2(100), T2B3(100), T2B4(100)
3     $     T3B1(100), T3B2(100), T3B3(100), T3B4(100)
3     $     T4B1(100), T4B2(100), T4B3(100), T4B4(100)
3      EQUIVALENCE (TABLE(1, 1) , T&EB1(1)),
3     $            (TABLE(1, 6) , T&EB2(1)),
3     $            (TABLE(1,11) , T&EB3(1)),
3     $            (TABLE(1,16) , T&EB4(1))
3      DATA NGRID, NGRDD, GRID /20, 20, 0.0000000, 0.0
3     $0.1052532, 0.1578947, 0.2105263, 0.2631579, 0.3
3     $0.3684211, 0.4210526, 0.4736842, 0.5263158, 0.5
3     $0.6315789, 0.6842105, 0.7368421, 0.7894737, 0.8
3     $0.8947368, 0.9473684, 1.0000000/
3C
3C     APPROXIMATE SOLUTION OF LINEARIZED PROBLEM USIN
3C     HODIE-ACF (METHOD=4, IORDER=41, 20 X 20 GRID)
3C
3      DATA T1B1/  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0096
3     $  .021191,  .024955,  .027830,  .029984,  .0315
3     $  .033001,  .033001,  .032514,  .031522,  .0299
3     $  .024955,  .021191,  .016256,  .009616,  .0000
3     $  .016256,  .028498,  .037893,  .045161,  .0507
3     $  .057968,  .059913,  .060866,  .060866,  .0599
3     $  .054957,  .050752,  .045161,  .037893,  .0284
3     $  .000000,  .000000,  .021191,  .037893,  .0510
3     $  .069435,  .075507,  .079869,  .082690,  .0840
3     $  .082690,  .079869,  .075507,  .069435,  .0614
3     $  .037893,  .021191,  .000000,  .000000,  .0249
3     $  .061406,  .074337,  .084461,  .092159,  .0977
3     $  .103078,  .103078,  .101308,  .097710,  .0921
3     $  .074337,  .061406,  .045161,  .024955,  .0000
3C
3      DATA T1B2/  .000000,  .027830,  .050752,  .0694
3     $  .096312,  .105368,  .111921,  .116179,  .1182
3     $  .116179,  .111921,  .105368,  .096312,  .0844
3     $  .050752,  .027830,  .000000,  .000000,  .0299
3     $  .075507,  .092159,  .105368,  .115506,  .1228
3     $  .130010,  .130010,  .127650,  .122862,  .1159
3     $  .092159,  .075507,  .054957,  .029984,  .0000
3     $  .031522,  .057968,  .079869,  .097710,  .1119
3     $  .130819,  .136006,  .138564,  .138564,  .1360
3     $  .122862,  .111921,  .097710,  .079869,  .0579
3     $  .000000,  .000000,  .032514,  .059913,  .0826
3     $  .116179,  .127650,  .136006,  .141459,  .1443
3     $  .141459,  .136006,  .127650,  .116179,  .1013
3     $  .059913,  .032514,  .000000,  .000000,  .0330
3     $  .084076,  .103078,  .118274,  .130010,  .1389
3     $  .146907,  .146907,  .144150,  .138564,  .1306
3     $  .103078,  .084076,  .060866,  .033001,  .0000
```

```
3     $  .127650,  .122862,  .115506,  .105368,  .0921
3     $  .054957,  .029984,  .000000,  .000000,  .0278
3     $  .069435,  .084461,  .096312,  .105368,  .1119
3     $  .118274,  .118274,  .116179,  .111921,  .1053
3     $  .084461,  .069435,  .050752,  .027830,  .0000
3C
3      DATA T1B4/  .000000,  .024955,  .045161,  .0614
3     $  .084461,  .092159,  .097710,  .101308,  .1030
3     $  .101308,  .097710,  .092159,  .084461,  .0743
3     $  .045161,  .024955,  .000000,  .000000,  .0211
3     $  .051061,  .061406,  .069435,  .075507,  .0798
3     $  .084076,  .084076,  .082690,  .079869,  .0755
3     $  .061406,  .051061,  .037893,  .021191,  .0000
3     $  .016256,  .028498,  .037893,  .045161,  .0507
3     $  .057968,  .059913,  .060866,  .060866,  .0599
3     $  .054957,  .050752,  .045161,  .037893,  .0284
3     $  .000000,  .000000,  .009616,  .016256,  .0211
3     $  .027830,  .029984,  .031522,  .032514,  .0330
3     $  .032514,  .031522,  .029984,  .027830,  .0249
3     $  .016256,  .009616,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3C
3      DATA T2B1/  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .2527
3     $  .422720,  .447250,  .458919,  .464413,  .4670
3     $  .468682,  .468682,  .468253,  .467014,  .4644
3     $  .447250,  .422720,  .367756,  .252731,  .0000
3     $  .367756,  .557305,  .650506,  .693899,  .7145
3     $  .728757,  .730865,  .731652,  .731652,  .7308
3     $  .724187,  .714513,  .693899,  .650506,  .5573
3     $  .000000,  .000000,  .422720,  .650506,  .7651
3     $  .845933,  .858258,  .864071,  .866731,  .8677
3     $  .866731,  .864071,  .858258,  .845933,  .8198
3     $  .650506,  .422720,  .000000,  .000000,  .4472
3     $  .819808,  .880741,  .910053,  .923977,  .9305
3     $  .934720,  .934720,  .933563,  .930554,  .9239
3     $  .880741,  .819808,  .693899,  .447250,  .0000
3C
3      DATA T2B2/  .000000,  .458919,  .714513,  .8459
3     $  .941055,  .955858,  .962869,  .966079,  .9673
3     $  .966079,  .962869,  .955858,  .941055,  .9100
3     $  .714513,  .458919,  .000000,  .000000,  .4644
3     $  .858258,  .923977,  .955858,  .971139,  .9783
3     $  .983009,  .983009,  .981719,  .978392,  .9711
3     $  .923977,  .858258,  .724187,  .464413,  .0000
3     $  .467014,  .728757,  .864071,  .930554,  .9628
3     $  .985772,  .989162,  .990480,  .990480,  .9891
3     $  .978392,  .962869,  .930554,  .864071,  .7287
3     $  .000000,  .000000,  .468253,  .730865,  .8667
3     $  .966079,  .981719,  .989162,  .992584,  .9939
3     $  .992584,  .989162,  .981719,  .966079,  .9335
3     $  .730865,  .468253,  .000000,  .000000,  .4686
3     $  .867747,  .934720,  .967321,  .983009,  .9904
3     $  .995254,  .995254,  .993916,  .990480,  .9830
3     $  .934720,  .867747,  .731652,  .468682,  .0000
3C
3      DATA T2B3/  .000000,  .468682,  .731652,  .8677
3     $  .967321,  .983009,  .990480,  .993916,  .9952
3     $  .993916,  .990480,  .983009,  .967321,  .9347
```

```
3     $  .910053.  .845933.  .714513.  .458919.  .0000
3C
3      DATA T2B4/  .000000.  .447250.  .693899.  .8198
3     $  .910053.  .923977.  .930554.  .933563.  .9347
3     $  .933563.  .930554.  .923977.  .910053.  .8807
3     $  .693899.  .447250.  .000000.  .000000.  .4227
3     $  .765119.  .819808.  .845933.  .858258.  .8640
3     $  .867747.  .867747.  .866731.  .864071.  .8582
3     $  .819808.  .765119.  .650506.  .422720.  .0000
3     $  .367756.  .557305.  .650506.  .693899.  .7145
3     $  .728757.  .730865.  .731652.  .731652.  .7308
3     $  .724187.  .714513.  .693899.  .650506.  .5573
3     $  .000000.  .000000.  .252731.  .367756.  .4227
3     $  .458919.  .464413.  .467014.  .468253.  .4686
3     $  .468253.  .467014.  .464413.  .458919.  .4472
3     $  .367756.  .252731.  .000000.  .000000.  .0000
3     $  .000000.  .000000.  .000000.  .000000.  .0000
3     $  .000000.  .000000.  .000000.  .000000.  .0000
3     $  .000000.  .000000.  .000000.  .000000.  .0000
3C
3      DATA T3B1/  .000000.  .000000.  .000000.  .0000
3     $  .000000.  .000000.  .000000.  .000000.  .0000
3     $  .000000.  .000000.  .000000.  .000000.  .0000
3     $  .000000.  .000000.  .000000.  .000000.  .0091
3     $  .019938.  .023367.  .025957.  .027881.  .0292
3     $  .030550.  .030550.  .030122.  .029246.  .0278
3     $  .023367.  .019938.  .015386.  .009168.  .0000
3     $  .015386.  .026806.  .035451.  .042060.  .0470
3     $  .053517.  .055233.  .056072.  .056072.  .0552
3     $  .050847.  .047093.  .042060.  .035451.  .0268
3     $  .000000.  .000000.  .019938.  .035451.  .0475
3     $  .064138.  .069552.  .073417.  .075906.  .0771
3     $  .075906.  .073417.  .069552.  .064138.  .0569
3     $  .035451.  .019938.  .000000.  .000000.  .0233
3     $  .056920.  .068630.  .077715.  .084572.  .0894
3     $  .094214.  .094214.  .092659.  .089487.  .0845
3     $  .068630.  .056920.  .042060.  .023367.  .0000
3C
3      DATA T3B2/  .000000.  .025957.  .047093.  .0641
3     $  .088333.  .096390.  .102187.  .105938.  .1077
3     $  .105938.  .102187.  .096390.  .088333.  .0777
3     $  .047093.  .025957.  .000000.  .000000.  .0278
3     $  .069552.  .084572.  .096390.  .105399.  .1119
3     $  .118189.  .118189.  .116116.  .111901.  .1053
3     $  .084572.  .069552.  .050847.  .027881.  .0000
3     $  .029246.  .053517.  .073417.  .089487.  .1021
3     $  .118329.  .123493.  .125738.  .125738.  .1234
3     $  .111901.  .102187.  .089487.  .073417.  .0535
3     $  .000000.  .000000.  .030122.  .055233.  .0759
3     $  .105938.  .116116.  .123493.  .128288.  .1306
3     $  .128288.  .123493.  .116116.  .105938.  .0926
3     $  .055233.  .030122.  .000000.  .000000.  .0305
3     $  .077125.  .094214.  .107779.  .118189.  .1257
3     $  .133069.  .133069.  .130650.  .125738.  .1181
3     $  .094214.  .077125.  .056072.  .030550.  .0000
3C
3      DATA T3B3/  .000000.  .030550.  .056072.  .0771
3     $  .107779.  .118189.  .125738.  .130650.  .1330
3     $  .130650.  .125738.  .118189.  .107779.  .0942
3     $  .056072.  .030550.  .000000.  .000000.  .0301
3     $  .075906.  .092659.  .105938.  .116116.  .1234
3     $  .130650.  .130650.  .128288.  .123493.  .1161
3     $  .092659.  .075906.  .055233.  .030122.  .0000
```

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3     $  .092659,  .089487,  .084572,  .077715,  .0686
3     $  .042060,  .023367,  .000000,  .000000,  .0199
3     $  .047532,  .056920,  .064138,  .069552,  .0734
3     $  .077125,  .077125,  .075906,  .073417,  .0695
3     $  .056920,  .047532,  .035451,  .019938,  .0000
3     $  .015386,  .026806,  .035451,  .042060,  .0470
3     $  .053517,  .055233,  .056072,  .056072,  .0552
3     $  .050847,  .047093,  .042060,  .035451,  .0268
3     $  .000000,  .000000,  .009168,  .015386,  .0199
3     $  .025957,  .027881,  .029246,  .030122,  .0305
3     $  .030122,  .029246,  .027881,  .025957,  .0233
3     $  .015386,  .009168,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3C
3      DATA T4B1/  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0091
3     $  .019900,  .023316,  .025896,  .027811,  .0291
3     $  .030465,  .030465,  .030040,  .029169,  .0278
3     $  .023316,  .019900,  .015360,  .009154,  .0000
3     $  .015360,  .026754,  .035373,  .041959,  .0469
3     $  .053363,  .055069,  .055903,  .055903,  .0550
3     $  .050706,  .046971,  .041959,  .035373,  .0267
3     $  .000000,  .000000,  .019900,  .035373,  .0474
3     $  .063956,  .069343,  .073187,  .075662,  .0768
3     $  .075662,  .073187,  .069343,  .063956,  .0567
3     $  .035373,  .019900,  .000000,  .000000,  .0233
3     $  .056770,  .068432,  .077476,  .084298,  .0891
3     $  .093885,  .093885,  .092338,  .089185,  .0842
3     $  .068432,  .056770,  .041959,  .023316,  .0000
3C
3      DATA T4B2/  .000000,  .025896,  .046971,  .0639
3     $  .088044,  .096058,  .101821,  .105549,  .1073
3     $  .105549,  .101821,  .096058,  .088044,  .0774
3     $  .046971,  .025896,  .000000,  .000000,  .0278
3     $  .089343,  .084298,  .096058,  .105018,  .1114
3     $  .117729,  .117729,  .115670,  .111481,  .1050
3     $  .084298,  .069343,  .050706,  .027811,  .0000
3     $  .029169,  .053363,  .073187,  .089185,  .1018
3     $  .118466,  .123001,  .125232,  .125232,  .1230
3     $  .111481,  .101821,  .089185,  .073187,  .0533
3     $  .000000,  .000000,  .030040,  .055069,  .0756
3     $  .105549,  .115670,  .123001,  .127765,  .1301
3     $  .127765,  .123001,  .115670,  .105549,  .0923
3     $  .055069,  .030040,  .000000,  .000000,  .0304
3     $  .076874,  .093885,  .107379,  .117729,  .1252
3     $  .132513,  .132513,  .130110,  .125232,  .1177
3     $  .093885,  .076874,  .055903,  .030465,  .0000
3C
3      DATA T4B3/  .000000,  .030465,  .055903,  .0768
3     $  .107379,  .117729,  .125232,  .130110,  .1325
3     $  .130110,  .125232,  .117729,  .107379,  .0938
3     $  .055903,  .030465,  .000000,  .000000,  .0300
3     $  .075662,  .092338,  .105549,  .115670,  .1230
3     $  .130110,  .130110,  .127765,  .123001,  .1156
3     $  .092338,  .075662,  .055069,  .030040,  .0000
3     $  .029169,  .053363,  .073187,  .089185,  .1018
3     $  .118466,  .123001,  .125232,  .125232,  .1230
3     $  .111481,  .101821,  .089185,  .073187,  .0533
3     $  .000000,  .000000,  .027811,  .050706,  .0693
```

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3     $  .056770.  .047417.  .035373.  .019900.  .000000
3     $  .015360.  .026754.  .035373.  .041959.  .046973
3     $  .053363.  .055069.  .055903.  .055903.  .055069
3     $  .050706.  .046971.  .041959.  .035373.  .026754
3     $  .000000.  .000000.  .009154.  .015360.  .019900
3     $  .025896.  .027811.  .029169.  .030040.  .030469
3     $  .030040.  .029169.  .027811.  .025896.  .023316
3     $  .015360.  .009154.  .000000.  .000000.  .000000
3     $  .000000.  .000000.  .000000.  .000000.  .000000
3     $  .000000.  .000000.  .000000.  .000000.  .000000
3     $  .000000.  .000000.  .000000.  .000000.  .000000
3C
3C     INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRDI
3C
3      TRUE = DERVSL(6)
3C
3      RETURN
3      END
3      FUNCTION W(X,Y)
3C
3      REAL DERVSL(6), GRID(9), TAB1(9,9), TAB2(9,9),
3     $     TAB3(9,9), TAB4(9,9)
3      DATA NGRID, NGRDD, GRID /9, 9, 0., 0.125, 0.250,
3     $0.375, 0.500, 0.625, 0.750, 0.875, 1.00/
3C
3C     APPROXIMATE SOLUTION OF NONLINEAR PROBLEM FOR NOD
3C     COLLOCATION
3C
3      DATA TAB1/ .000000.  .000000.  .000000.  .000000
3     $  .000000.  .000000.  .000000.  .000000.  .000000
3     $  .056745.  .067036.  .070229.  .067036.  .056745
3     $  .000000.  .000000.  .056745.  .091025.  .109316
3     $  .109316.  .091025.  .056745.  .000000.  .000000
3     $  .109316.  .132414.  .139739.  .132414.  .109316
3     $  .000000.  .000000.  .070228.  .115060.  .139739
3     $  .139739.  .115060.  .070228.  .000000.  .000000
3     $  .109316.  .132414.  .139738.  .132414.  .109316
3     $  .000000.  .000000.  .056744.  .091025.  .109316
3     $  .109316.  .091025.  .056744.  .000000.  .000000
3     $  .056744.  .067036.  .070228.  .067036/ .056744
3     $  .000000.  .000000.  .000000.  .000000.  .000000
3     $  .000000.  .000000.  .000000.  .000000/
3C
3      DATA TAB2/ .000000.  .000000.  .000000.  .000000
3     $  .000000.  .000000.  .000000.  .000000.  .000000
3     $  .770994.  .791956.  .794924.  .791956.  .770994
3     $  .000000.  .000000.  .770993.  .929309.  .957243
3     $  .957243.  .929309.  .770994.  .000000.  .000000
3     $  .957243.  .986836.  .991138.  .986836.  .957242
3     $  .000000.  .000000.  .794924.  .961261.  .991138
3     $  .991138.  .961261.  .794924.  .000000.  .000000
3     $  .957242.  .986836.  .991138.  .986836.  .957242
3     $  .000000.  .000000.  .770993.  .929309.  .957242
3     $  .957243.  .929309.  .770993.  .000000.  .000000
3     $  .770993.  .791956.  .794923.  .791956.  .770993
3     $  .000000.  .000000.  .000000.  .000000.  .000000
3     $  .000000.  .000000.  .000000.  .000000/
3C
3      DATA TAB3/ .000000.  .000000.  .000000.  .000000
3     $  .000000.  .000000.  .000000.  .000000.  .000000
3     $  .052605.  .061771.  .064583.  .061771.  .052605
```

```
3     $  .000000,   .000000,   .000000,   .000000,   .000000,
3     $  .052466,   .061586,   .064383,   .061586,   .052465,
3     $  .000000,   .000000,   .052466,   .083345,   .099507,
3     $  .099507,   .083345,   .052465,   .000000,   .000000,
3     $  .099507,   .119864,   .126259,   .119864,   .099507,
3     $  .000000,   .000000,   .064382,   .104531,   .126259,
3     $  .126259,   .104531,   .064382,   .000000,   .000000,
3     $  .099507,   .119864,   .126259,   .119864,   .099507,
3     $  .000000,   .000000,   .052465,   .083345,   .099507,
3     $  .099507,   .083345,   .052465,   .000000,   .000000,
3     $  .052465,   .061586,   .064382,   .061586,   .052465,
3     $  .000000,   .000000,   .000000,   .000000,   .000000,
3     $  .000000,   .000000,   .000000,   .000000/
3C
3C    INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3     CALL QUADRD(X,Y,TAB&E,GRID,GRID,NGRID,NGRID,NGRDD,
3C
3     G = DERUSL(6)
3C
3     W = -(&A**2)*(1.-G)**(&B-1)*EXP(&C+&D*G/(1.+&C*G))
3C
3     RETURN
3     END
------------------------------------------------------------------
'EOR
*************
* MACRO  45 *
*************
*          2000001202000
1          TWO DIMENSIONS $ HOMOGENEOUS
1          UXX$ + UYY$ - (&A*R(X,Y))U$ = 0.0
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3     FUNCTION TRUE(X,Y)
3C
3C    ***************************************************
3C    *
3C    *                MACRO 45 PARAMETERS
3C    *
3C    ***************************************************
3C    *
3C    *                    A   I  B  I  C
3C    *                 -------I-----I-----
3C    *                      2  I  1  I  1
3C    *                         I     I
3C    *                   1000  I  1  I  2
3C    *                         I     I
3C    *                      2  I  2  I  3
3C    *
3C    ***************************************************
3C
3     REAL DERUSL(6), GRID(20), TABLE(20,20),
3     $     T1B1(100), T1B2(100), T1B3(100), T1B4(100),
3     $     T2B1(100), T2B2(100), T2B3(100), T2B4(100),
3     $     T3B1(100), T3B2(100), T3B3(100), T3B4(100)
3     EQUIVALENCE (TABLE(1, 1) , T&CB1(1)),
3     $           (TABLE(1, 6) , T&CB2(1)),
3     $           (TABLE(1,11) , T&CB3(1)),
3     $           (TABLE(1,16) , T&CB4(1))
```

```
3     $  .675985.  .667311.  .660220.  .654633.  .6504
3     $  .646386.  .646386.  .647745.  .650496.  .6546
3     $  .667312.  .675985.  .686376.  .698560.  .7126
3     $  .686375.  .673701.  .662897.  .653849.  .6464
3     $  .636314.  .633459.  .632039.  .632039.  .6334
3     $  .640631.  .646455.  .653849.  .662897.  .6737
3     $  .701080.  .691074.  .675984.  .662897.  .6517
3     $  .634703.  .628672.  .624202.  .621247.  .6197
3     $  .621247.  .624202.  .628672.  .634703.  .6423
3     $  .662897.  .675984.  .691074.  .682909.  .6673
3     $  .642360.  .632726.  .624845.  .618637.  .6140
3     $  .609482.  .609482.  .610995.  .614036.  .6186
3     $  .632726.  .642360.  .653849.  .667311.  .6829
3C
3      DATA T1B2/  .676140.  .660219.  .646454.  .6347
3     $  .616781.  .610426.  .605716.  .602602.  .6010
3     $  .602602.  .605716.  .610426.  .616781.  .6248
3     $  .646454.  .660219.  .676140.  .670803.  .6546
3     $  .628671.  .618637.  .610426.  .603954.  .5991
3     $  .594408.  .594408.  .595986.  .59.'157.  .6039
3     $  .618637.  .628671.  .640630.  .654632.  .6708
3     $  .650495.  .636314.  .624202.  .614036.  .6057
3     $  .594295.  .591080.  .589481.  .589481.  .5910
3     $  .599157.  .605715.  .614036.  .624202.  .6363
3     $  .666916.  .664211.  .647745.  .633459.  .6212
3     $  .602602.  .595986.  .591080.  .587837.  .5862
3     $  .587837.  .591080.  .595986.  .602602.  .6109
3     $  .633458.  .647745.  .664210.  .662967.  .6463
3     $  .619777.  .609482.  .601053.  .594408.  .5894
3     $  .584603.  .584603.  .586223.  .589481.  .5944
3     $  .609482.  .619777.  .632039.  .646386.  .6629
3C
3      DATA T1B3/  .662968.  .646387.  .632039.  .6197
3     $  .601053.  .594408.  .589481.  .586223.  .5846
3     $  .586223.  .589481.  .594408.  .601053.  .6094
3     $  .632039.  .646386.  .662966.  .664212.  .6477
3     $  .621247.  .610995.  .602602.  .595986.  .5910
3     $  .586223.  .586223.  .587837.  .591080.  .5959
3     $  .610995.  .621247.  .633459.  .647745.  .6642
3     $  .650496.  .636314.  .624202.  .614036.  .6057
3     $  .594295.  .591080.  .589481.  .589481.  .5910
3     $  .599157.  .605715.  .614036.  .624202.  .6363
3     $  .666917.  .670805.  .654633.  .640631.  .6286
3     $  .610426.  .603954.  .599157.  .595985.  .5944
3     $  .595985.  .599157.  .603954.  .610425.  .6186
3     $  .640630.  .654633.  .670804.  .676143.  .6602
3     $  .634703.  .624845.  .616780.  .610425.  .6057
3     $  .601053.  .601053.  .602602.  .605715.  .6104
3     $  .624845.  .634703.  .646454.  .660220.  .6761
3C
3      DATA T1B4/  .682910.  .667312.  .653849.  .6423
3     $  .624845.  .618637.  .614036.  .610995.  .6094
3     $  .610994.  .614036.  .618637.  .624845.  .6327
3     $  .653849.  .667311.  .682910.  .691075.  .6759
3     $  .651725.  .642360.  .634703.  .628671.  .6242
3     $  .619777.  .619777.  .621247.  .624201.  .6286
3     $  .642360.  .651725.  .662897.  .675985.  .6910
3     $  .686374.  .673701.  .662896.  .653848.  .6464
3     $  .636314.  .633458.  .632039.  .632039.  .6334
3     $  .640630.  .646454.  .653848.  .662897.  .6737
3     $  .701080.  .712678.  .698559.  .686374.  .6759
3     $  .660219.  .654632.  .650495.  .647745.  .6463
3     $  .647744.  .650495.  .654632.  .660219.  .667
```

```
3     $  .001214,  .001214,  .001214,  .001214,  .0012
3     $  .001215,  .001220,  .001251,  .001460,  .0025
3     $  .037356,  .031205,  .006469,  .001460,  .0004
3     $  .000239,  .000232,  .000231,  .000231,  .0002
3     $  .000231,  .000231,  .000232,  .000239,  .0002
3     $  .001460,  .006469,  .031205,  .033785,  .0063
3     $  .000275,  .000088,  .000052,  .000045,  .0000
3     $  .000044,  .000044,  .000044,  .000044,  .0000
3     $  .000088,  .000275,  .001251,  .006373,  .0337
3C
3      DATA T2B2/  .033551,  .006387,  .001220,  .0002
3     $  .000017,  .000010,  .000009,  .000008,  .0000
3     $  .000008,  .000009,  .000010,  .000017,  .0000
3     $  .001220,  .006387,  .033551,  .033579,  .0063
3     $  .000232,  .000045,  .000010,  .000003,  .0000
3     $  .000002,  .000002,  .000002,  .000002,  .0000
3     $  .000045,  .000232,  .001215,  .006385,  .0335
3     $  .006384,  .001214,  .000231,  .000044,  .0000
3     $  .000001,  .000000,  .000000,  .000000,  .0000
3     $  .000002,  .000009,  .000044,  .000231,  .0012
3     $  .033571,  .033566,  .006384,  .001214,  .0002
3     $  .000008,  .000002,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000002,  .000008,  .0000
3     $  .001214,  .006384,  .033566,  .033569,  .0063
3     $  .000231,  .000044,  .000008,  .000002,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000044,  .000231,  .001214,  .006384,  .0335
3C
3      DATA T2B3/  .033569,  .006384,  .001214,  .0002
3     $  .000008,  .000002,  .000000,  .000000,  .0000
3     $  .000000,  .000000,  .000002,  .000008,  .0000
3     $  .001214,  .006384,  .033569,  .033566,  .0063
3     $  .000231,  .000044,  .000008,  .000002,  .0000
3     $  .000000,  .000000,  .000000,  .000000,  .0000
3     $  .000044,  .000231,  .001214,  .006384,  .0335
3     $  .006384,  .001214,  .000231,  .000044,  .0000
3     $  .000001,  .000000,  .000000,  .000000,  .0000
3     $  .000002,  .000009,  .000044,  .000231,  .0012
3     $  .033571,  .033579,  .006385,  .001215,  .0002
3     $  .000010,  .000003,  .000002,  .000002,  .0000
3     $  .000002,  .000002,  .000003,  .000010,  .0000
3     $  .001215,  .006385,  .033579,  .033551,  .0063
3     $  .000239,  .000052,  .000017,  .000010,  .0000
3     $  .000008,  .000008,  .000008,  .000009,  .0000
3     $  .000052,  .000239,  .001220,  .006387,  .0335
3C
3      DATA T2B4/  .033785,  .006373,  .001251,  .0002
3     $  .000052,  .000045,  .000044,  .000044,  .0000
3     $  .000044,  .000044,  .000045,  .000052,  .0000
3     $  .001251,  .006373,  .033785,  .031205,  .0064
3     $  .000468,  .000275,  .000239,  .000232,  .0002
3     $  .000231,  .000231,  .000231,  .000231,  .0002
3     $  .000275,  .000468,  .001460,  .006469,  .0312
3     $  .008223,  .002597,  .001460,  .001251,  .0012
3     $  .001214,  .001214,  .001214,  .001214,  .0012
3     $  .001215,  .001220,  .001251,  .001460,  .0025
3     $  .037356,  .048804,  .014807,  .008223,  .0064
3     $  .006387,  .006385,  .006384,  .006384,  .0063
3     $  .006384,  .006384,  .006385,  .006387,  .0063
3     $  .008223,  .014807,  .048804,  .065852,  .0488
3     $  .031205,  .033785,  .033551,  .033579,  .0335
3     $  .033569,  .033569,  .033566,  .033571,  .0335
3     $  .033785,  .031205,  .037356,  .048804,  .0658
```

```
3     $  .699604.   .701881.   .705331.   .709998.   .71594
3     $  .731994.   .742302.   .754253.   .747913.   .73554
3     $  .715943.   .708432.   .702313.   .697507.   .69395
3     $  .690446.   .690445.   .691610.   .693954.   .69750
3     $  .708432.   .715943.   .724941.   .735544.   .74791
3C
3      DATA T3B2/   .742651.   .730041.   .719202.   .70999
3     $  .696049.   .691128.   .687491.   .685090.   .68389
3     $  .685090.   .687490.   .691128.   .696048.   .70231
3     $  .719201.   .730040.   .742649.   .738532.   .72572
3     $  .705331.   .697507.   .691128.   .686117.   .68241
3     $  .678750.   .678749.   .679965.   .682411.   .68611
3     $  .697506.   .705330.   .714695.   .725720.   .73853
3     $  .722529.   .711366.   .701881.   .693954.   .68749
3     $  .678654.   .676174.   .674942.   .674942.   .67617
3     $  .682410.   .687490.   .693953.   .701880.   .71136
3     $  .735528.   .733442.   .720410.   .709167.   .69960
3     $  .685090.   .679965.   .676174.   .673672.   .67242
3     $  .673672.   .676174.   .679964.   .685089.   .69160
3     $  .709166.   .720409.   .733440.   .732489.   .71936
3     $  .698472.   .690445.   .683895.   .678749.   .67494
3     $  .671179.   .671179.   .672428.   .674941.   .67874
3     $  .690444.   .698471.   .708073.   .719364.   .73248
3C
3      DATA T3B3/   .732489.   .719365.   .708074.   .69847
3     $  .683896.   .678749.   .674941.   .672428.   .67117
3     $  .672428.   .674941.   .678748.   .683896.   .69044
3     $  .708073.   .719364.   .732487.   .733441.   .72040
3     $  .699603.   .691609.   .685089.   .679964.   .67617
3     $  .672427.   .672427.   .673671.   .676173.   .67996
3     $  .691608.   .699602.   .709165.   .720408.   .73343
3     $  .722527.   .711365.   .701879.   .693953.   .68748
3     $  .678653.   .676173.   .674941.   .674941.   .67617
3     $  .682409.   .687489.   .693952.   .701879.   .71136
3     $  .735527.   .738529.   .725719.   .714695.   .70532
3     $  .691126.   .686115.   .682409.   .679963.   .67874
3     $  .679963.   .682409.   .686115.   .691126.   .69750
3     $  .714694.   .725718.   .738529.   .742648.   .73003
3     $  .709996.   .702310.   .696046.   .691126.   .68748
3     $  .683895.   .683895.   .685088.   .687488.   .69112
3     $  .702310.   .709995.   .719199.   .730038.   .74264
3C
3      DATA T3B4/   .747909.   .735541.   .724939.   .71594
3     $  .702310.   .697504.   .693952.   .691608.   .69044
3     $  .691608.   .693952.   .697504.   .702310.   .70842
3     $  .724939.   .735541.   .747909.   .754250.   .74229
3     $  .723240.   .715940.   .709995.   .705328.   .70187
3     $  .698470.   .698470.   .699601.   .701878.   .70532
3     $  .715940.   .723241.   .731991.   .742299.   .75425
3     $  .750441.   .740455.   .731991.   .724938.   .71919
3     $  .711363.   .709164.   .708071.   .708072.   .70916
3     $  .714693.   .719199.   .724938.   .731991.   .74045
3     $  .762107.   .771257.   .760047.   .750441.   .74229
3     $  .730037.   .725717.   .722525.   .720406.   .71936
3     $  .720407.   .722525.   .725717.   .730037.   .73554
3     $  .750441.   .760048.   .771258.   .781909.   .77125
3     $  .754249.   .747908.   .742646.   .738527.   .73552
3     $  .732485.   .732485.   .733438.   .735526.   .73852
3     $  .747908.   .754249.   .762106.   .771257.   .78191
3C
3C     INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRD
```

```
3C
3      DATA TAB3/  .781915,   .759066,   .743903,   .735237,
3     $   .735236,   .743903,   .759066,   .781915,   .759065,
3     $   .716957,   .707336,   .704204,   .707335,   .716957,
3     $   .759065,   .743902,   .716956,   .698974,   .688669,
3     $   .688668,   .698973,   .716955,   .743900,   .735236,
3     $   .688668,   .677956,   .674465,   .677955,   .688666,
3     $   .735234,   .732416,   .704204,   .685311,   .674464,
3     $   .674463,   .685309,   .704201,   .732414,   .735234,
3     $   .688666,   .677954,   .674463,   .677953,   .688665,
3     $   .735233,   .743899,   .716953,   .698970,   .688666,
3     $   .688665,   .698970,   .716952,   .743898,   .759061,
3     $   .716952,   .707331,   .704200,   .707332,   .716952,
3     $   .759062,   .781909,   .759060,   .743897,   .735231,
3     $   .735232,   .743897,   .759061,   .781911/
3C
3      IF (&B .EQ. 2) GO TO 10
3      R = 1.0
3      RETURN
3C
3C     INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3   10 CALL QUADRD(X,Y,TAB3,GRID,GRID,NGRID,NGRID,NGRDD,N
3C
3      R = DERUSL(6)
3C
3      RETURN
3      END
```

```
'EOR
*************
* MACRO  46 *
*************
*          2000220202000
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS $ HOMOG
1          UXX$ + UYY$ - &BUY$ = 0.
2          DIRICHLET
2          X=0. , U= 0.
2          X=&A , U= 0.
2          Y=0. , U=-1.
2          Y=1. , U= 1.
3      FUNCTION TRUE(X,Y)
3C
3C     ***************************************************
3C     *
3C     *                  MACRO 46 PARAMETERS
3C     *
3C     ***************************************************
3C     *
3C     *                    A  I   B  I  C
3C     *                   ----I------I----
3C     *                    1  I   2  I  1
3C     *                       I      I
3C     *                    4  I   2  I  2
3C     *                       I      I
3C     *                    4  I  10  I  3
3C     *                       I      I
3C     *                    8  I   2  I  4
3C     *
3C     ***************************************************
3C
3      REAL DERUSL(6), TABLE(20,20), GRID1(20),
3     $     GRID2(20), GRID3(20), GRID4(20),
```

```
3     $1.0000000/
3      DATA GRID2/0.0000000, 0.2105263, 0.4210526, 0.6
3     $0.8421053, 1.0526316, 1.2631579, 1.4736842, 1.6
3     $1.8947368, 2.1052632, 2.3157895, 2.5263158, 2.7
3     $2.9473684, 3.1578947, 3.3684211, 3.5789474, 3.7
3     $4.0000000/
3      DATA GRID3/0.0000000, 0.2105263, 0.4210526, 0.6
3     $0.8421053, 1.0526316, 1.2631579, 1.4736842, 1.6
3     $1.8947368, 2.1052632, 2.3157895, 2.5263158, 2.7
3     $2.9473684, 3.1578947, 3.3684211, 3.5789474, 3.7
3     $4.0000000/
3      DATA GRID4/0.0000000, 0.4210526, 0.8421053, 1.2
3     $1.6842105, 2.1052632, 2.5263158, 2.9473684, 3.3
3     $3.7894737, 4.2105263, 4.6315789, 5.0526316, 5.4
3     $5.8947368, 6.3157895, 6.7368421, 7.1578947, 7.5
3     $8.0000000/
3C
3C     APPROXIMATE SOLUTION OF PROBLEM USING
3C     P3-C1 COLLOCATION (8 X 8 GRID)
3C
3      DATA T1B1/-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,  .031332, -.2053
3     $ -.817443, -.857488, -.894001, -.909298, -.9192
3     $ -.928640, -.928751, -.926530, -.921708, -.9115
3     $ -.875902, -.827951, -.792383, -.619858,  .0313
3     $ -.219279, -.513384, -.673778, -.741340, -.7894
3     $ -.836165, -.847318, -.852362, -.852400, -.8474
3     $ -.818609, -.791142, -.747734, -.677427, -.5682
3     $ -.042140, -.047770, -.227276, -.415552, -.5476
3     $ -.687743, -.725177, -.750246, -.765570, -.7726
3     $ -.765706, -.751106, -.725896, -.688705, -.6328
3     $ -.425126, -.250824, -.047770,  .006366, -.1524
3     $ -.445435, -.530434, -.593233, -.635471, -.6646
3     $ -.691458, -.691540, -.683244, -.665984, -.6365
3     $ -.533931, -.447016, -.335403, -.189134,  .0063
3C
3      DATA T1B2/ -.008657, -.135812, -.262867, -.3670
3     $ -.507305, -.551224, -.581744, -.600866, -.6101
3     $ -.600900, -.581958, -.551403, -.507545, -.4470
3     $ -.265253, -.141679, -.008657, -.001713, -.1121
3     $ -.302230, -.374162, -.429104, -.471392, -.5012
3     $ -.529380, -.529314, -.519998, -.501114, -.4711
3     $ -.372592, -.301277, -.212279, -.111526, -.0017
3     $ -.089061, -.171761, -.245781, -.307823, -.3566
3     $ -.422671, -.440317, -.449200, -.449148, -.4403
3     $ -.394853, -.356424, -.306595, -.245036, -.1706
3     $  .000386, -.000909, -.069776, -.135734, -.1951
3     $ -.288236, -.321251, -.345454, -.361272, -.3691
3     $ -.361272, -.345452, -.321246, -.288229, -.2461
3     $ -.135706, -.069764, -.000909, -.000015, -.0550
3     $ -.149636, -.190299, -.222356, -.249766, -.2695
3     $ -.288862, -.288693, -.281920, -.268605, -.2488
3     $ -.187692, -.148376, -.100697, -.051419, -.0000
3C
3      DATA T1B3/ -.000008, -.037091, -.070953, -.1031
3     $ -.155868, -.176071, -.191055, -.200851, -.2060
3     $ -.200813, -.190640, -.175631, -.155406, -.1308
3     $ -.069654, -.035437, -.000008, -.000052, -.0198
3     $ -.055845, -.072664, -.086967, -.099741, -.1094
3     $ -.119490, -.119438, -.115931, -.109087, -.0993
3     $ -.071702, -.055412, -.036898, -.018570, -.0000
```

```
3     $  .309026,  .303747,  .293226,  .277171,  .2504
3     $  .156400,  .082855,  .000000, -.000001,  .1385
3     $  .324298,  .377583,  .410414,  .431180,  .4436
3     $  .453498,  .453502,  .450387,  .443745,  .4312
3     $  .377831,  .324414,  .244673,  .138975, -.0000
3     $  .182666,  .372736,  .479280,  .534312,  .5727
3     $  .605291,  .611630,  .614897,  .614676,  .6117
3     $  .593524,  .574053,  .538910,  .481864,  .3848
3     $  .000000,  .000000,  .364539,  .600069,  .6927
3     $  .768311,  .782081,  .790041,  .794434,  .7964
3     $  .794520,  .790289,  .782132,  .769066,  .7395
3     $  .607330,  .378554,  .000000,  .000000, 1.0313
3     $  .952230, 1.006366,  .991341,  .998282, 1.0003
3     $  .999977,  .999977,  .999039, 1.000391,  .9982
3     $ 1.006366,  .952230,  .957860, 1.031332,  .0000
3C
3      DATA T2B1/-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,  .031332, -.3373
3     $ -.913851, -.935099, -.955727, -.963381, -.9646
3     $ -.966591, -.965818, -.965444, -.966943, -.9646
3     $ -.973336, -.936573,-1.024956,-1.053698,  .0313
3     $ -.583009, -.820302, -.888801, -.907831, -.9208
3     $ -.925646, -.926188, -.926869, -.926601, -.9263
3     $ -.924854, -.923851, -.921108, -.896691, -.9179
3     $ -.042140, -.047770, -.662054, -.820058, -.8476
3     $ -.877901, -.881399, -.882810, -.883104, -.8840
3     $ -.883347, -.883194, -.881111, -.878838, -.8709
3     $ -.827376, -.677138, -.047770,  .006366, -.5534
3     $ -.791510, -.816764, -.828036, -.833113, -.8349
3     $ -.836699, -.836071, -.835586, -.835566, -.8326
3     $ -.819806, -.792775, -.753521, -.576948,  .0063
3C
3      DATA T2B2/ -.008657, -.475263, -.670619, -.7332
3     $ -.773474, -.778669, -.781104, -.782017, -.7826
3     $ -.782077, -.781200, -.778597, -.773707, -.7617
3     $ -.672442, -.479021, -.008657, -.001713, -.4178
3     $ -.668234, -.699453, -.712947, -.719571, -.7223
3     $ -.724377, -.723898, -.723272, -.722708, -.7191
3     $ -.700611, -.668502, -.598330, -.405151, -.0017
3     $ -.349512, -.525281, -.598374, -.631821, -.6452
3     $ -.656541, -.657452, -.658720, -.658345, -.6576
3     $ -.653070, -.646900, -.632727, -.598584, -.5229
3     $  .000386, -.000909, -.279552, -.445959, -.5232
3     $ -.572644, -.579762, -.583272, -.584677, -.5853
3     $ -.584682, -.583280, -.579754, -.572661, -.5577
3     $ -.445900, -.279298, -.000909, -.000015, -.2361
3     $ -.442412, -.476725, -.491462, -.499207, -.5027
3     $ -.505164, -.504852, -.504106, -.502988, -.4988
3     $ -.476819, -.442191, -.369005, -.224010, -.0000
3C
3      DATA T2B3/ -.000008, -.174360, -.289796, -.3544
3     $ -.401672, -.409212, -.412868, -.414247, -.4151
3     $ -.414331, -.412966, -.409066, -.401863, -.3870
3     $ -.288023, -.168780, -.000008, -.000052, -.1154
3     $ -.258949, -.288604, -.302458, -.309542, -.3131
3     $ -.315353, -.315351, -.314671, -.313138, -.3095
3     $ -.288308, -.258768, -.201567, -.111454, -.0000
3     $ -.061608, -.112896, -.154902, -.180712, -.1927
3     $ -.202706, -.204138, -.204799, -.204792, -.2041
3     $ -.199275, -.192614, -.179827, -.154362, -.1078
3     $  .000004, -.000005,  .016566, -.007429, -.0398
```

```
3     $  .382444,  .381600,  .384950,  326554, -.0000
3     $  .447745,  .538473,  .551996,  .562925,  .5624
3     $  .562762,  .561406,  .562954,  .562046,  .5619
3     $  .562623,  .564325,  .566423,  .553316,  .5583
3     $  .000000,  .000000,  .688631,  .735748,  .7422
3     $  .766365,  .769300,  .769854,  .768829,  .7700
3     $  .769152,  .770303,  .768836,  .767486,  .7715
3     $  .747681,  .716116,  .000000,  .000000, 1.0313
3     $  .952230, 1.006366,  .991341,  .998282, 1.0003
3     $  .999977,  .999977,  .999039, 1.000391,  .9982
3     $ 1.006366,  .952230,  .957860, 1.031332,  .0000
3C
3      DATA T3B1/-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,  .031332, -.3913
3     $ -.952148, -.972313, -.992515, -.998877, -.9997
3     $-1.001588,-1.000718,-1.000366,-1.002265,-1.0001
3     $-1.013562, -.976641,-1.088341,-1.156170,  .0313
3     $ -.733253, -.934475, -.973754, -.988436, -.9973
3     $ -.999649, -.999790,-1.000378,-1.000076,-1.0000
3     $ -.999692,-1.000618,-1.002758, -.982258,-1.0388
3     $ -.042140, -.047770, -.867063, -.995266, -.9840
3     $ -.999381,-1.000133,-1.000146, -.999665,-1.0006
3     $ -.999976,-1.000488, -.999562,-1.000217, -.9977
3     $-1.003428, -.886322, -.047770,  .006366, -.7855
3     $ -.981972, -.995069, -.998631,-1.000227,-1.0001
3     $-1.000876,-1.000032, -.999824,-1.000713, -.9993
3     $ -.996384, -.982043, -.982737, -.815523,  .0063
3C
3      DATA T3B2/ -.008657, -.734549, -.951262, -.9796
3     $ -.997615, -.998527, -.998770, -.998758, -.9990
3     $ -.999235, -.998855, -.999285, -.997823, -.9927
3     $ -.953296, -.739347, -.008657, -.001713, -.6958
3     $ -.974706, -.990749, -.996110, -.998562, -.9987
3     $ -.999426, -.998584, -.998329, -.999219, -.9976
3     $ -.991864, -.974667, -.928979, -.683361, -.0017
3     $ -.643618, -.903358, -.967714, -.987958, -.9939
3     $ -.996971, -.996308, -.997574, -.996915, -.9966
3     $ -.995862, -.994877, -.988840, -.967684, -.9015
3     $  .000386, -.000909, -.590183, -.872571, -.9584
3     $ -.990511, -.992719, -.993539, -.993634, -.9937
3     $ -.993644, -.993549, -.992700, -.990536, -.9837
3     $ -.872526, -.589933, -.000909, -.000015, -.5652
3     $ -.946655, -.977289, -.985404, -.989575, -.9903
3     $ -.991181, -.990251, -.989906, -.990867, -.9885
3     $ -.979124, -.947008, -.848901, -.555348, -.0000
3C
3      DATA T3B3/ -.000008, -.525620, -.819198, -.9317
3     $ -.977270, -.981166, -.982321, -.982009, -.9829
3     $ -.982252, -.982572, -.980699, -.977942, -.9688
3     $ -.818872, -.521027, -.000008, -.000052, -.4912
3     $ -.911476, -.953237, -.963860, -.968035, -.9694
3     $ -.970045, -.969775, -.969564, -.969639, -.9677
3     $ -.953727, -.911553, -.787369, -.489058, -.0000
3     $ -.464670, -.753857, -.883824, -.930362, -.9426
3     $ -.949734, -.949093, -.950727, -.949919, -.9495
3     $ -.947349, -.943909, -.931826, -.884055, -.7538
3     $  .000004, -.000005, -.421154, -.705669, -.8424
3     $ -.906376, -.911457, -.913355, -.913482, -.9140
3     $ -.913595, -.913471, -.911237, -.906682, -.8930
3     $ -.705662, -.419556, -.000005, -.000002, -.3795
3     $ -.779598, -.832003, -.845481, -.852136, -.8541
```

```
3     $  .000000,  .000000,  .332085,  .247819,  .1889
3     $  .174398,  .174712,  .174340,  .173400,  .1745
3     $  .173718,  .174737,  .174192,  .175519,  .1854
3     $  .260071,  .359805,  .000000,  .000000, 1.0313
3     $  .952230, 1.006366,  .991341,  .998282, 1.0003
3     $  .999977,  .999977,  .999039, 1.000391,  .9982
3     $ 1.006366,  .952230,  .957860, 1.031332,  .0000
3C
3      DATA T4B1/-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,-1.000000,-1.0000
3     $-1.000000,-1.000000,-1.000000,  .031332, -.3898
3     $ -.914846, -.938208, -.956910, -.965106, -.9659
3     $ -.968304, -.966702, -.966078, -.969213, -.9659
3     $ -.984159, -.941488,-1.062985,-1.172767,  .0313
3     $ -.745919, -.871901, -.899702, -.919758, -.9238
3     $ -.926998, -.926564, -.927720, -.927163, -.9269
3     $ -.926854, -.927762, -.935713, -.908953, -.9801
3     $ -.042140, -.047770, -.879855, -.904387, -.8703
3     $ -.884009, -.885089, -.885004, -.883794, -.8853
3     $ -.884261, -.885423, -.884105, -.885060, -.8886
3     $ -.913185, -.900626, -.047770,  .006366, -.7854
3     $ -.824578, -.839884, -.836717, -.838284, -.8379
3     $ -.838543, -.837251, -.836928, -.838573, -.8367
3     $ -.840509, -.823891, -.861101, -.817824,  .0063
3C
3      DATA T4B2/ -.008657, -.716569, -.794651, -.7740
3     $ -.783934, -.783679, -.783549, -.783265, -.7835
3     $ -.783381, -.783654, -.783434, -.784196, -.7843
3     $ -.796844, -.721744, -.008657, -.001713, -.6571
3     $ -.718829, -.727195, -.725674, -.726301, -.7257
3     $ -.726221, -.725239, -.725048, -.726228, -.7250
3     $ -.726149, -.717362, -.731700, -.638213, -.0017
3     $ -.571192, -.665170, -.655241, -.661281, -.6604
3     $ -.660201, -.659171, -.660496, -.659728, -.6596
3     $ -.659763, -.661169, -.660463, -.654094, -.6605
3     $  .000386, -.000909, -.475176, -.582992, -.5835
3     $ -.587687, -.586974, -.586653, -.586553, -.5865
3     $ -.586564, -.586662, -.586950, -.587706, -.5870
3     $ -.582873, -.474797, -.000909, -.000015, -.4166
3     $ -.505907, -.508466, -.507456, -.507482, -.5068
3     $ -.507022, -.506385, -.506299, -.507169, -.5067
3     $ -.507916, -.505027, -.500147, -.394151, -.0000
3C
3      DATA T4B3/ -.000008, -.320672, -.412314, -.4180
3     $ -.417933, -.417433, -.416874, -.416337, -.4167
3     $ -.416502, -.417013, -.417073, -.418235, -.4185
3     $ -.408958, -.310286, -.000008, -.000052, -.2297
3     $ -.320682, -.319571, -.318531, -.317527, -.3169
3     $ -.316694, -.316689, -.316780, -.316993, -.3175
3     $ -.319410, -.320576, -.309297, -.221935, -.0000
3     $ -.150308, -.208674, -.213207, -.210190, -.2082
3     $ -.206484, -.206209, -.206132, -.206118, -.2062
3     $ -.207072, -.208218, -.209709, -.212891, -.2003
3     $  .000004, -.000005, -.028886, -.083017, -.0934
3     $ -.085678, -.084329, -.083612, -.083338, -.0832
3     $ -.083339, -.083610, -.084320, -.085661, -.0875
3     $ -.080944, -.023133, -.000005, -.000002,  .0693
3     $  .039158,  .048332,  .050450,  .053013,  .0536
3     $  .054444,  .053828,  .053528,  .053994,  .0523
3     $  .048312,  .039251,  .053038,  .094589, -.0000
3C
3      DATA T4B4/  .000000,  .200694,  .188269,  .1861
```

```
3     $  .952230,  1.006366,   .991341,   .998282,  1.000391,
3     $  .999977,   .999977,   .999039,  1.000391,   .998282,
3     $ 1.006366,   .952230,   .957860,  1.031332,   .000000/
3C
3C
3C    INTERPOLATE NUMERICAL SOLUTION BY QUADRATICS
3C
3     CALL QUADRD(X,Y,TABLE,GRID&C,GRID1,NGRID,NGRID,NGR
3    $            DERVSL)
3C
3     TRUE = DERVSL(6)
3C
3     RETURN
3     END
-------------------------------------------------------------
'EOR
••••••••••••
• MACRO  47 •
••••••••••••
•          2020021002000
1          TWO DIMENSIONS  $  CONSTANT COEFFICIENTS  $  P
1          UXX$ + UYY$ = F(X,Y)
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3     FUNCTION TRUE(X,Y)
3     TRUE = (X*Y)**(.5*(&A))
3     RETURN
3     END
3     FUNCTION F(X,Y)
3     IF ( X.EQ.0. .OR. Y.EQ.0. ) GO TO 10
3     F = (.5*(&A))*(.5*(&A)-1.)*(X*Y)**(.5*(&A)-2.)*(X*
3     RETURN
3  10 F = 0.
3     RETURN
3     END
-------------------------------------------------------------
'EOR
••••••••••••
• MACRO  48 •
••••••••••••
•          2000001202000
1          TWO DIMENSIONS
1          UXX$ + UYY$ + R(X,Y)U$ = 0.0
2          DIRICHLET
2          X=0. , U=1.
2          X=1. , U=1.
2          Y=0. , U=1.
2          Y=1. , U=1.
3     FUNCTION TRUE(X,Y)
3C
3C    ••••••••••••••••••••••••••••••••••••••••••••••••••••••
3C    •
3C    •                 MACRO 48 PARAMETERS
3C    •
3C    ••••••••••••••••••••••••••••••••••••••••••••••••••••••
3C    •
3C    •                A  I    B  I  C  I  D
3C    •               ----I-------I-----I-----
3C    •                2  I  .04  I  1  I  1
3C    •                   I       I     I
```

```
3     $     T3B1(100), T3B2(100), T3B3(100), T3B4(100)
3     $     T4B1(100), T4B2(100), T4B3(100), T4B4(100)
3     $     T5B1(100), T5B2(100), T5B3(100), T5B4(100)
3      EQUIVALENCE (TABLE(1, 1) , T&DB1(1)),
3     $            (TABLE(1, 6) , T&DB2(1)),
3     $            (TABLE(1,11) , T&DB3(1)),
3     $            (TABLE(1,16) , T&DB4(1))
3      DATA NGRID, NGRDD, GRID /20, 20, 0.0000000, 0.0
3     $0.1052632, 0.1578947, 0.2105263, 0.2631579, 0.3
3     $0.3684211, 0.4210526, 0.4736842, 0.5263158, 0.5
3     $0.6315789, 0.6842105, 0.7368421, 0.7894737, 0.8
3     $0.8947368, 0.9473684, 1.0000000/
3C
3C     APPROXIMATE SOLUTION OF LINEARIZED PROBLEM USIN
3C     HODIE-ACF (METHOD=4, IORDER=41, 20 X 20 GRID)
3C
3      DATA T1B1/ 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000,  .9934
3     $  .985662,  .983162,  .981263,  .979847,  .9788
3     $  .977874,  .977874,  .978191,  .978839,  .9798
3     $  .983162,  .985662,  .988963,  .993444, 1.0000
3     $  .988963,  .980721,  .974445,  .969621,  .9659
3     $  .961193,  .959923,  .959301,  .959301,  .9599
3     $  .963166,  .965930,  .969621,  .974445,  .9807
3     $ 1.000000, 1.000000,  .985662,  .974445,  .9656
3     $  .953504,  .949515,  .946657,  .944813,  .9439
3     $  .944813,  .946657,  .949515,  .953504,  .9588
3     $  .974445,  .985662, 1.000000, 1.000000,  .9831
3     $  .958802,  .950236,  .943561,  .938504,  .9348
3     $  .931362,  .931362,  .932517,  .934869,  .9385
3     $  .950236,  .958802,  .969621,  .983162, 1.0000
3C
3      DATA T1B2/ 1.000000,  .981263,  .965930,  .9535
3     $  .935752,  .929805,  .925515,  .922733,  .9213
3     $  .922733,  .925515,  .929805,  .935752,  .9435
3     $  .965930,  .981263, 1.000000, 1.000000,  .9798
3     $  .949515,  .938504,  .929805,  .923152,  .9183
3     $  .913667,  .913667,  .915207,  .918336,  .9231
3     $  .938504,  .949515,  .963166,  .979847, 1.0000
3     $  .978839,  .961193,  .946657,  .934869,  .9255
3     $  .913128,  .909739,  .908069,  .908069,  .9097
3     $  .918336,  .925515,  .934869,  .946657,  .9611
3     $ 1.000000, 1.000000,  .978191,  .959923,  .9448
3     $  .922733,  .915207,  .909739,  .906176,  .9044
3     $  .906176,  .909739,  .915207,  .922733,  .9325
3     $  .959923,  .978191, 1.000000, 1.000000,  .9778
3     $  .943909,  .931362,  .921365,  .913667,  .9080
3     $  .902620,  .902620,  .904420,  .908069,  .9136
3     $  .931362,  .943909,  .959301,  .977874, 1.0000
3C
3      DATA T1B3/ 1.000000,  .977874,  .959301,  .9439
3     $  .921365,  .913667,  .908069,  .904420,  .9026
3     $  .904420,  .908069,  .913667,  .921365,  .9313
3     $  .959301,  .977874, 1.000000, 1.000000,  .9781
3     $  .944813,  .932517,  .922733,  .915207,  .9097
3     $  .904420,  .904420,  .906176,  .909739,  .9152
3     $  .932517,  .944813,  .959923,  .978191, 1.0000
3     $  .978839,  .961193,  .946657,  .934869,  .9255
3     $  .913128,  .909739,  .908069,  .908069,  .9097
3     $  .918336,  .925515,  .934869,  .946657,  .9611
3     $ 1.000000, 1.000000,  .979847,  .963166,  .9495
```

```
3     $  .958802,  .965662,  .974445,  .985662, 1.0000
3     $  .988963,  .980721,  .974445,  .969621,  .9659
3     $  .961193,  .959923,  .959301,  .959301,  .9599
3     $  .963166,  .965930,  .969621,  .974445,  .9807
3     $ 1.000000, 1.000000,  .993444,  .988963,  .9856
3     $  .981263,  .979847,  .978339,  .978191,  .9778
3     $  .978191,  .978839,  .979847,  .981263,  .9831
3     $  .988963,  .993444, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3C
3      DATA T2B1/ 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000,  .9932
3     $  .985062,  .982399,  .980363,  .978836,  .9777
3     $  .976693,  .976693,  .977039,  .977744,  .9788
3     $  .982399,  .985062,  .988547,  .993229, 1.0000
3     $  .988547,  .979910,  .973273,  .968131,  .9641
3     $  .959048,  .957667,  .956989,  .956989,  .9576
3     $  .961187,  .964170,  .968131,  .973273,  .9799
3     $ 1.000000, 1.000000,  .985062,  .973273,  .9639
3     $  .950953,  .946644,  .943545,  .941539,  .9405
3     $  .941539,  .943545,  .946644,  .950953,  .9566
3     $  .973273,  .985062, 1.000000, 1.000000,  .9823
3     $  .956644,  .947487,  .940308,  .934843,  .9308
3     $  .927079,  .927079,  .928338,  .930898,  .9348
3     $  .947487,  .956644,  .968131,  .982399, 1.0000
3C
3      DATA T2B2/ 1.000000,  .980363,  .964170,  .9509
3     $  .931901,  .925468,  .920809,  .917779,  .9162
3     $  .917779,  .920809,  .925468,  .931901,  .9403
3     $  .964170,  .980363, 1.000000, 1.000000,  .9788
3     $  .946644,  .934843,  .925468,  .918265,  .9130
3     $  .907942,  .907942,  .909622,  .913031,  .9182
3     $  .934843,  .946644,  .961187,  .978836, 1.0000
3     $  .977744,  .959048,  .943545,  .930898,  .9208
3     $  .907369,  .903675,  .901852,  .901852,  .9036
3     $  .913031,  .920809,  .930898,  .943545,  .9590
3     $ 1.000000, 1.000000,  .977039,  .957667,  .9415
3     $  .917779,  .909622,  .903675,  .899790,  .8978
3     $  .899790,  .903675,  .909622,  .917779,  .9283
3     $  .957667,  .977039, 1.000000, 1.000000,  .9766
3     $  .940554,  .927079,  .916287,  .907942,  .9018
3     $  .895907,  .895907,  .897872,  .901852,  .9079
3     $  .927079,  .940554,  .956989,  .976693, 1.0000
3C
3      DATA T2B3/ 1.000000,  .976693,  .956989,  .9405
3     $  .916287,  .907942,  .901852,  .897872,  .8959
3     $  .897872,  .901852,  .907942,  .916287,  .9270
3     $  .956989,  .976693, 1.000000, 1.000000,  .9770
3     $  .941539,  .928338,  .917779,  .909622,  .9036
3     $  .897872,  .897872,  .899790,  .903675,  .9096
3     $  .928338,  .941539,  .957667,  .977039, 1.0000
3     $  .977744,  .959048,  .943545,  .930898,  .9208
3     $  .907369,  .903675,  .901852,  .901852,  .9036
3     $  .913031,  .920809,  .930898,  .943545,  .9590
3     $ 1.000000, 1.000000,  .978836,  .961187,  .9466
3     $  .925468,  .918265,  .913031,  .909622,  .9079
3     $  .909622,  .913031,  .918265,  .925468,  .9348
3     $  .961187,  .978836, 1.000000, 1.000000,  .9803
3     $  .950953,  .940308,  .931901,  .925468,  .9208
```

```
3     $ 1.000000.  1.000000.   .993229.   .988547.   .9850
3     $  .980363.   .978836.   .977744.   .977039.   .9766
3     $  .977039.   .977744.   .978836.   .980363.   .9823
3     $  .988547.   .993229.  1.000000.  1.000000.  1.0000
3     $ 1.000000.  1.000000.  1.000000.  1.000000.  1.0000
3     $ 1.000000.  1.000000.  1.000000.  1.000000.  1.0000
3     $ 1.000000.  1.000000.  1.000000.  1.000000.  1.0000
3C
3      DATA T3B1/ 1.000000.  1.000000.  1.000000.  1.0000
3     $ 1.000000.  1.000000.  1.000000.  1.000000.  1.0000
3     $ 1.000000.  1.000000.  1.000000.  1.000000.  1.0000
3     $ 1.000000.  1.000000.  1.000000.  1.000000.   .9936
3     $  .986233.   .983885.   .982115.   .980804.   .9798
3     $  .978988.   .978988.   .979279.   .979874.   .9808
3     $  .983885.   .986233.   .989350.   .993649.  1.0000
3     $  .989360.   .981493.   .975558.   .971033.   .9675
3     $  .963217.   .962050.   .961479.   .961479.   .9620
3     $  .965035.   .967595.   .971033.   .975558.   .9814
3     $ 1.000000.  1.000000.   .986233.   .975558.   .9672
3     $  .955913.   .952221.   .949589.   .947895.   .9470
3     $  .947895.   .949589.   .952221.   .955913.   .9608
3     $  .975558.   .986233.  1.000000.  1.000000.   .9838
3     $  .960843.   .952832.   .946627.   .941951.   .9386
3     $  .935386.   .935386.   .936445.   .938604.   .9419
3     $  .952832.   .960843.   .971033.   .983885.  1.0000
3C
3      DATA T3B2/ 1.000000.   .982115.   .967595.   .9559
3     $  .939377.   .933883.   .929934.   .927381.   .9261
3     $  .927381.   .929934.   .933883.   .939377.   .9466
3     $  .967595.   .982115.  1.000000.  1.000000.   .9808
3     $  .952221.   .941951.   .933883.   .927740.   .9233
3     $  .919031.   .919031.   .920441.   .923311.   .9277
3     $  .941951.   .952221.   .965035.   .980804.  1.0000
3     $  .979874.   .963217.   .949589.   .938604.   .9299
3     $  .918523.   .915416.   .913887.   .913887.   .9154
3     $  .923311.   .929934.   .938604.   .949589.   .9632
3     $ 1.000000.  1.000000.   .979279.   .962050.   .9478
3     $  .927381.   .920441.   .915416.   .912151.   .9105
3     $  .912151.   .915416.   .920441.   .927381.   .9364
3     $  .962050.   .979279.  1.000000.  1.000000.   .9789
3     $  .947066.   .935386.   .926128.   .919031.   .9138
3     $  .908897.   .908897.   .910544.   .913887.   .9190
3     $  .935386.   .947066.   .961479.   .978988.  1.0000
3C
3      DATA T3B3/ 1.000000.   .978988.   .961479.   .9470
3     $  .926128.   .919031.   .913887.   .910544.   .9088
3     $  .910544.   .913887.   .919031.   .926128.   .9353
3     $  .961479.   .978988.  1.000000.  1.000000.   .9792
3     $  .947895.   .936445.   .927381.   .920441.   .9154
3     $  .910544.   .910544.   .912151.   .915416.   .9204
3     $  .936445.   .947895.   .962050.   .979279.  1.0000
3     $  .979874.   .963217.   .949589.   .938604.   .9299
3     $  .918523.   .915416.   .913887.   .913887.   .9154
3     $  .923311.   .929934.   .938604.   .949589.   .9632
3     $ 1.000000.  1.000000.   .980804.   .965035.   .9522
3     $  .933883.   .927740.   .923311.   .920441.   .9190
3     $  .920441.   .923311.   .927740.   .933883.   .9419
3     $  .965035.   .980804.  1.000000.  1.000000.   .9821
3     $  .955913.   .946627.   .939377.   .933883.   .9299
3     $  .926128.   .926128.   .927381.   .929934.   .9338
3     $  .946627.   .955913.   .967595.   .982115.  1.0000
3C
3      DATA T3B4/ 1.000000.   .983885.   .971033.   .9608
```

```
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3C
3      DATA T4B1/ 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000,  .9934
3     $  .985730,  .983248,  .981366,  .979963,  .9789
3     $  .978011,  .978011,  .978325,  .978966,  .9799
3     $  .983248,  .985730,  .989010,  .993468, 1.0000
3     $  .989010,  .980813,  .974578,  .969791,  .9661
3     $  .961443,  .960186,  .959571,  .959571,  .9601
3     $  .963395,  .966132,  .969791,  .974578,  .9808
3     $ 1.000000, 1.000000,  .985730,  .974578,  .9658
3     $  .953800,  .949849,  .947022,  .945198,  .9443
3     $  .945198,  .947022,  .949849,  .953800,  .9590
3     $  .974578,  .985730, 1.000000, 1.000000,  .9832
3     $  .959050,  .950555,  .943941,  .938935,  .9353
3     $  .931871,  .931871,  .933013,  .935338,  .9389
3     $  .950555,  .959050,  .969791,  .983248, 1.0000
3C
3      DATA T4B2/ 1.000000,  .981366,  .966132,  .9538
3     $  .936206,  .930320,  .926076,  .923325,  .9219
3     $  .923325,  .926076,  .930320,  .936206,  .9439
3     $  .966132,  .981366, 1.000000, 1.000000,  .9799
3     $  .949849,  .938935,  .930320,  .923735,  .9189
3     $  .914358,  .914358,  .915880,  .918973,  .9237
3     $  .938935,  .949849,  .963395,  .979963, 1.0000
3     $  .978966,  .961443,  .947022,  .935338,  .9260
3     $  .913823,  .910473,  .908823,  .908823,  .9104
3     $  .918973,  .926076,  .935338,  .947022,  .9614
3     $ 1.000000, 1.000000,  .978325,  .960186,  .9451
3     $  .923325,  .915880,  .910473,  .906953,  .9052
3     $  .906953,  .910473,  .915880,  .923325,  .9330
3     $  .960186,  .978325, 1.000000, 1.000000,  .9780
3     $  .944304,  .931871,  .921973,  .914358,  .9088
3     $  .903440,  .903440,  .905217,  .908823,  .9143
3     $  .931871,  .944304,  .959571,  .978011, 1.0000
3C
3      DATA T4B3/ 1.000000,  .978011,  .959571,  .9443
3     $  .921973,  .914358,  .908823,  .905217,  .9034
3     $  .905217,  .908823,  .914358,  .921973,  .9318
3     $  .959571,  .978011, 1.000000, 1.000000,  .9783
3     $  .945198,  .933013,  .923325,  .915880,  .9104
3     $  .905217,  .905217,  .906953,  .910473,  .9158
3     $  .933013,  .945198,  .960186,  .978325, 1.0000
3     $  .978966,  .961443,  .947022,  .935338,  .9260
3     $  .913823,  .910473,  .908823,  .908823,  .9104
3     $  .918973,  .926076,  .935338,  .947022,  .9614
3     $ 1.000000, 1.000000,  .979963,  .963395,  .9498
3     $  .930320,  .923735,  .918973,  .915880,  .9143
3     $  .915880,  .918973,  .923735,  .930320,  .9389
3     $  .963395,  .979963, 1.000000, 1.000000,  .9813
3     $  .953800,  .943941,  .936206,  .930320,  .9260
3     $  .921973,  .921973,  .923325,  .926076,  .9303
3     $  .943941,  .953800,  .966132,  .981366, 1.0000
3C
3      DATA T4B4/ 1.000000,  .983248,  .969791,  .9590
3     $  .943941,  .938935,  .935338,  .933013,  .9318
3     $  .933013,  .935338,  .938935,  .943941,  .9505
3     $  .969791,  .983248, 1.000000, 1.000000,  .9857
3     $  .965855,  .959050,  .953800,  .949849,  .9470
```

```
3      DATA T5B1/ 1.000000,  1.000000,  1.000000,  1.0000
3     $ 1.000000,  1.000000,  1.000000,  1.000000,  1.0000
3     $ 1.000000,  1.000000,  1.000000,  1.000000,  1.0000
3     $ 1.000000,  1.000000,  1.000000,  1.000000,   .9934
3     $  .985745,   .983268,   .981390,   .979991,   .9789
3     $  .978044,   .978044,   .978357,   .978997,   .9799
3     $  .983268,   .985745,   .989020,   .993473,  1.0000
3     $  .989020,   .980834,   .974609,   .969831,   .9661
3     $  .961504,   .960252,   .959638,   .959638,   .9602
3     $  .963451,   .966181,   .969831,   .974609,   .9808
3     $ 1.000000,  1.000000,   .985745,   .974609,   .9659
3     $  .953873,   .949933,   .947114,   .945296,   .9444
3     $  .945296,   .947114,   .949933,   .953873,   .9591
3     $  .974609,   .985745,  1.000000,  1.000000,   .9832
3     $  .959110,   .950634,   .944036,   .939045,   .9354
3     $  .932004,   .932004,   .933142,   .935459,   .9390
3     $  .950634,   .959110,   .969831,   .983268,  1.0000
3C
3      DATA T5B2/ 1.000000,   .981390,   .966181,   .9538
3     $  .936322,   .930453,   .926223,   .923481,   .9221
3     $  .923481,   .926223,   .930453,   .936322,   .9440
3     $  .966181,   .981390,  1.000000,  1.000000,   .9799
3     $  .949933,   .939045,   .930453,   .923889,   .9191
3     $  .914543,   .914543,   .916060,   .919142,   .9238
3     $  .939045,   .949933,   .963451,   .979991,  1.0000
3     $  .978997,   .961504,   .947114,   .935459,   .9262
3     $  .914009,   .910671,   .909027,   .909027,   .9106
3     $  .919142,   .926223,   .935459,   .947114,   .9615
3     $ 1.000000,  1.000000,   .978357,   .960252,   .9452
3     $  .923481,   .916060,   .910671,   .907163,   .9054
3     $  .907163,   .910671,   .916060,   .923481,   .9331
3     $  .960252,   .978357,  1.000000,  1.000000,   .9780
3     $  .944405,   .932004,   .922134,   .914543,   .9090
3     $  .903664,   .903664,   .905435,   .909027,   .9145
3     $  .932004,   .944405,   .959638,   .978044,  1.0000
3C
3      DATA T5B3/ 1.000000,   .978044,   .959638,   .9444
3     $  .922134,   .914543,   .909027,   .905435,   .9036
3     $  .905435,   .909027,   .914543,   .922134,   .9320
3     $  .959638,   .978044,  1.000000,  1.000000,   .9783
3     $  .945296,   .933142,   .923481,   .916060,   .9106
3     $  .905435,   .905435,   .907163,   .910671,   .9160
3     $  .933142,   .945296,   .960252,   .978357,  1.0000
3     $  .978997,   .961504,   .947114,   .935459,   .9262
3     $  .914009,   .910671,   .909027,   .909027,   .9106
3     $  .919142,   .926223,   .935459,   .947114,   .9615
3     $ 1.000000,  1.000000,   .979991,   .963451,   .9499
3     $  .930453,   .923889,   .919142,   .916060,   .9145
3     $  .916060,   .919142,   .923889,   .930453,   .9390
3     $  .963451,   .979991,  1.000000,  1.000000,   .9813
3     $  .953873,   .944036,   .936322,   .930453,   .9262
3     $  .922134,   .922134,   .923481,   .926223,   .9304
3     $  .944036,   .953873,   .966181,   .981390,  1.0000
3C
3      DATA T5B4/ 1.000000,   .983268,   .969831,   .9591
3     $  .944036,   .939045,   .935459,   .933142,   .9320
3     $  .933142,   .935459,   .939045,   .944036,   .9506
3     $  .969831,   .983268,  1.000000,  1.000000,   .9857
3     $  .965901,   .959110,   .953873,   .949933,   .9471
3     $  .944405,   .944405,   .945296,   .947114,   .9499
3     $  .959110,   .965901,   .974609,   .985745,  1.0000
3     $  .989020,   .980834,   .974609,   .969831,   .9661
3     $  .961504,   .960252,   .959638,   .959638,   .9602
```

```
3      TRUE = DERVSL(6)
3C
3      RETURN
3      END
3      FUNCTION R(X,Y)
3C
3      REAL DERVSL(6), GRID(9), TAB1(9,9), TAB2(9,9),
3     $     TAB3(9,9), TAB4(9,9), TAB5(9,9)
3      DATA NGRID, NGRDD, GRID /9, 9, 0., 0.125, 0.250,
3     $0.375, 0.500, 0.625, 0.750, 0.875, 1.00/
3C
3C     APPROXIMATE SOLUTION OF NONLINEAR PROBLEM FOR NON
3C     COLLOCATION
3C
3      DATA TAB1/ 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $  .961931,  .955174,  .953091,  .955175,  .961931
3     $ 1.000000, 1.000000,  .961931,  .939237,  .927238
3     $  .927237,  .939237,  .961931, 1.000000, 1.000000
3     $  .927238,  .912095,  .907312,  .912095,  .927237
3     $ 1.000000, 1.000000,  .953091,  .923487,  .907313
3     $  .907312,  .923487,  .953091, 1.000000, 1.000000
3     $  .927238,  .912095,  .907312,  .912095,  .927238
3     $ 1.000000, 1.000000,  .961932,  .939238,  .927238
3     $  .927238,  .939237,  .961932, 1.000000, 1.000000
3     $  .961932,  .955175,  .953091,  .955175,  .961931
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3      DATA TAB2/ 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $  .959940,  .952637,  .950369,  .952638,  .959940
3     $ 1.000000, 1.000000,  .959939,  .935551,  .922658
3     $  .922669,  .935662,  .959941, 1.000000, 1.000000
3     $  .922668,  .906244,  .901028,  .906245,  .922668
3     $ 1.000000, 1.000000,  .950368,  .918580,  .901028
3     $  .901028,  .918580,  .950368, 1.000000, 1.000000
3     $  .922668,  .906245,  .901028,  .906245,  .922668
3     $ 1.000000, 1.000000,  .959940,  .935662,  .922669
3     $  .922668,  .935661,  .959940, 1.000000, 1.000000
3     $  .959940,  .952637,  .950369,  .952637,  .959940
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3      DATA TAB3/ 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $  .963816,  .957569,  .955656,  .957570,  .963816
3     $ 1.000000, 1.000000,  .963816,  .942608,  .931532
3     $  .931533,  .942609,  .963817, 1.000000, 1.000000
3     $  .931532,  .917577,  .913196,  .917578,  .931534
3     $ 1.000000, 1.000000,  .955656,  .928093,  .913196
3     $  .913197,  .928094,  .955657, 1.000000, 1.000000
3     $  .931532,  .917577,  .913196,  .917578,  .931534
3     $ 1.000000, 1.000000,  .963815,  .942607,  .931533
3     $  .931533,  .942609,  .963817, 1.000000, 1.000000
3     $  .963816,  .957569,  .955656,  .957569,  .963817
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3      DATA TAB4/ 1.000000, 1.000000, 1.000000, 1.000000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000
3     $  .962161,  .955471,  .953410,  .955471,  .962161
3     $ 1.000000, 1.000000,  .962161,  .939658,  .927783
```

```
3     $  .962216,  .955545,  .953491,  .955546,  .962217,
3     $ 1.000000, 1.000000,  .962217,  .939764,  .927924,
3     $  .927925,  .939764,  .962217, 1.000000, 1.000000,
3     $  .927925,  .912992,  .908283,  .912992,  .927924,
3     $ 1.000000, 1.000000,  .953491,  .924229,  .908283,
3     $  .908282,  .924228,  .953490, 1.000000, 1.000000,
3     $  .927925,  .912993,  .908283,  .912991,  .927923,
3     $ 1.000000, 1.000000,  .962217,  .939765,  .927925,
3     $  .927923,  .939763,  .962216, 1.000000, 1.000000,
3     $  .962217,  .955545,  .953490,  .955544,  .962216,
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.000000,
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3C     INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TAB&D,GRID,GRID,NGRID,NGRID,NGRDD,
3C
3      G = DERUSL(6)
3C
3      R = -1.425*G**(&C-1)*EXP(&A*&B*(1.-G)/(1.+&B*(1.-G
3C
3      RETURN
3      END
```

```
'EOR
************
* MACRO  49 *
************
*          2000001002000
1          TWO DIMENSIONS
1          UXX$ + UYY$ + W(X,Y)U$ = F(X,Y)
2          DIRICHLET
2          X=0. , U=1.
2          X=1. , U=1.
2          Y=0. , U=1.
2          Y=1. , U=1.
3      FUNCTION TRUE(X,Y)
3C
3C     ***************************************************
3C     *
3C     *                 MACRO 49 PARAMETERS
3C     *
3C     ***************************************************
3C     *
3C     *               A  I   B   I   C  I  D
3C     *              ----I------I-----I-----
3C     *               1  I  .50  I   2  I  1
3C     *                  I       I      I
3C     *               1  I  .50  I  25  I  2
3C     *                  I       I      I
3C     *               2  I  .04  I   2  I  3
3C     *                  I       I      I
3C     *               2  I  .50  I   2  I  4
3C     *
3C     ***************************************************
3C
3      REAL DERUSL(6), GRID(20), TABLE(20,20),
3     $     T1B1(100), T1B2(100), T1B3(100), T1B4(100),
3     $     T2B1(100), T2B2(100), T2B3(100), T2B4(100),
3     $     T3B1(100), T3B2(100), T3B3(100), T3B4(100),
3     $     T4B1(100), T4B2(100), T4B3(100), T4B4(100)
3      EQUIVALENCE (TABLE(1, 1) , T&DB1(1)),
3     $            (TABLE(1, 6) , T&DB2(1)),
```

```
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0048
3     $ 1.010595, 1.012478, 1.013915, 1.014992, 1.0157
3     $ 1.016500, 1.016500, 1.016257, 1.015761, 1.0149
3     $ 1.012478, 1.010595, 1.008128, 1.004808, 1.0000
3     $ 1.008128, 1.014249, 1.018947, 1.022581, 1.0253
3     $ 1.028985, 1.029957, 1.030433, 1.030433, 1.0299
3     $ 1.027479, 1.025376, 1.022581, 1.018947, 1.0142
3     $ 1.000000, 1.000000, 1.010595, 1.018947, 1.0255
3     $ 1.034718, 1.037754, 1.039935, 1.041346, 1.0420
3     $ 1.041346, 1.039935, 1.037754, 1.034718, 1.0307
3     $ 1.018947, 1.010595, 1.000000, 1.000000, 1.0124
3     $ 1.030703, 1.037169, 1.042231, 1.046080, 1.0488
3     $ 1.051540, 1.051540, 1.050655, 1.048856, 1.0460
3     $ 1.037169, 1.030703, 1.022581, 1.012478, 1.0000
3C
3      DATA T1B2/ 1.000000, 1.013915, 1.025376, 1.0347
3     $ 1.048157, 1.052685, 1.055961, 1.058090, 1.0591
3     $ 1.058090, 1.055961, 1.052685, 1.048157, 1.0422
3     $ 1.025376, 1.013915, 1.000000, 1.000000, 1.0149
3     $ 1.037754, 1.046080, 1.052685, 1.057754, 1.0614
3     $ 1.065006, 1.065006, 1.063826, 1.061432, 1.0577
3     $ 1.046080, 1.037754, 1.027479, 1.014992, 1.0000
3     $ 1.015761, 1.028985, 1.039935, 1.048856, 1.0559
3     $ 1.065410, 1.068004, 1.069283, 1.069283, 1.0680
3     $ 1.061432, 1.055961, 1.048856, 1.039935, 1.0289
3     $ 1.000000, 1.000000, 1.016257, 1.029957, 1.0413
3     $ 1.058090, 1.063826, 1.068004, 1.070731, 1.0720
3     $ 1.070731, 1.068004, 1.063826, 1.058090, 1.0506
3     $ 1.029957, 1.016257, 1.000000, 1.000000, 1.0165
3     $ 1.042038, 1.051540, 1.059138, 1.065006, 1.0692
3     $ 1.073455, 1.073455, 1.072076, 1.069283, 1.0650
3     $ 1.051540, 1.042038, 1.030433, 1.016500, 1.0000
3C
3      DATA T1B3/ 1.000000, 1.016500, 1.030433, 1.0420
3     $ 1.059138, 1.065006, 1.069283, 1.072076, 1.0734
3     $ 1.072076, 1.069283, 1.065006, 1.059138, 1.0515
3     $ 1.030433, 1.016500, 1.000000, 1.000000, 1.0162
3     $ 1.041346, 1.050655, 1.058090, 1.063826, 1.0680
3     $ 1.072076, 1.072076, 1.070731, 1.068004, 1.0638
3     $ 1.050655, 1.041346, 1.029957, 1.016257, 1.0000
3     $ 1.015761, 1.028985, 1.039935, 1.048856, 1.0559
3     $ 1.065410, 1.068004, 1.069283, 1.069283, 1.0680
3     $ 1.061432, 1.055961, 1.048856, 1.039935, 1.0289
3     $ 1.000000, 1.000000, 1.014992, 1.027479, 1.0377
3     $ 1.052685, 1.057754, 1.061432, 1.063826, 1.0650
3     $ 1.063826, 1.061432, 1.057754, 1.052685, 1.0460
3     $ 1.027479, 1.014992, 1.000000, 1.000000, 1.0139
3     $ 1.034718, 1.042231, 1.048157, 1.052685, 1.0559
3     $ 1.059138, 1.059138, 1.058090, 1.055961, 1.0526
3     $ 1.042231, 1.034718, 1.025376, 1.013915, 1.0000
3C
3      DATA T1B4/ 1.000000, 1.012478, 1.022581, 1.0307
3     $ 1.042231, 1.046080, 1.048856, 1.050655, 1.0515
3     $ 1.050655, 1.048856, 1.046080, 1.042231, 1.0371
3     $ 1.022581, 1.012478, 1.000000, 1.000000, 1.0105
3     $ 1.025531, 1.030703, 1.034718, 1.037754, 1.0399
3     $ 1.042038, 1.042038, 1.041346, 1.039935, 1.0377
3     $ 1.030703, 1.025531, 1.018947, 1.010595, 1.0000
3     $ 1.008128, 1.014249, 1.018947, 1.022581, 1.0253
3     $ 1.028985, 1.029957, 1.030433, 1.030433, 1.0299
3     $ 1.027479, 1.025376, 1.022581, 1.018947, 1.0142
3     $ 1.000000, 1.000000, 1.004808, 1.008128, 1.0105
```

```
3     $ 1.348374,  1.373187,  1.323240,  1.198997,  1.00000
3     $ 1.323240,  1.476115,  1.496001,  1.491537,  1.49055
3     $ 1.490426,  1.490415,  1.490420,  1.490420,  1.4904
3     $ 1.490452,  1.490550,  1.491537,  1.496001,  1.4761
3     $ 1.000000,  1.000000,  1.373187,  1.496001,  1.5001
3     $ 1.500079,  1.500076,  1.500076,  1.500075,  1.5000
3     $ 1.500075,  1.500076,  1.500076,  1.500079,  1.5000
3     $ 1.496001,  1.373187,  1.000000,  1.000000,  1.3483
3     $ 1.500072,  1.499981,  1.499991,  1.499991,  1.4999
3     $ 1.499991,  1.499991,  1.499991,  1.499991,  1.4999
3     $ 1.499981,  1.500072,  1.491537,  1.348374,  1.0000
3C
3      DATA T2B2/ 1.000000,  1.340131,  1.490550,  1.5000
3     $ 1.500001,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500001,  1.4999
3     $ 1.490550,  1.340131,  1.000000,  1.000000,  1.3390
3     $ 1.500076,  1.499991,  1.500000,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.499991,  1.500076,  1.490452,  1.339086,  1.0000
3     $ 1.338927,  1.490426,  1.500076,  1.499991,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.499991,  1.500076,  1.4904
3     $ 1.000000,  1.000000,  1.338874,  1.490415,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.4999
3     $ 1.490415,  1.338874,  1.000000,  1.000000,  1.3388
3     $ 1.500075,  1.499991,  1.500000,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.499991,  1.500075,  1.490420,  1.338890,  1.0000
3C
3      DATA T2B3/ 1.000000,  1.338890,  1.490420,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.4999
3     $ 1.490420,  1.338890,  1.000000,  1.000000,  1.3388
3     $ 1.500075,  1.499991,  1.500000,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.499991,  1.500075,  1.490415,  1.338874,  1.0000
3     $ 1.338927,  1.490426,  1.500076,  1.499991,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.499991,  1.500076,  1.4904
3     $ 1.000000,  1.000000,  1.339086,  1.490452,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.4999
3     $ 1.490452,  1.339086,  1.000000,  1.000000,  1.3401
3     $ 1.500079,  1.499991,  1.500001,  1.500000,  1.5000
3     $ 1.500000,  1.500000,  1.500000,  1.500000,  1.5000
3     $ 1.499991,  1.500079,  1.490550,  1.340131,  1.0000
3C
3      DATA T2B4/ 1.000000,  1.348374,  1.491537,  1.5000
3     $ 1.499991,  1.499991,  1.499991,  1.499991,  1.4999
3     $ 1.499991,  1.499991,  1.499991,  1.499991,  1.4999
3     $ 1.491537,  1.348374,  1.000000,  1.000000,  1.3731
3     $ 1.500139,  1.500072,  1.500079,  1.500076,  1.5000
3     $ 1.500075,  1.500075,  1.500075,  1.500076,  1.5000
3     $ 1.500072,  1.500139,  1.496001,  1.373187,  1.0000
3     $ 1.323240,  1.476115,  1.496001,  1.491537,  1.4905
3     $ 1.490426,  1.490415,  1.490420,  1.490420,  1.4904
3     $ 1.490452,  1.490550,  1.491537,  1.496001,  1.4761
3     $ 1.000000,  1.000000,  1.198997,  1.323240,  1.3731
3     $ 1.340131,  1.339086,  1.338927,  1.338874,  1.3388
3     $ 1.338874,  1.338927,  1.339086,  1.340131,  1.3483
3     $ 1.323240,  1.198997,  1.000000,  1.000000,  1.0000
3     $ 1.000000,  1.000000,  1.000000,  1.000000,  1.0000
```

```
3     $ 1.000000, 1.000000, 1.000761, 1.001346, 1.0017
3     $ 1.002412, 1.002610, 1.002751, 1.002841, 1.0028
3     $ 1.002841, 1.002751, 1.002610, 1.002412, 1.0021
3     $ 1.001346, 1.000761, 1.000000, 1.000000, 1.0008
3     $ 1.002146, 1.002580, 1.002914, 1.003165, 1.0033
3     $ 1.003515, 1.003515, 1.003458, 1.003343, 1.0031
3     $ 1.002580, 1.002146, 1.001592, 1.000888, 1.0000
3C
3      DATA T3B2/ 1.000000, 1.000984, 1.001777, 1.0024
3     $ 1.003304, 1.003598, 1.003809, 1.003945, 1.0040
3     $ 1.003945, 1.003809, 1.003598, 1.003304, 1.0029
3     $ 1.001777, 1.000984, 1.000000, 1.000000, 1.0010
3     $ 1.002610, 1.003165, 1.003598, 1.003927, 1.0041
3     $ 1.004391, 1.004391, 1.004316, 1.004164, 1.0039
3     $ 1.003164, 1.002610, 1.001915, 1.001054, 1.0000
3     $ 1.001104, 1.002012, 1.002751, 1.003343, 1.0038
3     $ 1.004419, 1.004585, 1.004666, 1.004666, 1.0045
3     $ 1.004164, 1.003809, 1.003343, 1.002751, 1.0020
3     $ 1.000000, 1.000000, 1.001136, 1.002074, 1.0028
3     $ 1.003945, 1.004316, 1.004585, 1.004758, 1.0048
3     $ 1.004758, 1.004584, 1.004316, 1.003945, 1.0034
3     $ 1.002074, 1.001135, 1.000000, 1.000000, 1.0011
3     $ 1.002885, 1.003515, 1.004012, 1.004391, 1.0046
3     $ 1.004931, 1.004931, 1.004844, 1.004666, 1.0043
3     $ 1.003515, 1.002885, 1.002104, 1.001151, 1.0000
3C
3      DATA T3B3/ 1.000000, 1.001151, 1.002105, 1.0028
3     $ 1.004012, 1.004391, 1.004666, 1.004844, 1.0049
3     $ 1.004844, 1.004666, 1.004391, 1.004012, 1.0035
3     $ 1.002104, 1.001151, 1.000000, 1.000000, 1.0011
3     $ 1.002841, 1.003458, 1.003945, 1.004316, 1.0045
3     $ 1.004844, 1.004844, 1.004758, 1.004584, 1.0043
3     $ 1.003458, 1.002841, 1.002074, 1.001135, 1.0000
3     $ 1.001104, 1.002012, 1.002751, 1.003343, 1.0038
3     $ 1.004419, 1.004585, 1.004666, 1.004666, 1.0045
3     $ 1.004164, 1.003809, 1.003343, 1.002751, 1.0020
3     $ 1.000000, 1.000000, 1.001054, 1.001915, 1.0026
3     $ 1.003598, 1.003927, 1.004164, 1.004316, 1.0043
3     $ 1.004316, 1.004164, 1.003927, 1.003598, 1.0031
3     $ 1.001915, 1.001054, 1.000000, 1.000000, 1.0009
3     $ 1.002412, 1.002914, 1.003304, 1.003598, 1.0038
3     $ 1.004012, 1.004012, 1.003945, 1.003809, 1.0035
3     $ 1.002914, 1.002412, 1.001777, 1.000984, 1.0000
3C
3      DATA T3B4/ 1.000000, 1.000888, 1.001592, 1.0021
3     $ 1.002914, 1.003165, 1.003343, 1.003458, 1.0035
3     $ 1.003458, 1.003343, 1.003165, 1.002914, 1.0025
3     $ 1.001592, 1.000888, 1.000000, 1.000000, 1.0007
3     $ 1.001798, 1.002146, 1.002412, 1.002610, 1.0027
3     $ 1.002885, 1.002885, 1.002841, 1.002751, 1.0026
3     $ 1.002146, 1.001798, 1.001546, 1.000761, 1.0000
3     $ 1.000590, 1.001022, 1.001346, 1.001592, 1.0017
3     $ 1.002012, 1.002074, 1.002105, 1.002105, 1.0020
3     $ 1.001915, 1.001777, 1.001592, 1.001346, 1.0010
3     $ 1.000000, 1.000000, 1.000353, 1.000590, 1.0007
3     $ 1.000984, 1.001054, 1.001104, 1.001136, 1.0011
3     $ 1.001136, 1.001104, 1.001054, 1.000984, 1.0008
3     $ 1.000590, 1.000353, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.0000
3C
3      DATA T4B1/ 1.000000, 1.000000, 1.000000, 1.0000
```

```
3     $ 1.028385, 1.034216, 1.038738, 1.042149, 1.04459
3     $ 1.046942, 1.046942, 1.046169, 1.044592, 1.04214
3     $ 1.034216, 1.028385, 1.020979, 1.011658, 1.00000
3C
3      DATA T4B2/ 1.000000, 1.012948, 1.023485, 1.03197
3     $ 1.044022, 1.048029, 1.050911, 1.052774, 1.05368
3     $ 1.052774, 1.050911, 1.048029, 1.044022, 1.03873
3     $ 1.023485, 1.012948, 1.000000, 1.000000, 1.01390
3     $ 1.034671, 1.042149, 1.048029, 1.052509, 1.05574
3     $ 1.058864, 1.058864, 1.057835, 1.055740, 1.05250
3     $ 1.042149, 1.034671, 1.025353, 1.013905, 1.00000
3     $ 1.014584, 1.026681, 1.036594, 1.044592, 1.05091
3     $ 1.059233, 1.061500, 1.062616, 1.062616, 1.06150
3     $ 1.055740, 1.050911, 1.044592, 1.036594, 1.02668
3     $ 1.000000, 1.000000, 1.015020, 1.027534, 1.03783
3     $ 1.052774, 1.057835, 1.061500, 1.063882, 1.06505
3     $ 1.063882, 1.061500, 1.057835, 1.052774, 1.04618
3     $ 1.027534, 1.015020, 1.000000, 1.000000, 1.01523
3     $ 1.038437, 1.046942, 1.053689, 1.058864, 1.06261
3     $ 1.066256, 1.066256, 1.065055, 1.062616, 1.05888
3     $ 1.046942, 1.038437, 1.027951, 1.015233, 1.00000
3C
3      DATA T4B3/ 1.000000, 1.015233, 1.027951, 1.03843
3     $ 1.053689, 1.058864, 1.062616, 1.065055, 1.06625
3     $ 1.065055, 1.062616, 1.058864, 1.053689, 1.04694
3     $ 1.027951, 1.015233, 1.000000, 1.000000, 1.01502
3     $ 1.037831, 1.046169, 1.052774, 1.057835, 1.06150
3     $ 1.065055, 1.065055, 1.063882, 1.061500, 1.05783
3     $ 1.046169, 1.037831, 1.027534, 1.015020, 1.00000
3     $ 1.014584, 1.026681, 1.036594, 1.044592, 1.05091
3     $ 1.059233, 1.061500, 1.062616, 1.062616, 1.06150
3     $ 1.055740, 1.050911, 1.044592, 1.036594, 1.02668
3     $ 1.000000, 1.000000, 1.013905, 1.025353, 1.03467
3     $ 1.048029, 1.052509, 1.055740, 1.057835, 1.05886
3     $ 1.057835, 1.055740, 1.052509, 1.048029, 1.04214
3     $ 1.025353, 1.013905, 1.000000, 1.000000, 1.01294
3     $ 1.031978, 1.038738, 1.044022, 1.048029, 1.05091
3     $ 1.053689, 1.053689, 1.052774, 1.050911, 1.04802
3     $ 1.038738, 1.031978, 1.023485, 1.012948, 1.00000
3C
3      DATA T4B4/ 1.000000, 1.011658, 1.020979, 1.02838
3     $ 1.038738, 1.042149, 1.044592, 1.046169, 1.04694
3     $ 1.046169, 1.044592, 1.042149, 1.038738, 1.03421
3     $ 1.020979, 1.011658, 1.000000, 1.000000, 1.00999
3     $ 1.023708, 1.028385, 1.031978, 1.034671, 1.03659
3     $ 1.038437, 1.038437, 1.037831, 1.036594, 1.03467
3     $ 1.028385, 1.023708, 1.017687, 1.009950, 1.00000
3     $ 1.007680, 1.013377, 1.017687, 1.020979, 1.02348
3     $ 1.026681, 1.027534, 1.027951, 1.027951, 1.02753
3     $ 1.025353, 1.023485, 1.020979, 1.017687, 1.01337
3     $ 1.000000, 1.000000, 1.004577, 1.007680, 1.00999
3     $ 1.012948, 1.013905, 1.014584, 1.015020, 1.01523
3     $ 1.015020, 1.014584, 1.013905, 1.012948, 1.01169
3     $ 1.007680, 1.004577, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3C
3C     INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRI
3C
3      TRUE = DERVSL(6)
```

```
3      DATA TAB1/ 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.028380, 1.033520, 1.035120, 1.033520, 1.02838
3     $ 1.000000, 1.000000, 1.028380, 1.045520, 1.05467
3     $ 1.054670, 1.045520, 1.028380, 1.000000, 1.00000
3     $ 1.054670, 1.066220, 1.069880, 1.066220, 1.05467
3     $ 1.000000, 1.000000, 1.035120, 1.057540, 1.06988
3     $ 1.069880, 1.057540, 1.035120, 1.000000, 1.00000
3     $ 1.054670, 1.066220, 1.069880, 1.066220, 1.05467
3     $ 1.000000, 1.000000, 1.028380, 1.045520, 1.05467
3     $ 1.054670, 1.045520, 1.028380, 1.000000, 1.00000
3     $ 1.028380, 1.033520, 1.035120, 1.033520, 1.02838
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3      DATA TAB2/ 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.511040, 1.510490, 1.510470, 1.510490, 1.51104
3     $ 1.000000, 1.000000, 1.511040, 1.501050, 1.50055
3     $ 1.500550, 1.501050, 1.511040, 1.000000, 1.00000
3     $ 1.500550, 1.500060, 1.500030, 1.500050, 1.50055
3     $ 1.000000, 1.000000, 1.510470, 1.500530, 1.50003
3     $ 1.500030, 1.500530, 1.510470, 1.000000, 1.00000
3     $ 1.500550, 1.500060, 1.500030, 1.500050, 1.50055
3     $ 1.000000, 1.000000, 1.511040, 1.501050, 1.50055
3     $ 1.500550, 1.501050, 1.511040, 1.000000, 1.00000
3     $ 1.511040, 1.510490, 1.510470, 1.510490, 1.51104
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3      DATA TAB3/ 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.001990, 1.002320, 1.002430, 1.002320, 1.00199
3     $ 1.000000, 1.000000, 1.001990, 1.003140, 1.00373
3     $ 1.003730, 1.003140, 1.001990, 1.000000, 1.00000
3     $ 1.003730, 1.004480, 1.004720, 1.004480, 1.00373
3     $ 1.000000, 1.000000, 1.002430, 1.003920, 1.00472
3     $ 1.004720, 1.003920, 1.002430, 1.000000, 1.00000
3     $ 1.003730, 1.004480, 1.004720, 1.004480, 1.00373
3     $ 1.000000, 1.000000, 1.001990, 1.003140, 1.00373
3     $ 1.003730, 1.003140, 1.001990, 1.000000, 1.00000
3     $ 1.001990, 1.002320, 1.002430, 1.002320, 1.00199
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3      DATA TAB4/ 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.026240, 1.030800, 1.032200, 1.030800, 1.02624
3     $ 1.000000, 1.000000, 1.026240, 1.041680, 1.04976
3     $ 1.049760, 1.041680, 1.026240, 1.000000, 1.00000
3     $ 1.049760, 1.059940, 1.063140, 1.059940, 1.04976
3     $ 1.000000, 1.000000, 1.032200, 1.052280, 1.06314
3     $ 1.063140, 1.052270, 1.032200, 1.000000, 1.00000
3     $ 1.049760, 1.059940, 1.063140, 1.059940, 1.04976
3     $ 1.000000, 1.000000, 1.026240, 1.041680, 1.04976
3     $ 1.049760, 1.041680, 1.026240, 1.000000, 1.00000
3     $ 1.026240, 1.030800, 1.032200, 1.030800, 1.02624
3     $ 1.000000, 1.000000, 1.000000, 1.000000, 1.00000
3     $ 1.000000, 1.000000, 1.000000, 1.000000/
3C
3C     INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TAB&D,GRID,GRID,NGRID,NGRID,NGRD
```

```
3C
3       T = R(X,Y)
3       F = -(1.425**2)*((1.+&B-T)/&B)**(&A-1)*EXP(&C*(T-1
3C
3       RETURN
3       END
```

```
'EOR
*************
* MACRO  50 *
*************
```

```
'EOR
*************
* MACRO  51 *
*************
*           2000200000222
1           TWO DIMENSIONS
1           UXX$ + C(X)UYY$ + D(X)UX$ = -10.
2           MIXED $ HOMOGENEOUS
2           X=0. , UX = 0.
2           X=1. , U  = 0.
2           Y=0. , UY = 0.
2           Y=1. , MIXED = (A(X))UY + (B(X))U = 0.
3       FUNCTION TRUE(X,Y)
3C
3C      ************************************************
3C      *
3C      *                  MACRO 51 PARAMETERS
3C      *
3C      ************************************************
3C      *
3C      *                        A  I  B
3C      *                      -----I-----
3C      *                       .3  I  1
3C      *                           I
3C      *                       .5  I  2
3C      *                           I
3C      *                       .6  I  3
3C      *                           I
3C      *                       .7  I  4
3C      *
3C      ************************************************
3C
3       REAL DERUSL(6), GRID(20), TABLE(20,20),
3      $     T1B1(100), T1B2(100), T1B3(100), T1B4(100),
3      $     T2B1(100), T2B2(100), T2B3(100), T2B4(100),
3      $     T3B1(100), T3B2(100), T3B3(100), T3B4(100),
3      $     T4B1(100), T4B2(100), T4B3(100), T4B4(100)
3       EQUIVALENCE (TABLE(1, 1) , T&BB1(1)),
3      $            (TABLE(1, 6) , T&BB2(1)),
3      $            (TABLE(1,11) , T&BB3(1)),
3      $            (TABLE(1,16) , T&BB4(1))
3       DATA NGRID, NGRDD/20, 20/
3       DATA  GRID/0.0000000, 0.0526316, 0.1052632, 0.1578
3      $0.2105263, 0.2631579, 0.3157895, 0.3684211, 0.4210
3      $0.4736842, 0.5263158, 0.5789474, 0.6315789, 0.6842
3      $0.7368421, 0.7894737, 0.8421053, 0.8947368, 0.9473
3      $1.0000000/
3C
3C      APPROXIMATE SOLUTION OF PROBLEM USING
3C      P3-C1 COLLOCATION (8 X 8 GRID)
3C
```

```
3     $  .301130,  .159979,  .000000,  .669625,  .6736
3     $  .686760,  .695247,  .708437,  .727384,  .7467
3     $  .770140,  .766748,  .750120,  .718431,  .6701
3     $  .520992,  .418912,  .298119,  .158500,  .0000
3C
3      DATA T1B2/  .669522,  .673021,  .679219,  .6825
3     $  .697221,  .713056,  .730797,  .745681,  .7534
3     $  .735181,  .704770,  .658226,  .594527,  .5129
3     $  .294190,  .156573,  .000000,  .669393,  .6728
3     $  .678140,  .678781,  .683660,  .696481,  .7119
3     $  .733258,  .731090,  .716684,  .687977,  .6434
3     $  .502760,  .405399,  .289032,  .154016,  .0000
3     $  .672271,  .675508,  .672563,  .667954,  .6674
3     $  .688635,  .700935,  .708671,  .707293,  .6943
3     $  .625505,  .566757,  .490542,  .399248,  .2830
3     $  .000000,  .669049,  .671531,  .673227,  .6661
3     $  .648698,  .651339,  .661057,  .672266,  .6795
3     $  .667947,  .643446,  .604164,  .548688,  .4759
3     $  .275953,  .147607,  .000000,  .668830,  .6712
3     $  .660066,  .642678,  .627636,  .624896,  .6305
3     $  .646584,  .646836,  .637210,  .615202,  .5791
3     $  .458431,  .372291,  .266865,  .143078,  .0000
3C
3      DATA T1B3/  .668578,  .670459,  .668393,  .6526
3     $  .603843,  .592989,  .594296,  .601358,  .6082
3     $  .601847,  .582638,  .550089,  .502356,  .4383
3     $  .257017,  .138252,  .000000,  .668293,  .6697
3     $  .645243,  .611389,  .577647,  .557703,  .5537
3     $  .564951,  .567070,  .561437,  .545178,  .5165
3     $  .414707,  .339172,  .245079,  .132349,  .0000
3     $  .669288,  .663268,  .638163,  .595269,  .5491
3     $  .509019,  .510576,  .516588,  .519366,  .5155
3     $  .477936,  .439711,  .386938,  .318252,  .2305
3     $  .000000,  .667620,  .668393,  .660617,  .6301
3     $  .518209,  .473631,  .456913,  .456995,  .4621
3     $  .463890,  .453883,  .433748,  .401386,  .3552
3     $  .214696,  .117287,  .000000,  .667232,  .6678
3     $  .623231,  .560201,  .485416,  .425806,  .4010
3     $  .402089,  .405923,  .405732,  .398593,  .3831
3     $  .317470,  .264768,  .194293,  .107025,  .0000
3C
3      DATA T1B4/  .666804,  .667188,  .655797,  .6162
3     $  .450884,  .370110,  .337157,  .332298,  .3357
3     $  .340697,  .336459,  .325303,  .304603,  .2736
3     $  .170795,  .095230,  .000000,  .666320,  .6663
3     $  .609210,  .526637,  .415349,  .304361,  .2637
3     $  .262713,  .266164,  .268252,  .256454,  .2592
3     $  .222396,  .189195,  .143083,  .081192,  .0000
3     $  .665997,  .651691,  .603636,  .514208,  .3800
3     $  .188040,  .184084,  .183723,  .185676,  .1876
3     $  .183811,  .174727,  .160351,  .139054,  .1056
3     $  .000000,  .665059,  .665455,  .650086,  .5972
3     $  .346587,  .148142,  .095668,  .102680,  .0966
3     $  .098742,  .099018,  .097839,  .093930,  .0873
3     $  .060674,  .037806,  .000000,  .664065,  .6651
3     $  .589969,  .503893,  .318433,  .037348, -.0084
3     $  .000332,  .000176,  .001137, -.000092,  .0001
3     $ -.000009,  .000015,  .000001,  .000000,  .0000
3C
3      DATA T2B1/ 1.812462, 1.803797, 1.779917, 1.7439
3     $ 1.642354, 1.580187, 1.512403, 1.439459, 1.3614
3     $ 1.185056, 1.083620,  .971225,  .846356,  .7077
3     $  .385833,  .201110,  .000000, 1.812464, 1.8045
```

```
3C
3      DATA T2B2/ 1.812512, 1.803998, 1.779871, 1.74308
3     $ 1.635131, 1.566748, 1.491150, 1.410235, 1.32577
3     $ 1.143289, 1.042938,  .933683,  .813622,  .6809
3     $  .372335,  .194452,  .000000, 1.812535, 1.8046
3     $ 1.743105, 1.693890, 1.632332, 1.561908, 1.4825
3     $ 1.310432, 1.218922, 1.123876, 1.024166,  .9160
3     $  .668203,  .524629,  .365703,  .191152,  .0000
3     $ 1.804525, 1.779978, 1.742651, 1.692450, 1.6291
3     $ 1.471824, 1.381976, 1.290590, 1.196132, 1.0997
3     $  .894125,  .778990,  .652497,  .512755,  .3578
3     $  .000000, 1.812594, 1.804034, 1.779775, 1.7419
3     $ 1.625704, 1.547840, 1.459412, 1.364298, 1.2666
3     $ 1.070434,  .971163,  .867321,  .755669,  .6333
3     $  .348474,  .182683,  .000000, 1.812631, 1.8048
3     $ 1.742102, 1.690069, 1.622446, 1.541769, 1.4473
3     $ 1.241302, 1.136928, 1.035634,  .937070,  .8353
3     $  .610123,  .480906,  .336277,  .176597,  .0000
3C
3      DATA T2B3/ 1.812674, 1.804514, 1.779908, 1.7415
3     $ 1.619129, 1.533957, 1.433217, 1.322339, 1.2094
3     $  .993703,  .895172,  .796782,  .694005,  .5828
3     $  .322895,  .170037,  .000000, 1.812722, 1.8044
3     $ 1.741236, 1.687334, 1.616149, 1.526916, 1.4192
3     $ 1.174161, 1.053505,  .943827,  .845505,  .7509
3     $  .550195,  .435645,  .306527,  .161958,  .0000
3     $ 1.805058, 1.780128, 1.741380, 1.686997, 1.6137
3     $ 1.406313, 1.273462, 1.135947, 1.001613,  .8846
3     $  .696794,  .606762,  .511439,  .406689,  .2865
3     $  .000000, 1.812839, 1.804563, 1.780002, 1.7409
3     $ 1.611738, 1.515692, 1.393605, 1.248107, 1.0895
3     $  .813031,  .715716,  .632441,  .551984,  .4668
3     $  .264830,  .141390,  .000000, 1.812907, 1.8050
3     $ 1.741255, 1.686051, 1.610650, 1.512808, 1.3843
3     $ 1.042833,  .865472,  .727029,  .631918,  .5569
3     $  .413473,  .332842,  .237065,  .127525,  .0000
3C
3      DATA T2B4/ 1.812984, 1.805192, 1.780383, 1.7414
3     $ 1.610360, 1.511702, 1.378934, 1.202751,  .9893
3     $  .621900,  .531492,  .468656,  .411699,  .3523
3     $  .205740,  .111963,  .000000, 1.813069, 1.8046
3     $ 1.741397, 1.686091, 1.610869, 1.512334, 1.3784
3     $  .930359,  .661607,  .490722,  .412699,  .3667
3     $  .282469,  .231625,  .170024,  .094141,  .0000
3     $ 1.805519, 1.780751, 1.742164, 1.687505, 1.6124
3     $ 1.386757, 1.188946,  .877400,  .520356,  .3243
3     $  .251313,  .226507,  .198371,  .167254,  .1228
3     $  .000000, 1.813269, 1.805296, 1.780826, 1.7425
3     $ 1.614840, 1.522489, 1.398628, 1.213697,  .8307
3     $  .113950,  .124332,  .125649,  .116255,  .1048
3     $  .068948,  .041773,  .000000, 1.813440, 1.8048
3     $ 1.743384, 1.687783, 1.618464, 1.530878, 1.4001
3     $  .821258, -.032682, -.151509,  .012283, -.0146
3     $  .001195, -.002022, -.000161,  .000030,  .0000
3C
3      DATA T3B1/ 1.773236, 1.766725, 1.748071, 1.7186
3     $ 1.633266, 1.579784, 1.519841, 1.453204, 1.3794
3     $ 1.205985, 1.103758,  .989539,  .862149,  .7206
3     $  .392213,  .204250,  .000000, 1.773236, 1.7675
3     $ 1.718952, 1.680384, 1.633000, 1.579827, 1.5195
3     $ 1.378866, 1.296418, 1.204793, 1.102946,  .9884
3     $  .720050,  .563550,  .391987,  .204161,  .0000
3     $ 1.766983, 1.748062, 1.718408, 1.679334, 1.6318
```

```
3     $  .380355,  .198387,  .000000, 1.773224, 1.7673
3     $ 1.716115, 1.673707, 1.620573, 1.560086, 1.4912
3     $ 1.335315, 1.247932, 1.154145, 1.053206,  .9421
3     $  .686148,  .537935,  .374496,  .195468,  .0000
3     $ 1.767121, 1.747116, 1.715063, 1.671286, 1.6162
3     $ 1.480704, 1.401588, 1.318295, 1.228761, 1.1338
3     $  .923331,  .803746,  .672297,  .527379,  .3674
3     $  .000000, 1.773214, 1.766549, 1.746629, 1.7136
3     $ 1.611431, 1.544289, 1.468434, 1.385678, 1.2978
3     $ 1.109405, 1.008261,  .900242,  .783280,  .6552
3     $  .358974,  .187805,  .000000, 1.773207, 1.7672
3     $ 1.713108, 1.666456, 1.606479, 1.536763, 1.4562
3     $ 1.276349, 1.179610, 1.080647,  .979657,  .8726
3     $  .634579,  .498666,  .347884,  .182253,  .0000
3C
3      DATA T3B3/ 1.773200, 1.766814, 1.746110, 1.7117
3     $ 1.601104, 1.527036, 1.441591, 1.347707, 1.2496
3     $ 1.046286,  .944397,  .839293,  .728751,  .6097
3     $  .335456,  .176127,  .000000, 1.773191, 1.7666
3     $ 1.710522, 1.650490, 1.595683, 1.517432, 1.4264
3     $ 1.220417, 1.112360, 1.006002,  .902860,  .7994
3     $  .579813,  .456730,  .320019,  .168466,  .0000
3     $ 1.767118, 1.745610, 1.709769, 1.658354, 1.5904
3     $ 1.411412, 1.302522, 1.189105, 1.071797,  .9589
3     $  .751949,  .649863,  .543716,  .429216,  .3008
3     $  .000000, 1.773172, 1.766514, 1.745120, 1.7083
3     $ 1.585137, 1.498306, 1.394964, 1.278019, 1.1522
3     $  .903245,  .794602,  .694648,  .598701,  .5009
3     $  .279503,  .148374,  .000000, 1.773160, 1.7668
3     $ 1.707694, 1.653350, 1.580377, 1.489935, 1.3799
3     $ 1.115450,  .971980,  .838405,  .725462,  .6264
3     $  .448780,  .356518,  .251666,  .134418,  .0000
3C
3      DATA T3B4/ 1.773148, 1.766886, 1.744770, 1.7068
3     $ 1.576061, 1.481777, 1.365440, 1.228351, 1.0753
3     $  .761811,  .640660,  .543899,  .462778,  .3870
3     $  .219547,  .118413,  .000000, 1.773135, 1.7662
3     $ 1.705891, 1.649078, 1.572301, 1.473805, 1.3517
3     $ 1.033233,  .846789,  .668069,  .534340,  .4437
3     $  .313987,  .252003,  .182072,  .099697,  .0000
3     $ 1.766964, 1.744468, 1.705732, 1.648443, 1.5695
3     $ 1.341577, 1.185259,  .997438,  .777634,  .5587
3     $  .323718,  .269571,  .225297,  .183201,  .1320
3     $  .000000, 1.773107, 1.766642, 1.744260, 1.7051
3     $ 1.567542, 1.464628, 1.333501, 1.169114,  .9659
3     $  .415359,  .235466,  .178828,  .148860,  .1227
3     $  .074529,  .044213,  .000000, 1.773086, 1.7661
3     $ 1.704671, 1.646615, 1.566450, 1.461852, 1.3306
3     $  .950860,  .681654,  .214989, -.017429,  .0207
3     $ -.001696,  .002869,  .000228, -.000042,  .0000
3C
3      DATA T4B1/ 2.170821, 2.163987, 2.143856, 2.1109
3     $ 2.009438, 1.941830, 1.863493, 1.774542, 1.6750
3     $ 1.442931, 1.309491, 1.163695, 1.004949,  .8326
3     $  .445800,  .230392,  .000000, 2.170821, 2.1649
3     $ 2.111492, 2.066755, 2.009426, 1.942369, 1.8637
3     $ 1.675211, 1.564298, 1.442411, 1.309369, 1.1631
3     $  .832533,  .646076,  .445769,  .230402,  .0000
3     $ 2.164315, 2.143935, 2.111047, 2.065983, 2.0089
3     $ 1.862298, 1.772663, 1.672951, 1.562037, 1.4400
3     $ 1.160872, 1.002466,  .830622,  .644766,  .4447
3     $  .000000, 2.170820, 2.164526, 2.143966, 2.1110
3     $ 2.008224, 1.940180, 1.860721, 1.770317, 1.6702
```

```
3     $ 2.164608, 2.143686, 2.109968, 2.063200, 2.00297
3     $ 1.847434, 1.751836, 1.646789, 1.530574, 1.40484
3     $ 1.124503,  .968880,  .801829,  .622193,  .42928
3     $  .000000, 2.170816, 2.163945, 2.143354, 2.10921
3     $ 2.001097, 1.927777, 1.842216, 1.745099, 1.63720
3     $ 1.391932, 1.255739, 1.110388,  .955489,  .79011
3     $  .423046,  .219010,  .000000, 2.170815, 2.16487
3     $ 2.109384, 2.061619, 1.999350, 1.925450, 1.83781
3     $ 1.628260, 1.507046, 1.377197, 1.240026, 1.09352
3     $  .775977,  .601606,  .414888,  .214879,  .00000
3C
3      DATA T4B3/ 2.170814, 2.164347, 2.143280, 2.10868
3     $ 1.997326, 1.921510, 1.831946, 1.729613, 1.61650
3     $ 1.359565, 1.219798, 1.072601,  .919024,  .75783
3     $  .405026,  .209919,  .000000, 2.170812, 2.16419
3     $ 2.108206, 2.059072, 1.995349, 1.917902, 1.82620
3     $ 1.604316, 1.476192, 1.339462, 1.196371, 1.04730
3     $  .735197,  .568763,  .392173,  .203410,  .00000
3     $ 2.164797, 2.143227, 2.108187, 2.058696, 1.99352
3     $ 1.821011, 1.712356, 1.592315, 1.458899, 1.31685
3     $ 1.016823,  .863034,  .705960,  .545518,  .37547
3     $  .000000, 2.170809, 2.164094, 2.142888, 2.10742
3     $ 1.991610, 1.911075, 1.814909, 1.703567, 1.57814
3     $ 1.291027, 1.136420,  .979244,  .823818,  .67043
3     $  .354935,  .184513,  .000000, 2.170807, 2.16458
3     $ 2.107393, 2.056834, 1.990022, 1.908513, 1.81006
3     $ 1.565655, 1.420163, 1.263374, 1.100096,  .93403
3     $  .624343,  .476731,  .326596,  .170005,  .00000
3C
3      DATA T4B4/ 2.170805, 2.164616, 2.142908, 2.10713
3     $ 1.988573, 1.905844, 1.805380, 1.687196, 1.55301
3     $ 1.233503, 1.057230,  .878645,  .712617,  .56389
3     $  .290954,  .151878,  .000000, 2.170804, 2.16390
3     $ 2.106497, 2.054970, 1.987269, 1.902875, 1.80082
3     $ 1.540537, 1.381154, 1.202803, 1.008545,  .81148
3     $  .482260,  .359282,  .245319,  .128858,  .00000
3     $ 2.164764, 2.142853, 2.106823, 2.055320, 1.98649
3     $ 1.798338, 1.674820, 1.532175, 1.365132, 1.17578
3     $  .729965,  .526280,  .377719,  .269795,  .18174
3     $  .000000, 2.170800, 2.164388, 2.142693, 2.10650
3     $ 1.985861, 1.900494, 1.796024, 1.671037, 1.52490
3     $ 1.151884,  .913571,  .639924,  .368356,  .22084
3     $  .105409,  .057354,  .000000, 2.170798, 2.16382
3     $ 2.106133, 2.054137, 1.986547, 1.899508, 1.79494
3     $ 1.521450, 1.346740, 1.128583,  .897988,  .57287
3     $ -.018436,  .031183,  .002476, -.000457,  .00000
3C
3C     INTERPOLATE NUMERICAL SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRD
3C
3      TRUE = DERVSL(6)
3C
3      RETURN
3      END
3      FUNCTION D(X)
3      D = 0.
3      IF (X .NE. 0.) D=1./X
3      RETURN
3      END
3      FUNCTION C(X)
3      C = 0.
3      IF (X .NE. 0.) C=1./X**2
```

```
*EOR
************
* MACRO  52 *
************
*          2000000200020
1          TWO DIMENSIONS $ HOMOGENEOUS
1          R(X,Y)UXX$ + R(X,Y)UYY$ + R1(X,Y)UX$ +
1.         R2(X,Y)UY$ - (&A)U$ = 0.0
2          MIXED
2          X=0. , MIXED = (1.)U + (-1.)UX = 1.
2          X=1. , MIXED = (1.)U + ( 1.)UX = 1.
2          Y=0. , MIXED = (1.)U + (-1.)UY = 1.
2          Y=1. , MIXED = (1.)U + ( 1.)UY = 1.
3      FUNCTION TRUE(X,Y)
3C
3C     ***************************************************
3C     *
3C     *                MACRO 52 PARAMETERS
3C     *
3C     ***************************************************
3C     *
3C     *                        A  I  B
3C     *                      -----I-----
3C     *                        2  I  1
3C     *                           I
3C     *                        4  I  2
3C     *                           I
3C     *                       49  I  3
3C     *
3C     ***************************************************
3C
3      REAL DERVSL(6), GRID(20), TABLE(20,20),
3     $     T1B1(100), T1B2(100), T1B3(100), T1B4(100),
3     $     T2B1(100), T2B2(100), T2B3(100), T2B4(100),
3     $     T3B1(100), T3B2(100), T3B3(100), T3B4(100)
3      EQUIVALENCE (TABLE(1, 1) , T&BB1(1)),
3     $            (TABLE(1, 6) , T&BB2(1)),
3     $            (TABLE(1,11) , T&BB3(1)),
3     $            (TABLE(1,16) , T&BB4(1))
3      DATA NGRID, NGRDD, GRID /20, 20, 0.0000000, 0.0526
3     $0.1052632, 0.1578947, 0.2105263, 0.2631579, 0.3157
3     $0.3684211, 0.4210526, 0.4736842, 0.5263158, 0.5789
3     $0.6315789, 0.6842105, 0.7368421, 0.7894737, 0.8421
3     $0.8947368, 0.9473684, 1.0000000/
3C
3C     APPROXIMATE SOLUTION OF LINEARIZED PROBLEM USING
3C     P3-C1 COLLOCATION (8 X 8 GRID)
3C
3      DATA T1B1/-2.295261,-2.477411,-2.674675,-2.883587,
3     $-3.319594,-3.536248,-3.744104,-3.936499,-4.106518,
3     $-4.352371,-4.415667,-4.433288,-4.402111,-4.321443,
3     $-4.022659,-3.815790,-3.583680,-2.477411,-2.669367,
3     $-3.097772,-3.326190,-3.557712,-3.786302,-4.005670,
3     $-4.388890,-4.537280,-4.648595,-4.716580,-4.734307,
3     $-4.618365,-4.481900,-4.304610,-4.085016,-3.838016,
3     $-2.877542,-3.097131,-3.329554,-3.570681,-3.814999,
3     $-4.288487,-4.503424,-4.693803,-4.851472,-4.969623,
3     $-5.061752,-5.028399,-4.939129,-4.796625,-4.607167,
3     $-4.116688,-2.883585,-3.097765,-3.329551,-3.574802,
3     $-4.087612,-4.343046,-4.588738,-4.816699,-5.018982,
3     $-5.313044,-5.390557,-5.413020,-5.379412,-5.285998,
3     $-4.936543,-4.692186,-4.416933,-3.100256,-3.326178,
3     $-3.829375,-4.098132,-4.371139,-4.641469,-4.901875,
```

```
3     $-5.731579,-3.936482,-4.208748,-4.503405,-4.8166
3     $-5.478153,-5.811935,-6.136262,-6.440792,-6.7148
3     $-7.126991,-7.243976,-7.290879,-7.261707,-7.1539
3     $-6.717139,-6.404841,-6.049837,-4.106496,-4.3886
3     $-5.018842,-5.359151,-5.707101,-6.055849,-6.3952
3     $-7.004259,-7.249851,-7.441904,-7.569564,-7.6219
3     $-7.489698,-7.299751,-7.041875,-6.715955,-6.3438
3C
3      DATA T1B3/-4.247239,-4.537166,-4.851413,-5.1866
3     $-5.898587,-6.260363,-6.613568,-6.947099,-7.2498
3     $-7.711945,-7.848491,-7.908426,-7.887137,-7.7790
3     $-7.319862,-6.984879,-6.601692,-4.352339,-4.6484
3     $-5.312937,-5.673655,-6.043918,-6.416831,-6.7816
3     $-7.441813,-7.711902,-7.925710,-8.071085,-8.1379
3     $-8.013640,-7.820049,-7.547003,-7.204432,-6.8115
3     $-4.716214,-5.041344,-5.390302,-5.757608,-6.1345
3     $-6.889737,-7.243942,-7.569313,-7.848370,-8.0710
3     $-8.297419,-8.285615,-8.180966,-7.985598,-7.7120
3     $-6.961731,-4.433248,-4.734183,-5.061680,-5.4129
3     $-6.164781,-6.550599,-6.929851,-7.290843,-7.6218
3     $-8.137895,-8.297412,-8.376323,-8.367628,-8.2658
3     $-7.796413,-7.446483,-7.043072,-4.402068,-4.7028
3     $-5.379106,-5.749601,-6.130437,-6.518531,-6.8992
3     $-7.597524,-7.886929,-8.120401,-8.285385,-8.3675
3     $-8.265618,-8.074885,-7.801117,-7.451618,-7.0475
3C
3      DATA T1B4/-4.321397,-4.617910,-4.938938,-5.2856
3     $-6.029568,-6.414758,-6.793152,-7.153933,-7.4892
3     $-8.013537,-8.180719,-8.265745,-8.265612,-8.1715
3     $-7.716002,-7.370826,-6.970863,-4.193810,-4.4818
3     $-5.135366,-5.493972,-5.865238,-6.242077,-6.6143
3     $-7.299675,-7.586987,-7.820012,-7.985555,-8.0732
3     $-7.985171,-7.806041,-7.541693,-7.205032,-6.8144
3     $-4.303958,-4.606906,-4.936086,-5.284853,-5.6427
3     $-6.372695,-6.717078,-7.041131,-7.319499,-7.5468
3     $-7.796268,-7.800902,-7.715782,-7.541672,-7.2875
3     $-6.581414,-3.815741,-4.084604,-4.376095,-4.6918
3     $-5.371237,-5.724659,-6.072391,-6.404786,-6.7154
3     $-7.204297,-7.362887,-7.446387,-7.451482,-7.3706
3     $-6.960951,-6.647758,-6.283551,-3.583633,-3.8379
3     $-4.416881,-4.734953,-5.064746,-5.399747,-5.7315
3     $-6.343780,-6.601654,-6.811564,-6.961704,-7.0430
3     $-6.970853,-6.814473,-6.581412,-6.283551,-5.9372
3C
3      DATA T2B1/  .478938,  .448750,  .413085,  .3720
3     $  .275479,  .221884,  .166543,  .111361,  .0583
3     $ -.031690, -.064102, -.085586, -.094622, -.0904
3     $ -.043993, -.004295,  .043099,  .448750,  .4171
3     $  .335726,  .286911,  .233106,  .176590,  .1177
3     $  .002853, -.048907, -.093287, -.127655, -.1507
3     $ -.156159, -.138052, -.107269, -.065052, -.0145
3     $  .379139,  .338638,  .292047,  .239587,  .1820
3     $  .057764, -.005507, -.066099, -.121836, -.1697
3     $ -.232144, -.242898, -.238474, -.219272, -.1861
3     $ -.087125,  .372015,  .335722,  .292045,  .2418
3     $  .122677,  .056652, -.012216, -.081321, -.1471
3     $ -.260841, -.301842, -.329721, -.341994, -.3376
3     $ -.281823, -.233080, -.174720,  .325853,  .2869
3     $  .185165,  .123550,  .055251, -.016754, -.0922
3     $ -.240467, -.307984, -.366265, -.411801, -.4431
3     $ -.453449, -.431711, -.393647, -.340704, -.2771
3C
3      DATA T2B2/  .275481,  .233107,  .182076,  .1226
```

```
3     $ -.147116, -.240422, -.345242, -.457338, -.5764
3     $ -.815491, -.927263,-1.025861,-1.105371,-1.1633
3     $-1.195935,-1.168805,-1.114903,-1.036203, -.9400
3C
3      DATA T2B3/  .009920, -.048889, -.121823, -.2082
3     $ -.419750, -.540598, -.668649, -.799436, -.9272
3     $-1.155702,-1.243353,-1.307288,-1.342371,-1.3459
3     $-1.261056,-1.177632,-1.075312, -.031679, -.0932
3     $ -.260810, -.366224, -.484457, -.613001, -.7492
3     $-1.025829,-1.155687,-1.271499,-1.366755,-1.4366
3     $-1.481661,-1.453930,-1.394011,-1.306258,-1.1981
3     $ -.127585, -.207084, -.301774, -.411703, -.5357
3     $ -.813515, -.960835,-1.105281,-1.243309,-1.3667
3     $-1.544190,-1.587655,-1.595538,-1.567967,-1.5068
3     $-1.301869, -.035572, -.150704, -.232123, -.3296
3     $ -.570920, -.710603, -.859249,-1.012019,-1.1633
3     $-1.436625,-1.544187,-1.624282,-1.671116,-1.6810
3     $-1.591147,-1.497287,-1.380616, -.094606, -.1604
3     $ -.341905, -.457313, -.587453, -.730096, -.8821
3     $-1.194117,-1.342292,-1.475904,-1.587562,-1.6710
3     $-1.732707,-1.706363,-1.643403,-1.548036,-1.4289
3C
3      DATA T2B4/ -.090415, -.156056, -.238425, -.3375
3     $ -.584014, -.727479, -.880622,-1.038559,-1.1957
3     $-1.481618,-1.595438,-1.681051,-1.732704,-1.7460
3     $-1.658304,-1.562861,-1.443236, -.073341, -.1380
3     $ -.317218, -.431681, -.561025, -.703338, -.8553
3     $-1.168773,-1.318375,-1.453913,-1.567948,-1.6540
3     $-1.720745,-1.696789,-1.635080,-1.541078,-1.4229
3     $ -.107112, -.186057, -.281689, -.393448, -.5193
3     $ -.807352, -.960437,-1.114624,-1.260919,-1.3939
3     $-1.591087,-1.643312,-1.658212,-1.635070,-1.5755
3     $-1.368326, -.004278, -.064950, -.140978, -.2329
3     $ -.462005, -.596317, -.739546, -.887434,-1.0390
3     $-1.306202,-1.415146,-1.497246,-1.547978,-1.5628
3     $-1.483883,-1.395736,-1.284420,  .043116, -.0145
3     $ -.174700, -.277158, -.393073, -.520818, -.6574
3     $ -.940035,-1.075294,-1.198136,-1.301856,-1.3806
3     $-1.443231,-1.422908,-1.368325,-1.284419,-1.1784
3C
3      DATA T3B1   .126998,  .075205,  .017325, -.0381
3     $ -.097458, -.079581, -.024216,  .060504,  .1628
3     $  .332962,  .359965,  .342591,  .287119,  .2126
3     $  .103549,  .100115,  .130246,  .075205,  .0203
3     $ -.101862, -.147469, -.167279, -.149574, -.0916
3     $  .103916,  .206778,  .280741,  .305683,  .2837
3     $  .140192,  .065262,  .022950,  .020287,  .0534
3     $ -.042050, -.110341, -.176784, -.228282, -.2517
3     $ -.174545, -.081441,  .029825,  .134458,  .2059
3     $  .189114,  .113010,  .018683, -.064518, -.1095
3     $ -.070020, -.038116, -.101831, -.176773, -.2503
3     $ -.336122, -.319666, -.256733, -.159295, -.0445
3     $  .125920,  .130340,  .077332, -.017064, -.1274
3     $ -.268508, -.263312, -.216610, -.080108, -.1474
3     $ -.308098, -.371130, -.402143, -.384584, -.3170
3     $ -.095618,  .007775,  .064722,  .051990, -.0233
3     $ -.265617, -.369370, -.415238, -.403400, -.3488
3C
3      DATA T3B2/ -.097453, -.167267, -.251769, -.3361
3     $ -.432453, -.410589, -.335666, -.224880, -.1015
3     $  .045318,  .012979, -.083526, -.219719, -.3576
3     $ -.501784, -.480710, -.419285, -.079575, -.1494
3     $ -.319592, -.384472, -.410570, -.381637, -.2957
```

```
3      DATA T3B3/  .261032,  .206867,  .134488,  .060190
3     $ -.000435,  .053339,  .163107,  .293475,  .400549
3     $  .397749,  .269392,  .120930,  .028570,  .046584
3     $  .333233,  .496164,  .581070,  .332953,  .280750
3     $  .125918,  .064725,  .045324,  .089512,  .187559
3     $  .396745,  .397728,  .315747,  .153868, -.011977
3     $ -.024029,  .158654,  .401369,  .614031,  .713694
3     $  .305744,  .223233,  .130337,  .051993,  .012984
3     $  .122636,  .219465,  .286367,  .269328,  .153868
3     $ -.237410, -.321575, -.230112,  .000162,  .291380
3     $  .675303,  .342575,  .283785,  .189109,  .077320
3     $ -.083522, -.075012, -.004580,  .086177,  .145440
3     $ -.011972, -.237410, -.458370, -.561218, -.473735
3     $  .094315,  .379656,  .515872,  .287103,  .221900
3     $ -.017075, -.138884, -.219716, -.221952, -.151055
3     $  .028694,  .028556, -.090070, -.321295, -.561147
3     $ -.641933, -.432564, -.115902,  .167791,  .309678
3C
3      DATA T3B4/  .212612,  .140202,  .018682, -.127450
3     $ -.357682, -.359694, -.272436, -.136537, -.009309
3     $ -.023935, -.229817, -.473657, -.641928, -.650817
3     $ -.244456,  .008817,  .145796,  .143724,  .065254
3     $ -.221921, -.369367, -.461891, -.455236, -.341655
3     $  .033586,  .161442,  .158656,  .000188, -.231533
3     $ -.507633, -.443508, -.257927, -.054149,  .068164
3     $  .023083, -.109489, -.268428, -.415111, -.501750
3     $ -.341024, -.115011,  .129941,  .333186,  .401491
3     $  .094426, -.115636, -.244178, -.257895, -.136800
3     $  .097410,  .100109,  .020366, -.109138, -.263260
3     $ -.480691, -.448062, -.288681, -.035947,  .248770
3     $  .614100,  .552076,  .379730,  .167962,  .008990
3     $  .005418,  .104693,  .179123,  .130242,  .053440
3     $ -.216606, -.348829, -.419277, -.381208, -.220240
3     $  .327529,  .581051,  .713681,  .675303,  .515880
3     $  .145820,  .068182,  .097421,  .179127,  .247230
3C
3C     INTERPOLATE NONLINEAR SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRID,GRID,NGRID,NGRID,NGRD
3C
3      TRUE = DERVSL(6)
3C
3      RETURN
3      END
3      FUNCTION W(NDERV,X,Y)
3C
3      REAL DERVSL(6), GRID(9), TAB(9,9)
3      DATA NGRID, NGRDD, GRID /9, 9, 0., 0.125, 0.250,
3     $0.375, 0.500, 0.625, 0.750, 0.875, 1.00/
3C
3C     APPROXIMATE SOLUTION OF NONLINEAR PROBLEM FOR NO
3C     COLLOCATION
3C
3      DATA  TAB/ -.849260,-1.089850,-1.342400,-1.591250
3     $-1.976840,-2.044640,-1.975460,-1.731410,-1.089850
3     $-1.649620,-1.933770,-2.188320,-2.377680,-2.458420
3     $-2.106880,-1.342390,-1.649610,-1.976700,-2.303520
3     $-2.824330,-2.926380,-2.849290,-2.538770,-1.591240
3     $-2.303510,-2.677900,-3.022520,-3.289980,-3.420640
3     $-3.001420,-1.813090,-2.188300,-2.599750,-3.022510
3     $-3.732750,-3.897260,-3.831490,-3.454320,-1.976810
3     $-2.824300,-3.289960,-3.732740,-4.092960,-4.292680
3     $-3.837950,-2.044610,-2.458380,-2.926340,-3.420610
```

```
3     FUNCTION R(X,Y)
3     R = 11./(1. + 10.*W(6,X,Y))
3     RETURN
3     END
3     FUNCTION R1(X,Y)
3     R1 = -110./(1.+10.*W(6,X,Y))**2*W(4,X,Y)
3     RETURN
3     END
3     FUNCTION R2(X,Y)
3     R2 = -110./(1.+10.*W(6,X,Y))**2*W(5,X,Y)
3     RETURN
3     END
```

```
'EOR
************
* MACRO  53 *
************
*          2000021002000
1          TWO DIMENSIONS $ CONSTANT COEFFICIENTS
1          UXX$ + UYY$ - (&A)U$ = F(X,Y)
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3     FUNCTION TRUE(X,Y)
3     TRUE = COS(&B*Y)+SIN(&B*(X-Y))
3     RETURN
3     END
3     FUNCTION F(X,Y)
3     BXMY = &B*(X-Y)
3     B2 = (&B)*(&B)
3     UXX = -B2*SIN(BXMY)
3     U = COS(&B*Y)+SIN(BXMY)
3     UYY = -B2*U
3     F = UXX + UYY - (&A)*U
3     RETURN
3     END
```

```
'EOR
************
* MACRO  54 *
************
*          2000000002000
1          TWO DIMENSIONS
1          (1.+X*X)UXX$ + (1.+A(Y)**2)UYY$ + (2.*X)UX$ +
1.         (16.*Y*A(Y))UY$ - (1.+(8.*Y-X-4.)**2)U$ = F(X,
2          DIRICHLET
2          X=0. , U=TRUE(X,Y)
2          X=1. , U=TRUE(X,Y)
2          Y=0. , U=TRUE(X,Y)
2          Y=1. , U=TRUE(X,Y)
3     FUNCTION TRUE(X,Y)
3     A0Y=A(Y)
3     R = AMAX1(0.,(3.-X/A0Y)**3)
3     S = AMAX1(0.,X-A0Y)
3     T = 0.
3     IF (S .GE. .02) T = EXP(-R/S)
3     G    = 2.25*X*(X-A0Y)**2/A0Y**3
3     H    = 1./(1. + (8.*Y-X-4.)**2)
3     TRUE = G*(1.-T) + H
3     RETURN
3     END
```

```
3      GD   = A0Y**3
3      GDY  = 3.*A0Y**2*A1Y
3      GDYY= 3.*(2.*A0Y*A1Y**2 + A0Y**2*A2Y)
3      G    = 2.25*GN/GD
3      GX   = 2.25*GNX/GD
3      GXX  = 2.25*GNXX/GD
3      GY   = 2.25*(GD*GNY-GN*GDY)/GD**2
3      GYY =2.25*((GD*GNYY-GN*GDYY)/GD**2
3     $      + 2.*(GN*GDY-GD*GNY)*GDY/GD**3)
3      HD   = 1. + (8.*Y-X-4.)**2
3      HDX  = -2.*(8.*Y-X-4.)
3      HDXX= 2.
3      HDY  = 16.*(8.*Y-X-4.)
3      HDYY= 128.
3      H    = 1./HD
3      HX   = -HDX/HD**2
3      HXX  = (-HDXX +2.*HDX*HDX/HD)/(HD*HD)
3      HY   = -HDY/HD**2
3      HYY =(-HDYY +2.*HDY*HDY/HD)/(HD*HD)
3      T    = 0.
3      TX   = 0.
3      TXX  = 0.
3      TY   = 0.
3      TYY  = 0.
3      S    = AMAX1(0.,X-A0Y)
3      IF (S .LT. .02) GO TO 10
3      R    = AMAX1(0.,(3.-X/A0Y)**3)
3      T    = 1.0
3      IF (R .EQ. 0.0) GO TO 10
3      S2   = S*S
3      S3   = S2*S
3      SX   = 1.
3      SXX  = 0.
3      SY   = -A1Y
3      SYY  = -A2Y
3      RX   =  -3.*(3.-X/A0Y)**2/A0Y
3      RXX  =  6.*(3.-X/A0Y)/A0Y**2
3      RY   =  3.*(3.-X/A0Y)**2*X*A1Y/A0Y**2
3      RYY  =  6.*(3.-X/A0Y)*(X*A1Y/A0Y**2)**2 +
3     $       3.*X*(3.-X/A0Y)**2*(A2Y-2.*A1Y**2/A0
3      Q    = R/S
3      QX   = (RX-Q)/S
3      QXX  = RXX/S + 2.*(R-S*RX)/S3
3      QY   = (RY-Q*SY)/S
3      QYY  = (S*RYY-R*SYY)/S2 + 2.*(R*SY-S*RY)*SY
3      T    = EXP(-Q)
3      TX   = -QX*T
3      TXX  = (QX*QX - QXX)*T
3      TY   = -QY*T
3      TYY  = (QY*QY - QYY)*T
3   10 U    = G*(1.-T) + H
3      UX   = GX*(1.-T) - G*TX + HX
3      UXX  = GXX*(1.-T) -2.*GX*TX - G*TXX + HXX
3      UY   = GY*(1.-T) - G*TY + HY
3      UYY  = GYY*(1.-T) -2.*GY*TY - G*TYY + HYY
3      F1   = (1.+X*X)*UXX
3      F2   = 2.*X*UX
3      F3   = (1.+A0Y**2)*UYY
3      F4   = 2.*A0Y*A1Y*UY
3      F5   = HD*U
3      F    = F1 + F2 + F3 + F4 - F5
3      RETURN
3      END
```

```
*         2022021200220
1         TWO DIMENSIONS $ CONSTANT COEFFICIENTS $ HOMOGE
1.        POISSON $ LAPLACE
1         UXX$ + UYY$ = 0.
2         MIXED
2         X=0. , UX = 0.
2         X=6. , U  = F(Y)
2         Y=0. , MIXED = (A(X))U + (B(X))UY = G(X)
2         Y=1. , MIXED = (C(X))U + (D(X))UY = H(X)
3       FUNCTION TRUE(X,Y)
3C
3C      ************************************************
3C      *
3C      *                  MACRO 55 PARAMETERS
3C      *
3C      ************************************************
3C      *
3C      *                 A  I  B  I  C  I  D
3C      *                ----I-----I-----I----
3C      *                 1  I  3  I  1  I  1
3C      *                    I     I     I
3C      *                 3  I  2  I  1  I  2
3C      *                    I     I     I
3C      *                 1  I  3  I  2  I  3
3C      *                    I     I     I
3C      *                 6  I  2  I  2  I  4
3C      *
3C      ************************************************
3C
3       REAL DERUSL(6), TABLE(20,20), GRIDX(20), GRIDY(20)
3      $     T1B1(100), T1B2(100), T1B3(100), T1B4(100),
3      $     T2B1(100), T2B2(100), T2B3(100), T2B4(100),
3      $     T3B1(100), T3B2(100), T3B3(100), T3B4(100),
3      $     T4B1(100), T4B2(100), T4B3(100), T4B4(100)
3       EQUIVALENCE (TABLE(1, 1) , T&DB1(1)),
3      $            (TABLE(1, 6) , T&DB2(1)),
3      $            (TABLE(1,11) , T&DB3(1)),
3      $            (TABLE(1,16) , T&DB4(1))
3       DATA NGRID, NGRDD/20, 20/
3       DATA GRIDX/0.0000000, 0.3157895, 0.6315789, 0.9473
3      $1.2631579, 1.5789474, 1.8947368, 2.2105263, 2.5263
3      $2.8421053, 3.1578947, 3.4736842, 3.7894737, 4.1052
3      $4.4210526, 4.7368421, 5.0526316, 5.3684211, 5.6842
3      $6.0000000/
3       DATA GRIDY/0.0000000, 0.0526316, 0.1052632, 0.1578
3      $0.2105263, 0.2631579, 0.3157895, 0.3684211, 0.4210
3      $0.4736842, 0.5263158, 0.5789474, 0.6315789, 0.6842
3      $0.7368421, 0.7894737, 0.8421053, 0.8947368, 0.9473
3      $1.0000000/
3C
3C      APPROXIMATE SOLUTION OF PROBLEM USING
3C      P3-C1 COLLOCATION (8 X 8 GRID)
3C
3       DATA T1B1/ 1.000000,  1.006667,   .963921,   .420922,
3      $  .035678,  -.003856,  -.015701,  -.009375,  -.002813,
3      $  .024493,   .047173,   .081810,   .135661,   .224512,
3      $  .574550,   .825183,   .199583,   .898518,   .901087,
3      $  .399040,   .090743,   .037577,   .000429,  -.012505,
3      $ -.001690,   .009149,   .024776,   .047198,   .081694,
3      $  .222411,   .338273,   .567131,   .801969,   .359473,
3      $  .855271,   .778675,   .400049,   .128898,   .046642,
3      $ -.007849,  -.006276,  -.000599,   .009640,   .024855,
3      $  .081055,   .134750,   .217468,   .342597,   .548524,
```

```
3     $  .002375,  .010237,  .023138,  .042455,  .0732
3     $  .197801,  .324684,  .504710,  .787854, 1.3945
3     $  .563805,  .480451,  .305067,  .151547,  .0631
3     $  .003682,  .000118,  .002759,  .010037,  .0222
3     $  .070012,  .116646,  .190069,  .313769,  .4924
3     $ 1.474962,  .530252,  .509554,  .432747,  .2834
3     $  .062931,  .021899,  .004730,  .000874,  .0029
3     $  .021080,  .038318,  .066273,  .110358,  .1808
3     $  .477234,  .801975, 1.525152,  .478274,  .4588
3     $  .256607,  .136461,  .060391,  .021829,  .0053
3     $  .003203,  .009284,  .019871,  .036070,  .0621
3     $  .171114,  .285418,  .461522,  .798957, 1.5481
3C
3      DATA T1B3/  .427264,  .408292,  .344767,  .2325
3     $  .057565,  .021465,  .005803,  .001848,  .0032
3     $  .018410,  .033292,  .057531,  .096368,  .1594
3     $  .439280,  .784438, 1.548305,  .377170,  .3594
3     $  .207574,  .116843,  .053616,  .020433,  .0059
3     $  .003148,  .008054,  .016821,  .030347,  .0525
3     $  .146711,  .248816,  .413428,  .759381, 1.5289
3     $  .312441,  .263089,  .180935,  .102626,  .0484
3     $  .005576,  .002171,  .003015,  .007308,  .0151
3     $  .047192,  .079746,  .133263,  .227699,  .3841
3     $ 1.493296,  .279529,  .265585,  .223708,  .1561
3     $  .043055,  .017082,  .005420,  .002131,  .0027
3     $  .013245,  .023834,  .041451,  .070033,  .1179
3     $  .347194,  .672061, 1.444266,  .231817,  .2201
3     $  .129589,  .075490,  .036645,  .014486,  .0045
3     $  .002426,  .005523,  .011323,  .020400,  .0354
3     $  .102122,  .180487,  .307418,  .607928, 1.3845
3C
3      DATA T1B4/  .184712,  .175369,  .147577,  .1035
3     $  .029854,  .011869,  .003814,  .001697,  .0020
3     $  .009280,  .016695,  .029188,  .050189,  .0847
3     $  .261757,  .527022, 1.316522,  .138097,  .1308
3     $  .078277,  .046749,  .022887,  .009412,  .0032
3     $  .001588,  .003485,  .007127,  .012660,  .0226
3     $  .065446,  .128368,  .210137,  .426129, 1.2422
3     $  .087008,  .073283,  .051189,  .029749,  .0151
3     $  .001745,  .000943,  .001077,  .002372,  .0050
3     $  .015939,  .028951,  .044988,  .101410,  .1555
3     $ 1.163707,  .045865,  .043436,  .036585,  .0254
3     $  .007608,  .002869,  .000843,  .000503,  .0005
3     $  .002924,  .004366,  .008967,  .018440,  .0213
3     $  .098913,  .153649, 1.082479,  .000000,  .0000
3     $  .000001,  .000000,  .000002,  .000005, -.0000
3     $  .000008,  .000015,  .000909, -.000336,  .0017
3     $ -.006366,  .047770,  .042140, -.031332, 1.0000
3C
3      DATA T2B1/  .037037,  .037036,  .037042,  .0370
3     $  .037222,  .037454,  .036687,  .041360,  .0379
3     $  .017817,  .038075,  .067847,  .114664,  .1913
3     $  .492011,  .707037,  .171054,  .035093,  .0351
3     $  .035185,  .035190,  .035286,  .035623,  .0354
3     $  .035624,  .019012,  .019813,  .038881,  .0681
3     $  .189642,  .289247,  .485670,  .687150,  .3080
3     $  .033174,  .033162,  .033195,  .033217,  .0333
3     $  .033700,  .035789,  .032282,  .019556,  .0209
3     $  .067717,  .114138,  .185424,  .292984,  .4705
3     $  .582390,  .031193,  .031235,  .031218,  .0312
3     $  .031373,  .031604,  .031899,  .033217,  .0297
3     $  .021828,  .038137,  .066922,  .112655,  .1821
3     $  .458778,  .644883,  .797950,  .029245,  .0293
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3     $  .059163.  .099153.  .162231.  .268453.  .4217
3     $ 1.264135.  .021448.  .021451.  .021459.  .0214
3     $  .021590.  .021747.  .021976.  .021821.  .0197
3     $  .020710.  .032843.  .056105.  .093854.  .1543
3     $  .408752.  .687215. 1.307151.  .019499.  .0195
3     $  .019552.  .019602.  .019637.  .019811.  .0199
3     $  .018043.  .016316.  .019682.  .031039.  .0526
3     $  .146107.  .244231.  .395311.  .684640. 1.3268
3C
3      DATA T2B3/  .017549.  .017571.  .017566.  .0175
3     $  .017674.  .017813.  .017944.  .017685.  .0162
3     $  .018374.  .028801.  .048854.  .082037.  .1361
3     $  .376270.  .572207. 1.326995.  .015600.  .0156
3     $  .015630.  .015664.  .015712.  .015825.  .0159
3     $  .014463.  .013710.  .016878.  .026355.  .0446
3     $  .125308.  .212940.  .354137.  .650742. 1.3103
3     $  .013679.  .013668.  .013688.  .013724.  .0137
3     $  .013927.  .013672.  .012707.  .012253.  .0152
3     $  .040162.  .067941.  .113840.  .194880.  .3290
3     $ 1.279848.  .011700.  .011708.  .011710.  .0117
3     $  .011737.  .011867.  .011912.  .011596.  .0109
3     $  .013379.  .020822.  .035320.  .059684.  .1007
3     $  .297419.  .575927. 1.237826.  .009750.  .0097
3     $  .009774.  .009798.  .009825.  .009894.  .0099
3     $  .009124.  .009051.  .011438.  .017821.  .0302
3     $  .087260.  .154494.  .263354.  .520973. 1.1866
3C
3      DATA T2B4/  .007800.  .007813.  .007809.  .0078
3     $  .007861.  .007918.  .007934.  .007781.  .0073
3     $  .009372.  .014595.  .024899.  .042799.  .0724
3     $  .224244.  .451644. 1.128342.  .005850.  .0058
3     $  .005861.  .005874.  .005836.  .005931.  .0059
3     $  .005514.  .005584.  .007204.  .011115.  .0193
3     $  .055935.  .109904.  .180026.  .365183. 1.0646
3     $  .003901.  .003903.  .003908.  .003916.  .0039
3     $  .003961.  .003900.  .003677.  .003767.  .0050
3     $  .013602.  .024706.  .038451.  .086837.  .1332
3     $  .997370.  .001950.  .001950.  .001952.  .0019
3     $  .001967.  .001980.  .001980.  .001968.  .0018
3     $  .002867.  .003801.  .007656.  .015751.  .0182
3     $  .084751.  .131675.  .927752.  .000000.  .0000
3     $  .000001.  .000000.  .000002.  .000004. -.0000
3     $  .000007.  .000013.  .000779. -.000331.  .0014
3     $ -.005456.  .040942.  .036117. -.026854.  .8570
3C
3      DATA T3B1/ 1.000000. 1.004844.  .973786.  .5830
3     $  .448344.  .488383.  .537149.  .592334.  .6430
3     $  .743997.  .794079.  .843632.  .893525.  .9487
3     $ 1.070504. 1.099100.  .199583.  .917360.  .9211
3     $  .574650.  .409115.  .453123.  .494867.  .5416
3     $  .645894.  .695232.  .745000.  .796087.  .8442
3     $  .948910.  .983185. 1.065766. 1.077697.  .3594
3     $  .879926.  .834846.  .582935.  .445860.  .4639
3     $  .546054.  .596026.  .646089.  .695426.  .7449
3     $  .844079.  .894798.  .944807.  .990130. 1.0504
3     $  .679519.  .828193.  .832535.  .782219.  .5793
3     $  .472942.  .507929.  .550589.  .598088.  .6477
3     $  .745504.  .795499.  .844328.  .895294.  .9443
3     $ 1.042438. 1.034959.  .931028.  .777396.  .7780
3     $  .569470.  .474742.  .480628.  .513885.  .5545
3     $  .649529.  .697240.  .746060.  .796302.  .8445
3     $  .945127.  .997679. 1.039941. 1.049910. 1.1208
3C
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3     $ 1.048169, 1.140332, 1.525152,  .529037,  .5286
3     $  .506573,  .500610,  .512608,  .537772,  .5070
3     $  .655227,  .700398,  .747743,  .797290,  .8448
3     $  .946744, 1.002657, 1.054893, 1.158573, 1.5481
3C
3      DATA T3B3/  .479250,  .479665,  .482538,  .4924
3     $  .517597,  .540877,  .572832,  .611282,  .6552
3     $  .747666,  .796353,  .844519,  .894575,  .9455
3     $ 1.056993, 1.169097, 1.548305,  .429501,  .4308
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3     $  .382393,  .403315,  .461180,  .502948,  .5260
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3     $  .844628,  .895058,  .945775, 1.000858, 1.0627
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3     $  .748178,  .796107,  .844192,  .893519,  .9438
3     $ 1.061466, 1.172130, 1.444266,  .280770,  .2813
3     $  .433451,  .508280,  .534308,  .554118,  .5818
3     $  .658716,  .702357,  .748673,  .797130,  .8443
3     $  .944150;  .997884, 1.062656, 1.166017, 1.3845
3C
3      DATA T3B4/  .231759,  .228998,  .271615,  .4217
3     $  .538402,  .556710,  .583592,  .617504,  .6593
3     $  .748829,  .797220,  .844216,  .893893,  .9434
3     $ 1.061550, 1.156720, 1.316522,  .183566,  .1743
3     $  .414915,  .527255,  .542941,  .557938,  .5843
3     $  .658471,  .702308,  .748453,  .795813,  .8437
3     $  .941389,  .993342, 1.057515, 1.145287, 1.2422
3     $  .117594,  .164840,  .403226,  .532188,  .5451
3     $  .586512,  .618827,  .660350,  .703191,  .7490
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3     $ 1.053941, 1.130518, 1.082479,  .050736, -.0063
3     $  .411528,  .586934,  .550513,  .563540,  .5887
3     $  .659175,  .702642,  .748417,  .795696,  .8432
3     $  .940470,  .983578, 1.048731, 1.128374, 1.0000
3C
3      DATA T4B1/  .004630,  .004630,  .004630,  .0046
3     $  .004635,  .004542,  .004620,  .004754,  .0046
3     $  .006789,  .003712,  .008700,  .025193, -.0104
3     $  .104731, -.065799, 2.380075,  .004386,  .0043
3     $  .004395,  .004401,  .004404,  .004436,  .0044
3     $  .004929,  .005663,  .008098,  .011695,  .0234
3     $  .109370,  .286389,  .680538, 1.556083, 4.2869
3     $  .004147,  .004146,  .004149,  .004156,  .0041
3     $  .004298,  .004563,  .005031,  .006387,  .0101
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3     $ 8.103794,  .003900,  .003905,  .003903,  .0039
3     $  .003940,  .003994,  .004119,  .004477,  .0051
3     $  .012042,  .020936,  .050330,  .112552,  .2456
3     $ 1.395242, 3.546549,11.103243,  .003656,  .0036
3     $  .003668,  .003681,  .003707,  .003776,  .0039
3     $  .005223,  .007679,  .013750,  .025537,  .0621
3     $  .316061,  .865437, 1.786185, 4.569099,13.3674
3C
3      DATA T4B2/  .003413,  .003414,  .003416,  .0034
3     $  .003472,  .003549,  .003742,  .004266,  .0052
3     $  .015374,  .029597,  .072696,  .165031,  .3779
3     $ 2.157251, 5.465849,15.262661,  .003169,  .0031
```

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3      DATA T4B3/  .002195,  .002198,  .002200,  .0022
3     $  .002277,  .002386,  .002656,  .003387,  .0048
3     $  .018838,  .039129,  .098467,  .227774,  .5355
3     $ 3.137190, 7.762745,18.464763,  .001951,  .0019
3     $  .001964,  .001982,  .002033,  .002140,  .0024
3     $  .004556,  .008589,  .018491,  .038729,  .0975
3     $  .533144, 1.354719, 3.123691, 7.722816,18.2336
3     $  .001712,  .001713,  .001721,  .001740,  .0017
3     $  .002154,  .002858,  .004246,  .008161,  .0177
3     $  .094614,  .219869,  .518734, 1.316257, 3.0332
3     $17.808733,  .001464,  .001465,  .001468,  .0014
3     $  .001539,  .001638,  .001885,  .002555,  .0038
3     $  .016686,  .035398,  .089537,  .207883,  .4909
3     $ 2.853377, 7.103634,17.224007,  .001220,  .0012
3     $  .001231,  .001247,  .001290,  .001382,  .0016
3     $  .003448,  .006875,  .015346,  .032579,  .0828
3     $  .451440, 1.167923, 2.611325, 6.514965,16.5117
3C
3      DATA T4B4/  .000976,  .000978,  .000980,  .0009
3     $  .001039,  .001122,  .001323,  .001885,  .0029
3     $  .013756,  .028730,  .074168,  .173415,  .3964
3     $ 2.289928, 5.712786,15.700562,  .000732,  .0007
3     $  .000741,  .000751,  .000786,  .000856,  .0010
3     $  .002398,  .004994,  .012100,  .023562,  .0637
3     $  .322304,  .946691, 1.894989, 4.664829,14.8148
3     $  .000489,  .000490,  .000496,  .000502,  .0005
3     $  .000705,  .001181,  .001771,  .003784,  .0106
3     $  .051712,  .129230,  .226629,  .820912, 1.4467
3     $13.878120,  .000245,  .000244,  .000246,  .0002
3     $  .000280,  .000324,  .000357, -.000861,  .0010
3     $  .009950,  .008112,  .037563,  .111326,  .0990
3     $  .976288, 1.595291,12.909414,  .000001,  .0000
3     $  .000008, -.000005,  .000027,  .000060, -.0000
3     $  .000096,  .000182,  .010843, -.004607,  .0204
3     $ -.075919,  .569691,  .502551, -.373664,11.9257
3C
3C     INTERPOLATE NUMERICAL SOLUTION BY QUADRATICS
3C
3      CALL QUADRD(X,Y,TABLE,GRIDX,GRIDY,NGRID,NGRID,N
3     $            DERVSL)
3C
3      TRUE = DERVSL(6)
3C
3      RETURN
3      END
3      FUNCTION F(Y)
3      DATA BREAK,PI2/&A,1.570796327/
3      F = EXP(PI2*(BREAK-Y))*SIN(PI2*Y)/&A**3
3      RETURN
3      END
3      FUNCTION A(X)
3      DATA BREAK/&A/
3      A = 1.
3      IF (X .GE. BREAK) A=0.
3      RETURN
3      END
3      FUNCTION B(X)
3      DATA BREAK/&A/
3      B = 0.
3      IF (X .GE. BREAK) B=1.
3      RETURN
3      END
```

```
3      DATA BREAK,DECAY0/&A,2./
3      G = 1./&A**3
3      IF (X .GE. BREAK) G=G*EXP(DECAY0*(BREAK-X))
3      RETURN
3      END
3      FUNCTION H(X)
3      DATA BREAK,KASE,DECAY1/&A,&C,&B/
3      H = 0.
3      IF (KASE .EQ. 1) RETURN
3      IF (X .GE. BREAK) H=EXP(DECAY1*(BREAK-X))/&A**3
3      RETURN
3      END
-------------------------------------------------------------------
'EOR
************
* MACRO  S6 *
************
*          2000200202000
1          TWO DIMENSIONS $ HOMOGENEOUS
1          UXX$ + C(X)UYY$ + D(X)UX$ = 0.
2          DIRICHLET
2          X=0. ,              U=TRUE(X,Y)
2          X=1. ,              U=TRUE(X,Y)
2          Y=0. ,              U=TRUE(X,Y)
2          Y=6.28318530718 ,   U=TRUE(X,Y)
3      FUNCTION TRUE(X,Y)
3      DIMENSION ALPHA(8), W(4), Z(4)
3      DATA ALPHA/1., .5, .25, .16666666667, -.1, -.06666
3     $    .03333333333, -.02/
3      DATA W/0.34785485, .65214515, .65251S, .34785485/
3      DATA Z/-.86113631, -0.33998104, .33998104, .861136
3      IF (&A .EQ. 3) GO TO 10
3      W(1) = 2.0
3      Z(1) = 0.0
3      GO TO 20
3   10 CONTINUE
3      W(1) =  0.34785485
3      Z(1) = -0.86113631
3   20 CONTINUE
3      TRUE = 0.0
3      DO 30 I=1,&A+1
3         TRUE = TRUE + W(I)*(EXP(-Z(I)*X*SIN(Y))*
3     $                 COS(Z(I)*X*COS(Y))+EXP(-Z(I)*X*CO
3     $                 COS(Z(I)*X*SIN(Y)))
3   30 CONTINUE
3      DO 40 I=1,&B
3         TRUE = TRUE + ALPHA(I)*X**(I+I)*COS((I+I)*Y)
3   40 CONTINUE
3      RETURN
3      END
3      FUNCTION C(X)
3      C = 0.
3      IF (X .NE. 0.) C = 1./X**2
3      RETURN
3      END
3      FUNCTION D(X)
3      D = 0.
3      IF (X .NE. 0.) D = 1./X
3      RETURN
3      END
-------------------------------------------------------------------
$
'EOR
```

LINE 1 HAS THE NUMBER OF MODULES (NRMODS) IN I3 FOR
FOLLOWS NRMODS PAIRS OF LINES. THE FIRST OF EACH CONT
INTEGER IN THE RANGE 1,...,NRMODS IN I3 FORMAT AND AN
MODULE NAME STARTING IN COLUMN 6. THESE GIVE THE CORR
BETWEEN MODULE NUMBERS USED IN ENCODED PROGRAMS AND TH
OF MODULES TO BE USED IN THE GENERATED ELLPACK PROGRAM
LINE OF THE PAIR CONTAINS A LIST OF INTEGERS IN 4X,30I
THE MODULE NUMBERS THAT MAY IMMEDIATELY FOLLOW THIS MO
ELLPACK PROGRAMS. AFTER THIS SET OF LINES THERE IS ON
CONTAINING A SINGLE INTEGER IN I3 FORMAT GIVING THE NU
DISCRETIZATION MODULES (NRDIS). THIS IS FOLLOWED BY N
EACH CONTAINING A MODULE NUMBER IN I3 FORMAT AND 15 DI
STARTING IN COLUMN 5 GIVING MODULE COMPATIBILITY INFOR
(SEE ROUTINES GENPGM AND COMPAT).

RELATED FILES: GENPGM, EQNFIL, MACFIL, OPTFIL, GRDFIL

```
39
01  5-POINT STAR
    14 15 16 17 32
02  7-POINT 3D
    14 15 16 17
03  P3-C1 COLLOCATION
    14
04  P3-C1 GALERKIN
    14
05  HODIE-HELMHOLTZ
    14 16 17 32
06  HODIE-ACF
    14 16 17 32
07  HODIE-ACDEF
    14 16 17 32
08  HODIE 27-POINT 3D

09  FFT 9-POINT

10  2DEPEP

11  MARCHING ALGORITHM

12  DYAKANOV-CG

13  DYAKANOV-CG4

14  NATURAL
    20 21 22 30 31 33 34 35 36 37
15  RED-BLACK
    22 30 31 38 39
16  YALE MIN DEG
    30 31 33 34 35 36 37
17  YALE RCM
    20 21 22 30 31 33 34 35 36 37
18  BAND SOLVE                      (RETIRED)

19  SYMMETRIC BAND                  (RETIRED)

20  LINPACK BAND

21  LINPACK SPD BAND

22  SPARSE GE-PIVOTING
```

```
29  REDUCED SYSTEM CG                                  ITPACK 2.0

30  YALE SPARSE

31  YALE ENVELOPE

32  NESTED DISSECTION
    30 31 33 34 35 36 37
33  SOR

34  JACOBI SI

35  JACOBI CG

36  SYMMETRIC SOR SI

37  SYMMETRIC SOR CG

38  REDUCED SYSTEM SI

39  REDUCED SYSTEM CG

13
01  2011111110111112
02  0211012101111111
03  2011110111111111
04  2011012102002l2
05  2011010102001l2
06  2011010102001l2
07  2011110102001l2
08  0221020102002l2
09  2011121102001l2
10  2011012101111112
11  2011012101111112
12  2011012101111112
13  2011012101111112
*EOR
```

```
   OPTFIL IS DIVIDED UP INTO RECORDS WHICH ARE NUMBERE
ETC.  THE END-OF-RECORD IS DESIGNATED BY *EOR ('EOR ON
INSTALLATIONS).

   RECORD 0 CONTAINS A DESCRIPTION OF THE PURPOSE AND
OF OPTFIL.  RECORDS 1, 2, 3,... HAVE THE FOLLOWING FOR

   LINE 1:  LINE 1 CONTAINS THE RECORD NUMBER.

   LINE 2:  LINE 2 MUST CONTAIN THE 8 CHARACTERS OPTIO
   COLUMNS 1-8, FOLLOWED ON THE SAME LINE BY ALL OF TH
   SELECTED.

RELATED FILES:  GENPGM, EQNFIL, MACFIL, GRDFIL, OUTFIL
```

```
-------------------------------------------------------
*EOR
RECORD   1
OPTIONS.  TIME   $   MEMORY   $   LEVEL=1
-------------------------------------------------------
*EOR
RECORD   2
OPTIONS.  TIME   $   MEMORY   $   LEVEL=2
-------------------------------------------------------
*EOR
RECORD   3
OPTIONS.  TIME   $   MEMORY   $   LEVEL=3
-------------------------------------------------------
*EOR
RECORD   4
OPTIONS.  TIME   $   MEMORY   $   LEVEL=4
-------------------------------------------------------
*EOR
RECORD   5
OPTIONS.  TIME   $   MEMORY   $   LEVEL=5
-------------------------------------------------------
*EOR
$
*EOR
*EOF
```

OUTFIL IS DIVIDED UP INTO RECORDS WHICH ARE NUMBERE
ETC. THE END-OF-RECORD IS DESIGNATED BY *EOR ('EOR OI
INSTALLATIONS).

RECORD 0 CONTAINS A DESCRIPTION OF THE PURPOSE AND
OF OUTFIL. RECORDS 1, 2, 3,... HAVE THE FOLLOWING FO

LINE 1: LINE 1 CONTAINS THE RECORD NUMBER.

REMAINING LINES: THE REMAINING LINES OF THE RECORI
OUTPUT SEGMENT EXACTLY AS IT WILL APPEAR IN THE GE
PACK PROGRAM. TO SPECIFY AN ELLPACK CONTINUATION L
'.' (DOT) IN COLUMN 2. THE INFORMATION OF THIS REC
DIRECTLY INTO THE OUTPUT SEGMENT OF THE GENERATED E
PROGRAM. FOR PORTABILITY PURPOSES, THE END-OF-RECO
DETECTED BY THE PRESENCE OF A '-' (DASH) IN COLUMN

RELATED FILES: GENPGM, EQNFIL, MACFIL, OPTFIL, GRDFIL

```
'EOR
RECORD   1
            TABLE-SOLUTION  $  MAX-SOLUTION  $  MAX-ERRO
```

```
'EOR
RECORD   2
            TABLE-SOLUTION  $  PLOT-SOLUTION
```

```
'EOR
RECORD   3
            PLOT-TRUE
```

```
'EOR
RECORD   4
            TABLE-ERROR   $   MAX-ERROR   $   MAX-SOLUTI
```

```
'EOR
RECORD   5
            TABLE-TRUE   $   TABLE-SOLUTION   $   TABLE-
```

```
'EOR
RECORD   6
            TABLE-TRUE $ TABLE-SOLUTION $ TABLE-ERROR
            PLOT-TRUE  $ PLOT-SOLUTION  $ PLOT-ERROR
```

```
'EOR
RECORD   7
            TABLE-ERROR $ MAX-ERROR
            PLOT-TRUE $ PLOT-SOLUTION $ PLOT-ERROR
```

```
'EOR
$
'EOR
'EOF
```

```
     $         FATAL,CLOCKW,RECTAN
      REAL              AX,BX,AY,BY,AZ,BZ,HX,HY,HZ,CUXX,CUXY,
     $         CUX,CUY,CU,CUXZ,CUYZ,CUZZ,CUZ,EPSGRD
      INTEGER IROT(6),HORZ,VERT,BOTH,INTER,STAR,ONE,MINUS,XX
     $         PERIOD,CORNER
C
      COMMON / PROBL  / DIM2,DIM3,POISON,LAPLAC,CONSTC,SELF
     $                  CROSST,HOMOBC,DIRICH,NEUMAN,MIXED,U
      COMMON / PROBR  / AX,BX,AY,BY,AZ,BZ,HX,HY,HZ
      COMMON / PROBI  / NGRIDX,NGRIDY,NGRIDZ
      COMMON / CPDE   / CUXX,CUXY,CUYY,CUX,CUY,CU,CUXZ,CUYZ,
      COMMON / CONTRL / DEBUG,TIMER,SYMMET,FATAL,RECTAN
      COMMON / INTEGS / NUMBEQ,NUMCOE,NROW,NCOL,NBAND,IROT,I
     $                  INITL,INDIS,INSOL,MINPUT,
     $                  MOUTPT,MEMORY,ININD,NGRDXZ,NGRDYZ,
     $                  NGRDZZ,MXNEQZ,MXNCOZ,NROWDZ,NCOLDZ
      COMMON / REALS  / ERRMAX,SOLMAX,TRUMAX,RESMAX,TIMES(1
      COMMON / BNDRY  / IPIECE,NBOUND,NBNDPT,CLOCKW
      COMMON / SYMCON / HORZ,VERT,BOTH,INTER,STAR,ONE,BLANK
     $                  XXX,EXTER,PERIOD,CORNER
      COMMON / NUMCON / EPSGRD
      COMMON /GRIDXZ/ GRIDX(1)
      COMMON /GRIDYZ/ GRIDY(1)
      COMMON /GRIDZZ/ GRIDZ(1)
      COMMON /COEFZZ/ COEF(1)
      COMMON /IDCOZZ/ IDCOEF(1)
      COMMON /TABLZZ/ TABLE(1)
      COMMON /UNKNZZ/ UNKNWN(1)
      COMMON /NDXUZZ/ NDXUNK(1)
      COMMON /BCTYZZ/ BCTYPE(1)
      COMMON /INUNZZ/ INUNDX(1)
      COMMON /NDXEZZ/ NDXEQ(1)
      COMMON /IGRIZZ/ IGRID(1)
C
      REAL ERRL2,RESMAX,RESL2
      INTEGER BCTYPE
C
      DATA NRITNS /0/, INSOL/0/, NUMBEQ/0/
      COMMON /ITCOM1/ NRITNS
C       THIS IS THE NUMBER OF ITERATIONS FROM ITPACK
C
      NGRDXZ = NGRIDX
      NGRDYZ = NGRIDY
      NGRDZZ = NGRIDZ
C
C
C COMPUTE L-2 DISCRETE NORM AT THE NODES
C
      RNODES = 1.0/FLOAT(NGRIDX*NGRIDY*NGRIDZ)
      ERRL2 = 0.
      ERR = 0.0
      DO 100 IX=1,NGRIDX
         DO 100 IY=1,NGRIDY
            DO 100 IZ=1,NGRIDZ
               E = ERROR(GRIDX(IX),GRIDY(IY),GRIDZ(IZ),NGRD
     $                   NGRDYZ,NGRDZZ,GRIDX,GRIDY,GRIDZ,UN
     $                   MXNEQZ,NDXUNK,TABLE,COEF,IDCOEF,MX
     $                   BCTYPE,INUNDX,NDXEQ,IGRID)
C
               ERRL2 = ERRL2 + E*E
               ERR = AMAX1(ABS(E),ERR)
 100  CONTINUE
      ERRL2 = SQRT(ERRL2)*RNODES
```

```
     $         FATAL,CLOCKW,RECTAN
      REAL              AX,BX,AY,BY,AZ,BZ,HX,HY,HZ,CUXX,CUXY
     $         CUX,CUY,CU,CUXZ,CUYZ,CUZZ,CUZ,EPSGRD
      INTEGER IROT(6),HORZ,VERT,BOTH,INTER,STAR,ONE,MINUS,X
     $         PERIOD,CORNER
C
      COMMON / PROBL  / DIM2,DIM3,POISON,LAPLAC,CONSTC,SELF
     $                  CROSST,HOMOBC,DIRICH,NEUMAN,MIXED,U
      COMMON / PROBR  / AX,BX,AY,BY,AZ,BZ,HX,HY,HZ
      COMMON / PROBI  / NGRIDX,NGRIDY,NGRIDZ
      COMMON / CPDE   / CUXX,CUXY,CUYY,CUX,CUY,CU,CUXZ,CUYZ
      COMMON / CONTRL / DEBUG,TIMER,SYMMET,FATAL,RECTAN
      COMMON / INTEGS / NUMBEQ,NUMCOE,NROW,NCOL,NBAND,IROT,
     $                  INITL,INDIS,INSOL,MINPUT,
     $                  MOUTPT,MEMORY,ININD,NGRDXZ,NGRDYZ,
     $                  NGRDZZ,MXNEQZ,MXNCOZ,NROWDZ,NCOLDZ
      COMMON / REALS  / ERRMAX,SOLMAX,TRUMAX,RESMAX,TIMES(1
      COMMON / BNDRY  / IPIECE,NBOUND,NBNDPT,CLOCKW
      COMMON / SYMCON / HORZ,VERT,BOTH,INTER,STAR,ONE,BLANK
     $                  XXX,EXTER,PERIOD,CORNER
      COMMON / NUMCON / EPSGRD
      COMMON /GRIDXZ/ GRIDX(1)
      COMMON /GRIDYZ/ GRIDY(1)
      COMMON /GRIDZZ/ GRIDZ(1)
      COMMON /COEFZZ/ COEF(1)
      COMMON /IDCOZZ/ IDCOEF(1)
      COMMON /TABLZZ/ TABLE(1)
      COMMON /UNKNZZ/ UNKNWN(1)
      COMMON /NDXUZZ/ NDXUNK(1)
      COMMON /BCTYZZ/ BCTYPE(1)
      COMMON /INUNZZ/ INUNDX(1)
      COMMON /NDXEZZ/ NDXEQ(1)
      COMMON /IGRIZZ/ IGRID(1)
C
      REAL ERRL2,RESMAX,RESL2
      INTEGER BCTYPE
C
      DATA NRITNS /0/, INSOL/0/, NUMBEQ/0/
      COMMON /ITCOM1/ NRITNS
C       THIS IS THE NUMBER OF ITERATIONS FROM ITPACK
C
      NGRDXZ = NGRIDX
      NGRDYZ = NGRIDY
      NGRDZZ = NGRIDZ
C
C
C COMPUTE L-2 DISCRETE NORM AT THE NODES
C
      RNODES = 1.0/FLOAT(NGRIDX*NGRIDY*NGRIDZ)
      ERRL2 = 0.
      ERR = 0.0
      DO 100 IX=1,NGRIDX
         DO 100 IY=1,NGRIDY
            DO 100 IZ=1,NGRIDZ
               E = ERROR(GRIDX(IX),GRIDY(IY),GRIDZ(IZ),NGRD
     $                   NGRDYZ,NGRDZZ,GRIDX,GRIDY,GRIDZ,UN
     $                   MXNEQZ,NDXUNK,TABLE,COEF,IDCOEF,MX
     $                   BCTYPE,INUNDX,NDXEQ,IGRID)
C
               ERRL2 = ERRL2 + E*E
               ERR = AMAX1(ABS(E),ERR)
 100  CONTINUE
      ERRL2 = SQRT(ERRL2)*RNODES
```

```
C  TWO-DIMENSIONAL CASE
C
      RINTUL = 1.0/FLOAT(NX*NY)
      DO 200 IX = 1,NX
         XMIDL = (GRIDX(IX)+GRIDX(IX+1))*.5
         DO 200 IY = 1,NY
            YMIDL = (GRIDY(IY)+GRIDY(IY+1))*.5
            R = RESID(XMIDL,YMIDL,ZMIDL,NGRDXZ,NGRDYZ,NGRDZ
     $                GRIDY,GRIDZ,UNKNWN,MXNEQZ,NDXUNK,TABL
     $                IDCOEF,MXNCOZ,BCTYPE,INUNDX,NDXEQ,IGR
            RESMAX = AMAX1(RESMAX,ABS(R))
            RESL2 = RESL2 + R*R
            RHS = PDERHS(XMIDL,YMIDL)
            IF (RHS .NE. 0.0)  RESMXR = AMAX1(RESMXR,ABS(RA
 200  CONTINUE
      RESL2 = SQRT(RESL2)*RINTUL
                                          GO TO 300
C
C  THREE DIMENSIONAL CASE
C
  210 CONTINUE
      RINTUL = 1.0/FLOAT(NX*NY*NZ)
      DO 250 IX = 1,NX
         XMIDL = (GRIDX(IX)+GRIDX(IX+1))*.5
         DO 250 IY = 1,NY
            YMIDL = (GRIDY(IY)+GRIDY(IY+1))*.5
            DO 250 IZ = 1,NZ
               ZMIDL = (GRIDZ(IZ)+GRIDZ(IZ+1))*.5
            R = RESID(XMIDL,YMIDL,ZMIDL,NGRDXZ,NGRDYZ,NGRDZ
     $                GRIDY,GRIDZ,UNKNWN,MXNEQZ,NDXUNK,TABL
     $                IDCOEF,MXNCOZ,BCTYPE,INUNDX,NDXEQ,IGR
            RESMAX = AMAX1(RESMAX,ABS(R))
            RESL2 = RESL2 + R*R
            RHS = PDERHS(XMIDL,YMIDL,ZMIDL)
            IF (RHS .NE. 0.0)  RESMXR = AMAX1(RESMXR,ABS(RA
 250  CONTINUE
      RESL2 = SQRT(RESL2)*RINTUL
C
C COMPUTE THE TOTAL EXECUTION TIME
C
 300  CONTINUE
C
C     FIRST FIND NUMBER OF MODULES IN EXECUTION SEQUENCE
      GO TO (303,303,303,301,303,303,301,301,301,303,301,3
     $       301), INDIS
  301    CONTINUE
         NRMODL = 1
         GO TO 320
  303    CONTINUE
         NRMODL = 3
  320 CONTINUE
C
C     ADD TIME TO REFORMAT LINEAR SYSTEM (EXCEPT YALE SPARS
      IF ((INSOL .NE. 15) .AND. (NRMODL .EQ. 3))
     $       TIMES(3) = TIMES(3) + TIMES(4)
C
C     NOW COMPUTE THE TOTAL TIME
      TLTIME = 0.0
      DO 325 I=1,NRMODL
         TLTIME = TLTIME + TIMES(I)
  325 CONTINUE
C
C  FIND MAX DISTANCE BETWEEN ANY TWO GRID LINES
```

```
            IF (DIM2)  GO TO 360
               DO 350 I=1,NZ
                  DZ = GRIDZ(I+1)-GRIDZ(I)
                  HMAX = AMAX1(DZ,HMAX)
  350          CONTINUE
  360       CONTINUE
            GO TO 380
C
C           CASE OF UNIFORM GRID
  370       CONTINUE
            HMAX = AMAX1(HX,HY)
            IF (DIM3)  HMAX = AMAX1(HZ,HMAX)
  380 CONTINUE
C
C  WRITE SUMMARY OF ELLPACK RUN ON SAVE FILE
C
         WRITE(4,1000) NGRIDX,NGRIDY,HMAX,NUMBEQ,ERR,ERRMAX,ER
        $              RESMAX,RESMXR,RESL2,SOLMAX,NRITNS,MEMOR
        $              (TIMES(I),I=1,NRMODL)
 1000 FORMAT(1X,2I4,1X,E10.2,1X,I5,2X,7E9.2,1X,I3,1X,I5,1X,
C
         RETURN
         END
'EOR
```

```
C          AUX     - VECTOR OF ARGUMENT VALUES
C          VALUX   - VECTOR OF FUNCTION VALUES CORRESPONDING
C                    TO AUX.
C          N       - NUMBER OF INTERPOLATION POINTS
C
C
C      METHOD
C          DIVIDED DIFFERENCES ARE USED TO COMPUTE THE
C          EXTRAPOLATED VALUE.
C
C          P(X) = F1 + (X-AUX(1))*F2 + (X-AUX(1))*(X-AUX(2))
C
C      HISTORY
C          MODIFIED BY BILL WARD, MARCH, 1979.
C
C     ..................................................................
C
      REAL                AUX(N), VALUX(N),
     A                    F1, F2, F3, C1, T2
C
      F1     = VALUX(1)
      F2     = (VALUX(2) - VALUX(1)) / (AUX(2) - AUX(1))
      C1     = X - AUX(1)
      QUADR = F1 + C1*F2
      IF (N .EQ. 2) GO TO 999
      T2     = (VALUX(3) - VALUX(2)) / (AUX(3) - AUX(2))
      F3     = (T2 - F2) / (AUX(3) - AUX(1))
      QUADR = QUADR + C1*(X - AUX(2))*F3
C
  999 CONTINUE
      RETURN
      END
'EOR
```